Gravitational Wave Sources and Detectors

B.S. Sathyaprakash
Cardiff University, Cardiff, United Kingdom

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Resources for the Lecture

B.S. Sathyaprakash and Bernard F. Schutz, "Physics, Astrophysics and Cosmology with Gravitational Waves",
Living Rev. Relativity **12**, (2009), 2.
URL : http://www.livingreviews.org/lrr-2009-2

Michele Maggiore, Gravitational Waves: Volume 1 Theory and Experiments, Oxford University Press (2007)

Plan of the lectures

- Lecture 1: Sources of Gravitational Waves
  - Motivation
    - Why study gravitational waves?
  - Physics of gravitational waves
    - Polarizations, propagation and wave generation,
  - Estimating the amplitude of gravitational waves from typical sources
    - Supernovae, binary black holes, stochastic backgrounds, spinning neutron stars,
  - Modeling black hole binaries
    - Inspiral, merger and ring-down phases, post-Newtonian theory, effective one body formalism, numerical relativity simulations

Plan of the lectures

- Lecture 2: GW Detectors
  - Interferometric gravitational-wave detectors
    - Principle behind their operation
    - Response of an interferometer to incident signal
    - Antenna pattern, sky coverage, triangulation, source reconstruction
  - Current and planned detectors and their sensitivities
    - Ground-based detectors
      - LIGO, Virgo, GEO600, LCGT, IndIGO, LIGO-Australia, Einstein Telescope
    - Results from current detectors will be discussed in lecture 5
    - Space-based detectors
      - LISA, DECIGO, BBO, PTA
    - Sources and science from these detectors will be covered in lecture 6

Plan of the lectures

- Lectures 3: Data Analysis
  - Geometric formulation of signal analysis
    - Data as vectors, signal manifold, metric
  - Matched filtering
    - Detecting a signal of known shape but unknown parameters, examples from detection of CW and inspirals
  - Covariance matrix
    - Parameter estimation, principal components; examples
  - Choice of templates
    - The problem of template placement
  - Coincident and coherent detection
- Lecture 4: Current status of GW observations
  - Sensitivity of the current searches to various sources
  - Upper limits on GW emission from Crab, GRBs, Early-Universe

Plan of the lectures

- Lecture 5: Fundamental Physics and Cosmology with GW observations
  - Testing the properties of gravitational waves
    - Speed of gravitational waves and mass of the graviton, polarization states, alternative theories of gravity and testing string theory
  - Strong field tests of gravity
    - The no-hair theorem, binary black hole merger and ring-down phases, naked singularities and cosmic censorship hypothesis
  - Understanding supra-nuclear physics
    - Observation of the neutron stars and their equation-of-state
  - Standard sirens of gravity and cosmography
    - Dark matter and dark energy densities, dark energy equation of state

Plan of the lectures

- Lecture 6: Astrophysics and Cosmology with GW
  - Unveiling the origin of high energy transients
    - Gamma-ray bursts, magnetars, low-mass X-ray binaries
  - Understanding low-mass X-ray binaries
    - Stalled neutron stars, relativistic instabilities, r-modes, etc.
  - Seed black holes at galactic nuclei
    - How and when black hole seeds formed at galactic nuclei, what were their masses, spins, and how did they grow in size?
  - Stochastic backgrounds
    - Generation of a background in the early Universe; GUT phase transitions, cosmic strings, etc.

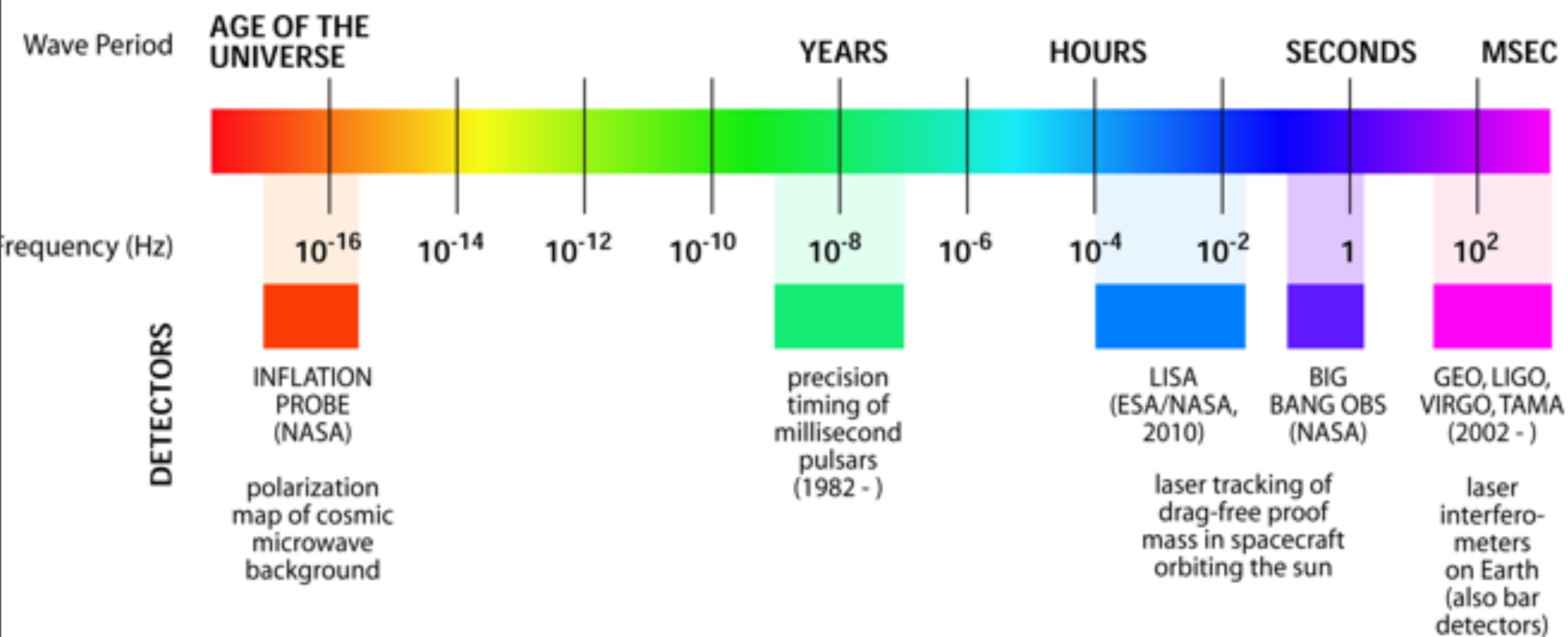
Why Study Gravitational Waves?

- In the early part of the 20th century Einstein's theory of gravity made three predictions
  - The Universe was born out of nothing in a big bang everywhere
  - Black holes are the ultimate fate of massive stars
  - Gravitational waves are an inevitable consequence of any theory of gravity that is consistent with special relativity
- Today we have indirect evidence for all but have directly observed none
- The key to observing the first two is the new tool that is provided by the last
  - In these lectures we will discuss what gravitational waves are and how they can be used to explore the dark and dense Universe

On Largest Scales Gravity Shapes the World

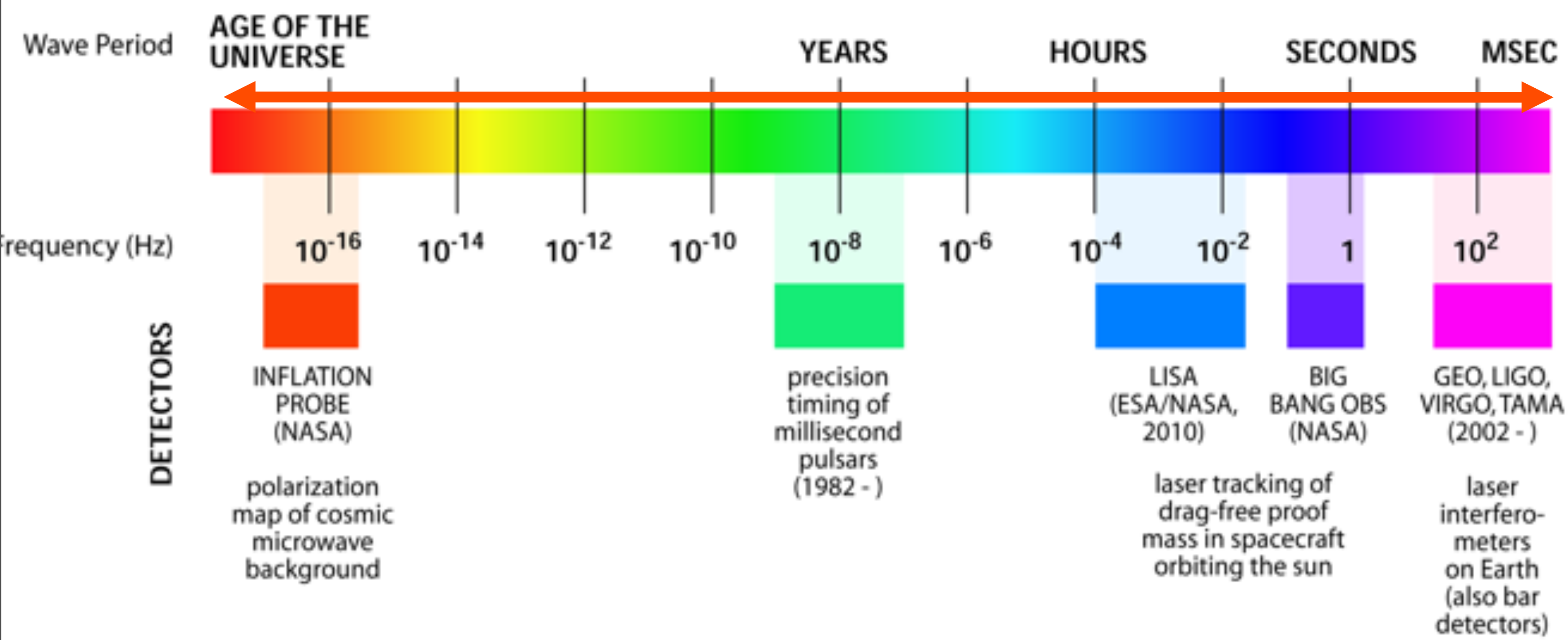
- On the largest scales matter is electrically neutral
  - Stars and galaxies feel only the gravitational field of other stars and galaxies
- So far, gravity has played a passive role in our exploration the Universe
  - But that is about to change
- Over the next decade we expect to open a new window on the Universe
  - The gravitational window
- These lectures will take you on a tour of what this window is all about and what it might tell us about the Universe

THE GRAVITATIONAL WAVE SPECTRUM



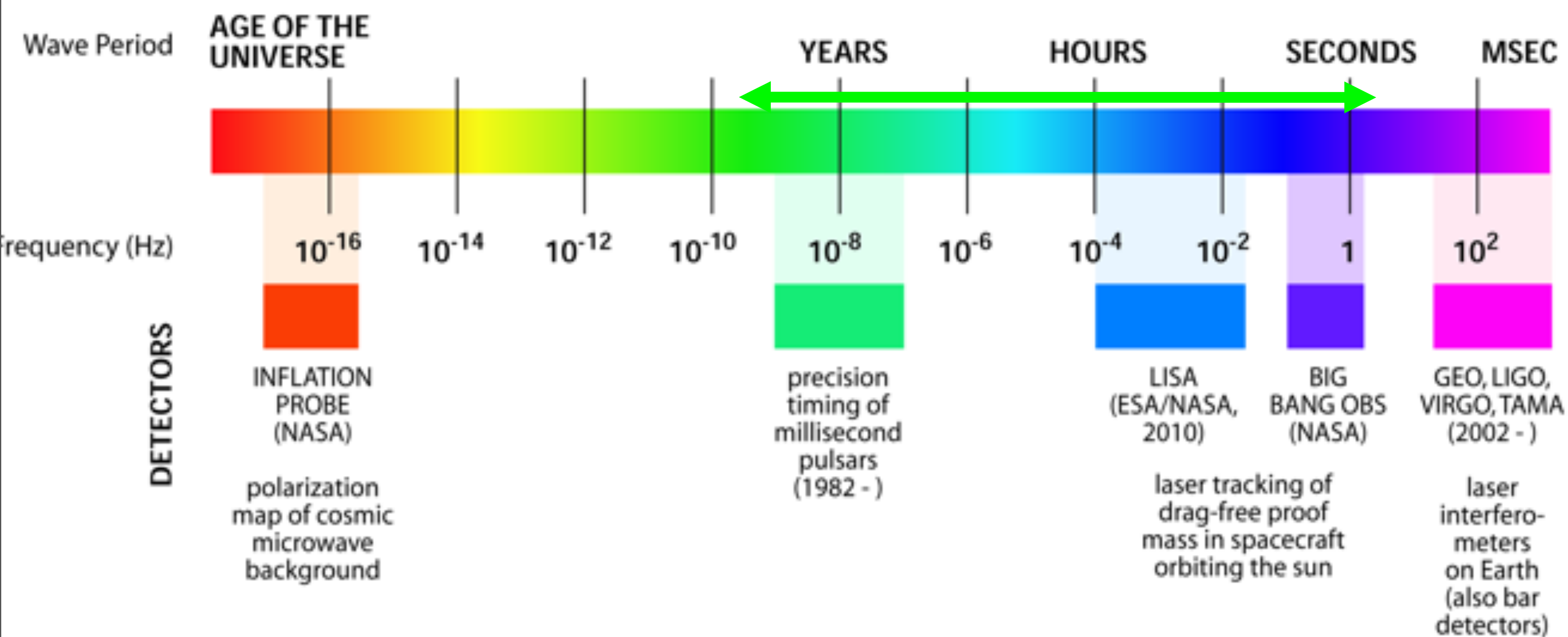
THE GRAVITATIONAL WAVE SPECTRUM

Quantum Fluctuations in the Early Universe



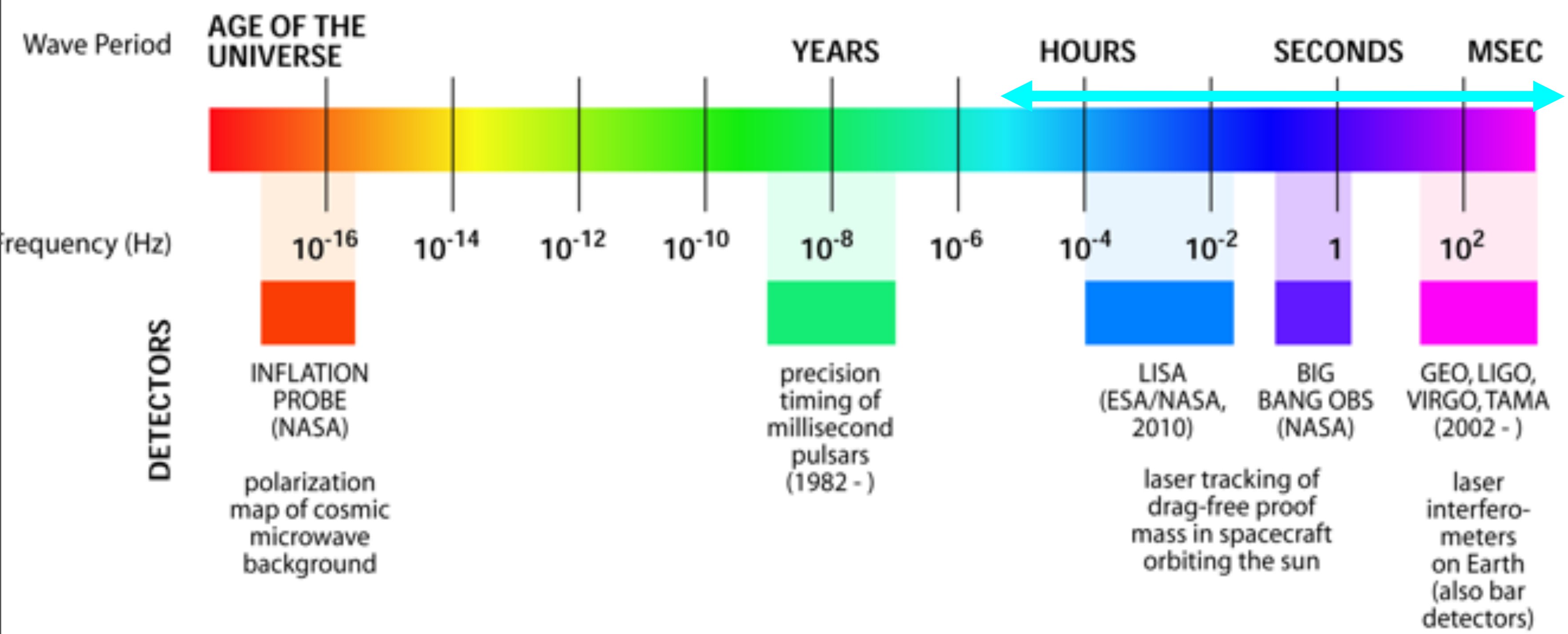
THE GRAVITATIONAL WAVE SPECTRUM

Merging super-massive black holes (SMBH) at galactic cores



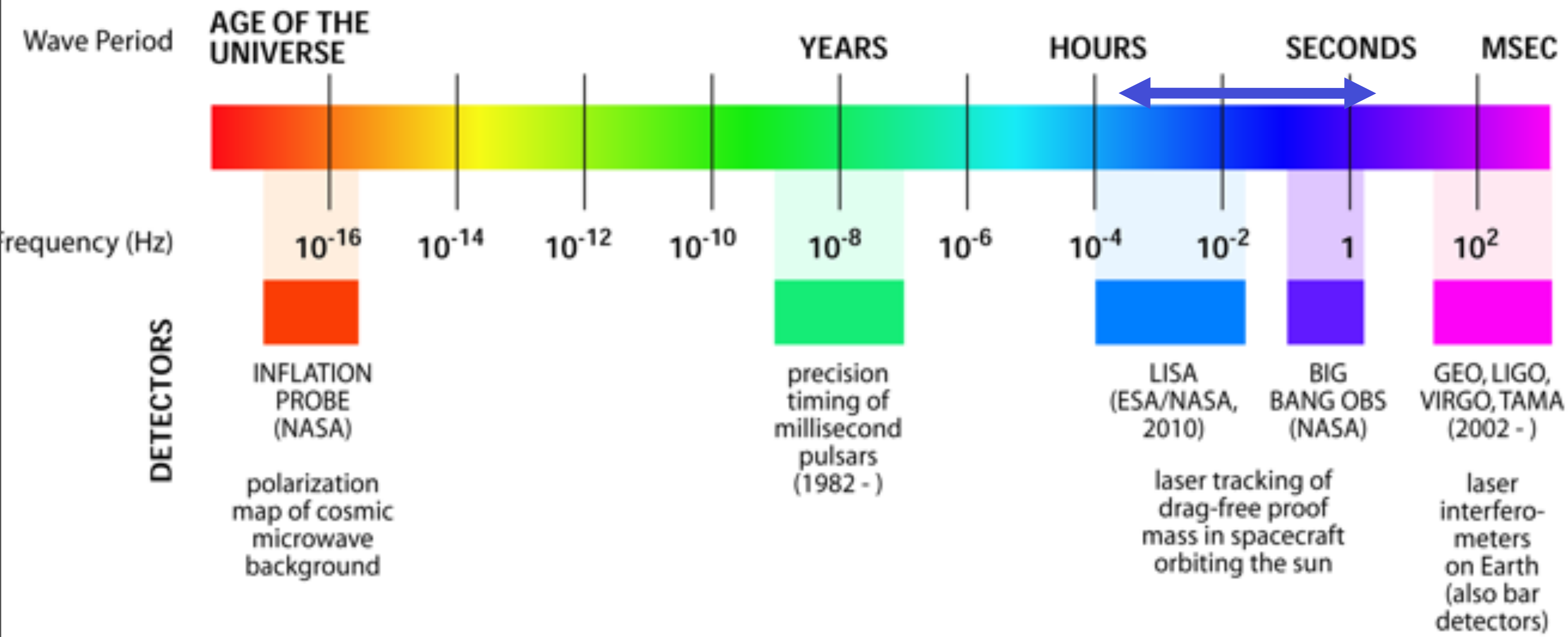
THE GRAVITATIONAL WAVE SPECTRUM

Phase transitions in the Early Universe



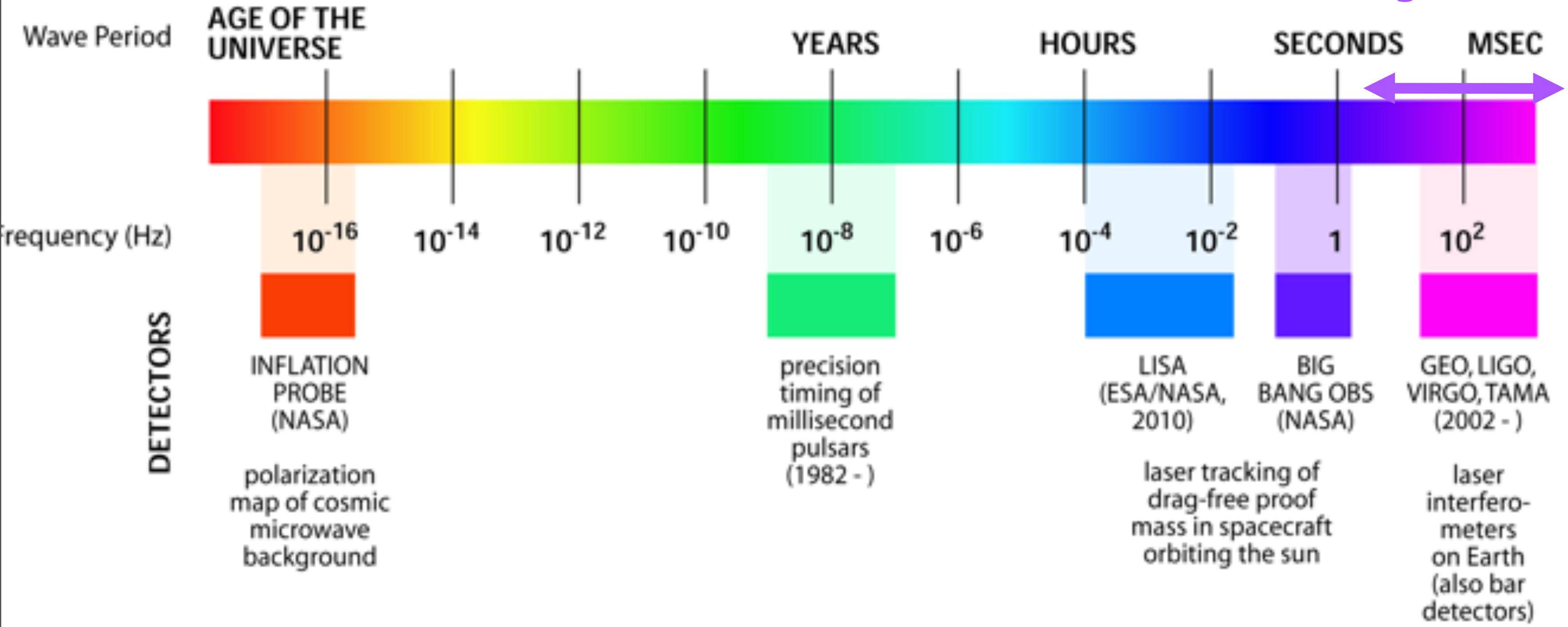
THE GRAVITATIONAL WAVE SPECTRUM

Capture of black holes and compact stars by SMBH



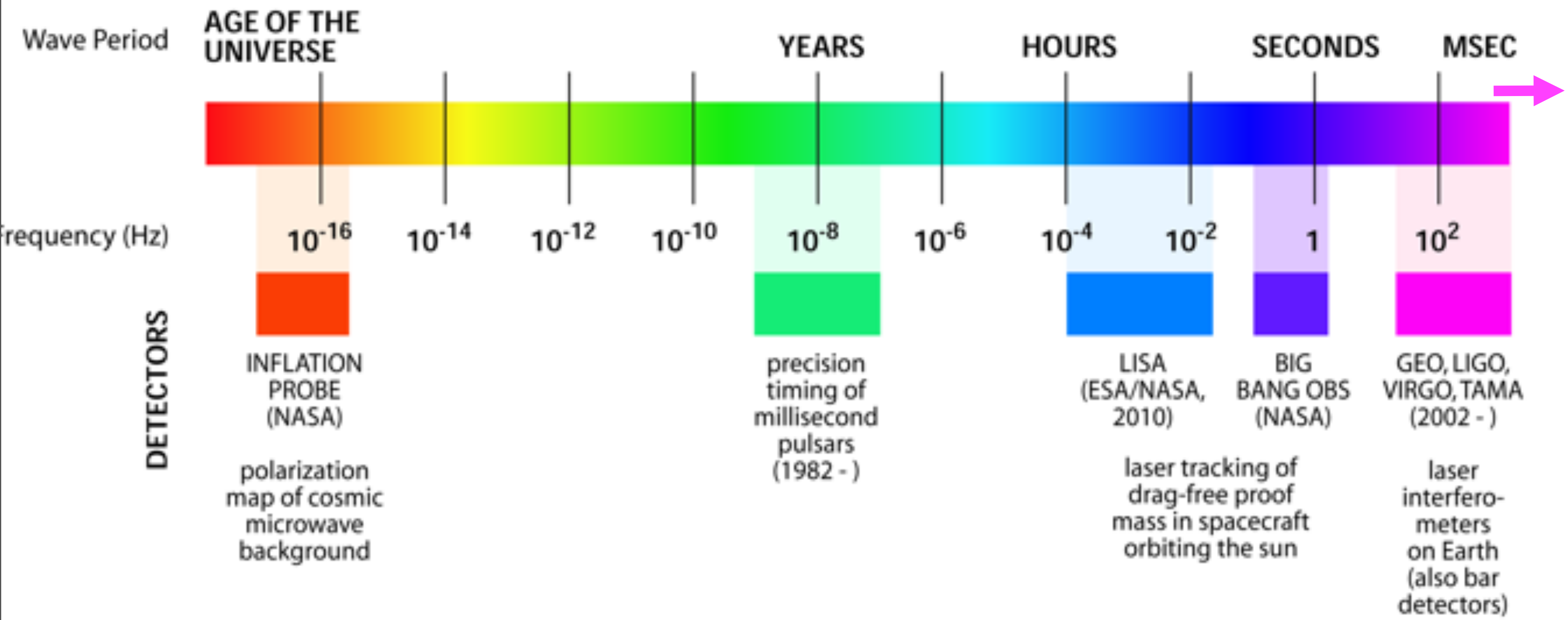
THE GRAVITATIONAL WAVE SPECTRUM

Merging binary neutron stars and black holes in distant galaxies



THE GRAVITATIONAL WAVE SPECTRUM

Neutron star quakes and magnetars



What are Gravitational Waves?

In Newton's law of gravity the gravitational field satisfies the Poisson equation:

$$\nabla^2\Phi(t,\mathbf{X}) = 4\pi G\rho(t,\mathbf{X})$$

Gravitational field is described by a scalar field, the interaction is instantaneous and no gravitational waves.

In general relativity for weak gravitational fields, i.e.

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}, \quad |h_{\alpha\beta}| \ll 1$$

in Lorentz gauge, i.e. $\bar{h}^{\alpha\beta}{}_{,\beta} = 0,$ Einstein's equations reduce to wave equations in the metric perturbation:

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right)\bar{h}^{\alpha\beta} = -16\pi T^{\alpha\beta}.$$

Here $\bar{h}_{\alpha\beta} = h_{\alpha\beta} - \frac{1}{2}\eta_{\alpha\beta}\eta^{\mu\nu}h_{\mu\nu}$ is the trace-reverse tensor.

Transverse-Traceless Gauge and Number of Degrees of Freedom

Plane-wave solutions:

$$\bar{h}^{\alpha\beta} = A^{\alpha\beta} \exp(2\pi\imath k_\mu x^\mu), \quad k_\alpha k^\alpha = 0$$

Gravitational waves travel at the speed of light.

Gauge conditions imply that $A^{\alpha\beta} k_\beta = 0$. Further gauge conditions

1. $A^{0\beta} = 0 \quad \Rightarrow \quad A^{ij} k_j = 0$: *Transverse* wave; and
2. $A^j{}_j = 0$: *Traceless* wave amplitude.

For a wave traveling in the z-direction then $k_z = k, \ k_x = k_y = 0.$

Gauge conditions, transversality and traceless conditions imply

$$A^{0\alpha} = A^{z\alpha} = 0, \ A^{xy} = A^{yx}, \ \ A^{yy} = -A^{xx}.$$

Only two independent amplitudes. Two independent degrees of freedom for polarization: plus-polarization and cross-polarization.

The Space-Time Metric of GW

A wave for which one of $A^{xy} = 0$ produces a metric of the form

$$ds^2 = -dt^2 + (1 + h_+)dx^2 + (1 - h_+)dy^2 + dz^2,$$
$$h_+ = A^{xx} \exp[ik(z - t)].$$

Note that the metric produces opposite effects on proper distance along x and y.

If $A^{xx} = 0$ then $h_{xy} = h_{\times}$, the corresponding metric is the same as before rotated by $\pi/4$:

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 + 2h_{xy}\, dx\, dy$$

Existence of two polarizations is the property of any non-zero spin field that propagates at the speed of light.

Tidal effect of GW

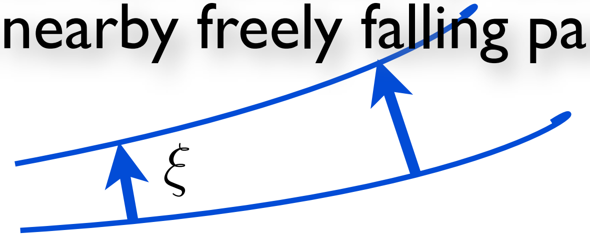
In the TT gauge, the effect of a wave on a particle at rest

$$\frac{d^2}{d\tau^2}x^i = -\Gamma^i{}_{00} = -\frac{1}{2}\left(2h_{i0,0} - h_{00,i}\right) = 0.$$

So a particle at rest remains at rest. TT gauge is a coordinate system that is comoving with freely falling particles.

The waves have a tidal effect which can be seen by looking at the change in distance between two nearby freely falling particles:

$$\frac{d^2}{d\tau^2}\xi^i = R^i{}_{0j0}\xi^j = \frac{1}{2}h_{ij,00}\xi^j.$$

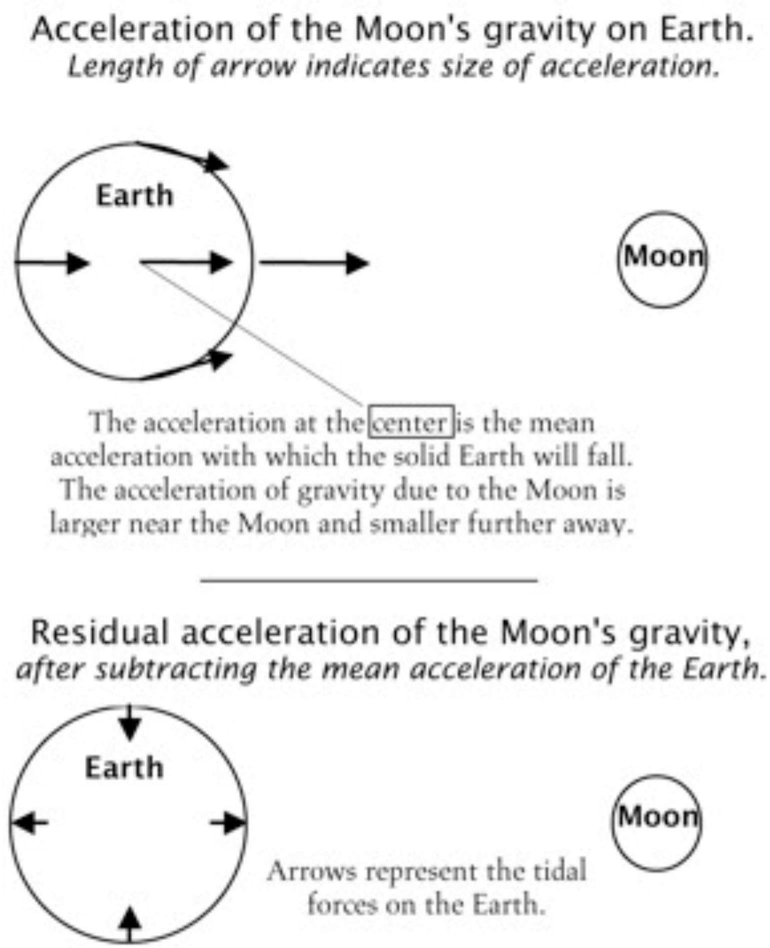


Isaacson showed that a spacetime with GW will have curvature with the corresponding Einstein tensor given by

$$G_{\alpha\beta} = 8\pi T^{(GW)}_{\alpha\beta} \qquad T^{(GW)}_{\alpha\beta} = \frac{1}{32\pi} h^{TT}_{\mu\nu}{}_{,\alpha} h^{TT\,\mu\nu}{}_{,\beta}.$$

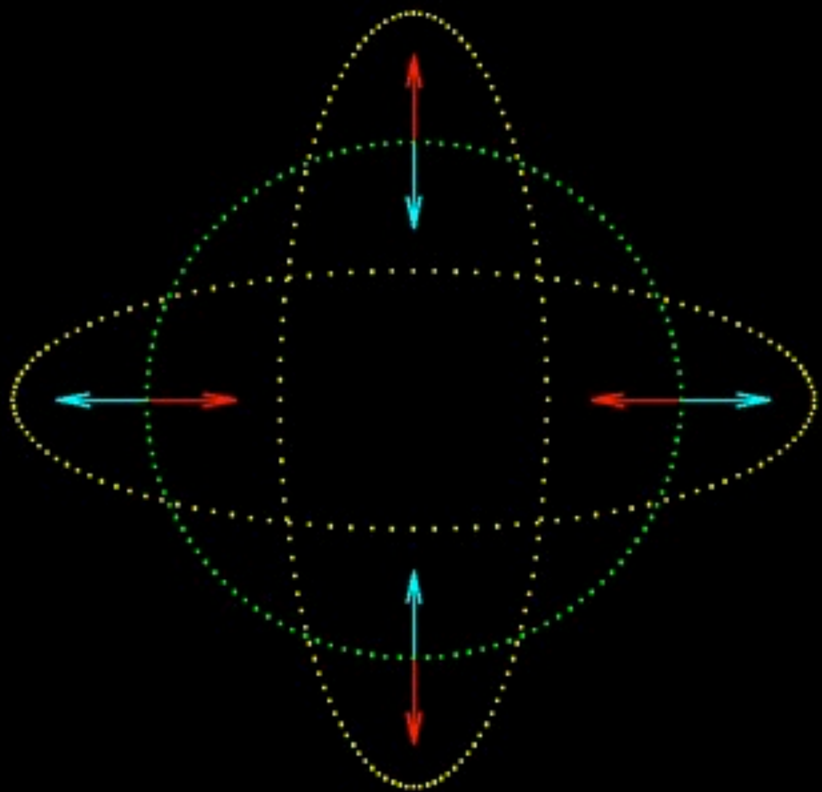
Tidal Gravitational Forces

- Gravitational effect of a distant source can only be felt through its tidal forces
- Gravitational waves are traveling, time-dependent tidal forces.
- Tidal forces scale with size, typically produce elliptical deformations.

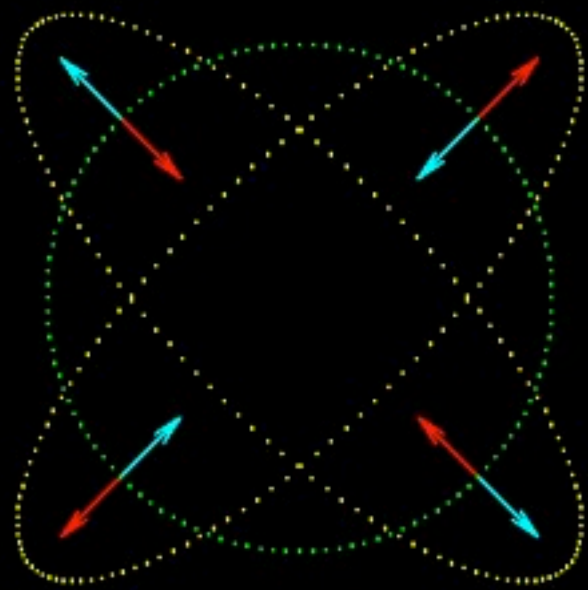


Tidal Action of Gravitational Waves

Tidal Action of Gravitational Waves



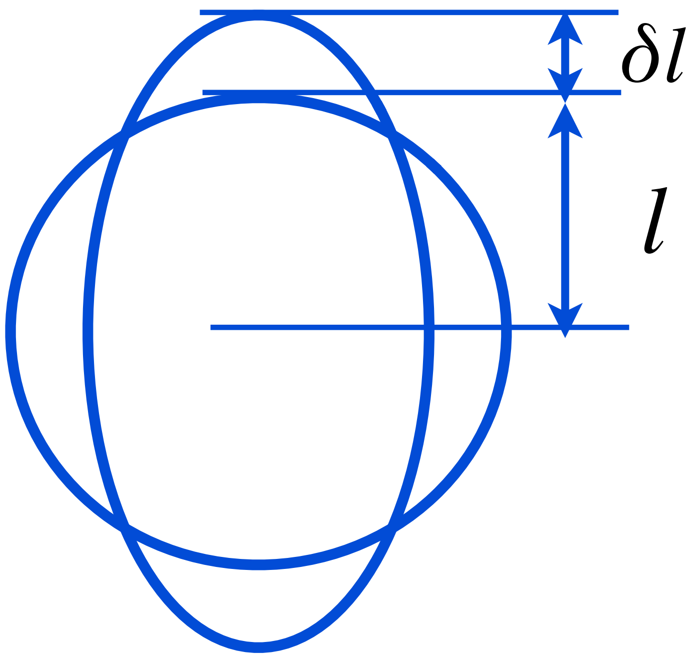
Plus polarization



Cross polarization

GW Amplitude - Measure of Strain

- Gravitational waves cause a strain in space as they pass
- Measurement of the strain gives the amplitude of gravitational waves



$$\delta l = \frac{h}{2} l$$

Gravitational Wave Flux

Flux of gravitational waves can be shown to be

$$\langle T^{(GW)0z} \rangle = \frac{k^2}{32\pi}(A_+^2 + A_\times^2)$$

where $k = 2\pi f$ is the wave number. For a wave with an amplitude h in both polarizations the energy flux is

$$F_{gw} = \frac{\pi}{4} f^2 h^2. \qquad F_{\rm gw} = 3 \text{ mW m}^{-2} \left[\frac{\rm h}{1 \times 10^{-22}}\right]^2 \left[\frac{\rm f}{1 \text{ kHz}}\right]^2$$

This is a large flux (twice that of full Moon) for even a source with a very small amplitude! Integrating over a sphere of radius r and assuming that the signal lasts for a duration τ gives the amplitude in terms of energy in GW

$$h = 10^{-21} \left[\frac{E_{gw}}{0.01 M_\odot c^2}\right]^{1/2} \left[\frac{r}{20 \text{ Mpc}}\right]^{-1} \left[\frac{f}{1 \text{ kHz}}\right]^{-1} \left[\frac{\tau}{1 \text{ ms}}\right]^{-1/2}.$$

Gravitational Wave Observables

- Luminosity = (Asymmetry factor) v^{10}
  - A strong function of velocity: During merger a binary black hole in gravitational waves outshines the entire Universe in *light*
- Amplitude from a source of size r at a distance D

$$h = \text{(Asymmetry factor)}\ (M/D)\ (M/r)$$

  - Amplitude gives strain in space as a wave propagates $h = \Delta L/L$
- Frequency of the waves is the dynamical frequency $f \sim \sqrt{\rho}$
  - For binaries dominant gravitational-wave frequency is twice the orbital frequency
- Polarization
  - In Einstein's theory two polarizations - plus and cross

Gravitational Vs EM Waves

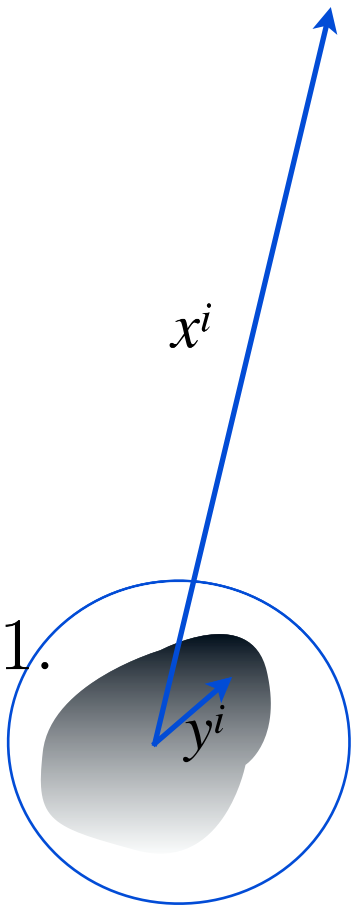
- EM waves are transverse waves, with two polarizations, travel at the speed of light
- Production: electronic transitions in atoms and accelerated charges – physics of small things
- Incoherent superposition of many, many waves
- Detectors sensitive to the intensity of the radiation
- Normally EM waves cannot be followed in phase
- Intensity falls off as inverse square of the distance to source
- Directional telescopes

- GW waves are also transverse waves, with two polarizations, travel at the speed of light
- Production: coherent motion stellar and super-massive black holes, supernovae, big bang, …
- Often, a single coherent wave, but stochastic background expected
- GW detectors are sensitive to the amplitude of the radiation
- Normally, waves followed in phase, great increase in signal visibility
- Amplitude falls off as inverse of the distance to source
- Sensitive to wide areas over the sky

Mass Quadrupole Radiation

General retarded solution of the field equation is

$$\bar{h}^{\alpha\beta}(x^i,t) = 4\int \frac{1}{R} T^{\alpha\beta}(t-R, y^i) d^3y,$$
$$R^2 = (x^i - y^i)(x_i - y_i).$$



At distances far from the source one can expand R around $r=|x|$

$$t - R = t - r + n^i y_i + O(1/r), \quad n^i = x^i/r, \quad n^i n_i = 1.$$

This gives for the field in terms of the moments of the energy momentum tensor

$$\bar{h}^{\alpha\beta} = \frac{4}{r}\int \left[T^{\alpha\beta}(t', y^i) + T^{\alpha\beta}{}_{,0}(t', y^i) n^j y_j + \frac{1}{2} T^{\alpha\beta}{}_{,00}(t', y^i) n^j n^k y_j y_k + \ldots \right] d^3y.$$

Radiation Zone Expansions

Together with the conservation law it follows that

$$\bar{h}^{00}(t, x^i) = \frac{4}{r}M + \frac{4}{r}P^j n_j + \frac{4}{r}S^{jk}(t') + \ldots;$$

$$\bar{h}^{0j}(t, x^i) = \frac{4}{r}P^j + \frac{4}{r}S^{jk}(t')n_k + \ldots;$$

$$\bar{h}^{jk}(t, x^i) = \frac{4}{r}S^{jk}(t') + \ldots.$$

The field depends only on the various moments of the source stress-energy tensor defined by

$$M(t') = \int T^{00}(t', y^i)d^3y, \quad M_j(t') = \int T^{00}(t', y^i)y_j d^3y,$$

$$M_{jk}(t') = \int T^{00}(t', y^i)y_j y_k d^3y;$$

$$P^\ell(t') = \int T^{0\ell}(t', y^i)d^3y, \quad P^\ell{}_j(t') = \int T^{0\ell}(t', y^i)y_j d^3y;$$

$$S^{\ell m}(t') = \int T^{\ell m}(t', y^i)d^3y.$$

The Quadrupole Formula

In TT gauge, the expressions simplify considerably

$$\bar{h}^{TT00} = \frac{4M}{r}; \quad \bar{h}^{TT0i} = 0; \quad \bar{h}^{TTij} = \frac{4}{r}\left[\perp^{ik}\perp^{j\ell} S_{k\ell} + \frac{1}{2}\perp^{ij}\left(S_{k\ell}n^k n^\ell - S^k{}_k\right)\right]$$

Here the projection operator is $\perp^{jk} = \delta^{jk} - n^j n^k$.

Note that the time-time part is the Newtonian field, the momentum part is zero, leaving only the spatial part which is explicitly traceless and transverse. In fact, using the conservation law the famous quadrupole formula follows

$$\frac{d^2 M^{jk}}{dt^2} = 2S^{jk} \quad \Rightarrow$$

$$\bar{h}^{TTij} = \frac{2}{r}\ddot{M}^{TTij}$$

$$M^{TT}_{ij} = \perp^k{}_i \perp^l{}_j M_{kl} - \frac{1}{2}\perp_{ij}\perp^{kl} M_{kl}$$

Luminosity in gravitational waves is given by

$$L^{mass}_{gw} = \frac{1}{5}\dddot{\not{M}}^{jk}\dddot{\not{M}}_{jk} \qquad \not{M}^{jk} = M^{jk} - \frac{1}{3}\delta^{jk}M^\ell{}_\ell$$

Application to a Binary System

- For a binary system of two compact stars orbiting in the x-y plane the quadrupole moments are

$$M_{xx} = mR^2 \cos(2\omega t), \quad M_{yy} = -mR^2 \cos(2\omega t), \quad M_{xy} = mR^2 \sin(2\omega t).$$

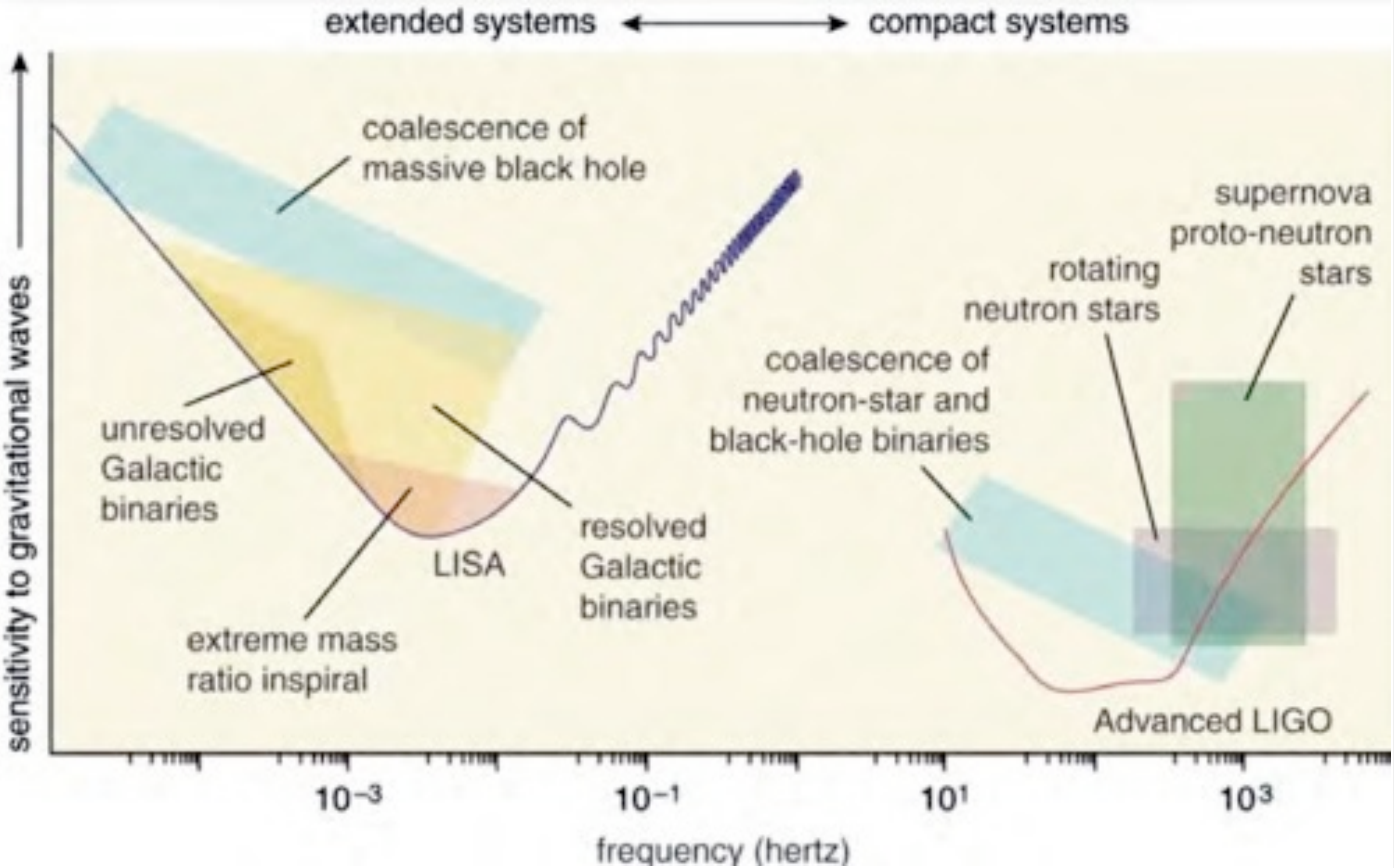
which shows that the radiation is emitted at twice the orbital frequency.

$$h_+ = \frac{2\nu M}{D} v^2 (1 + \cos^2 \iota) \cos[2\varphi(t)], \quad h_\times = \frac{4\nu M}{D} v^2 \cos \iota \sin[2\varphi(t)],$$

- Here D is the distance to the binary, M and ν are the total mass and symmetric mass ratio, φ(t) is the orbital phase, currently known to a high order in post-Newtonian approximation, ι is the inclination of the binary with the line-of-sight, v is the velocity of the stars

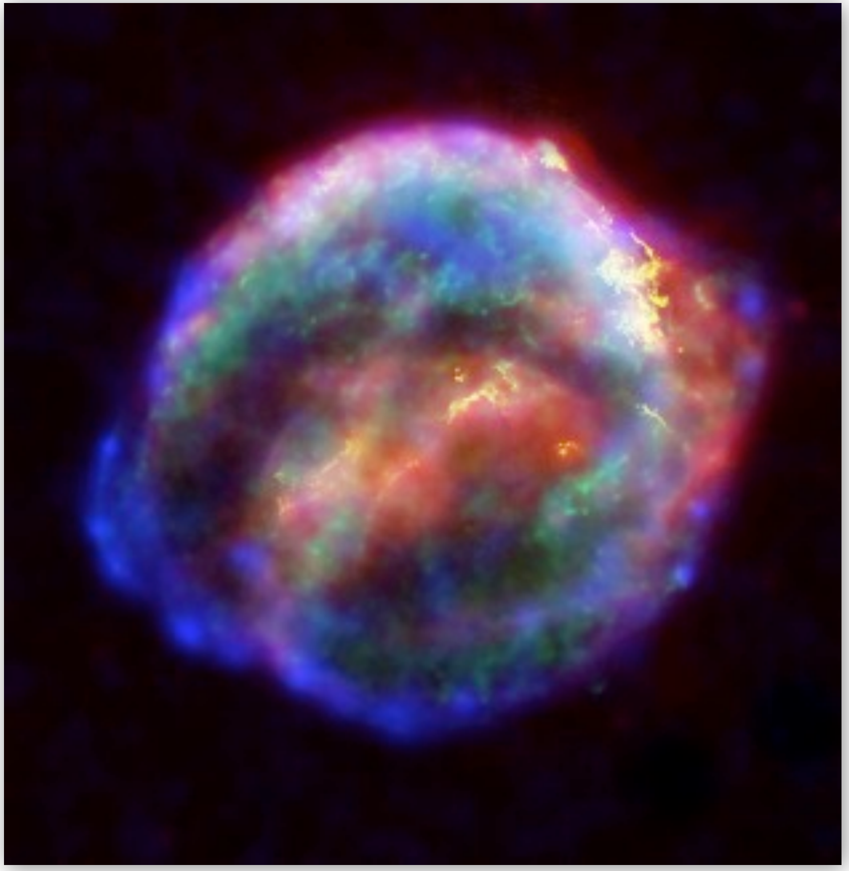
Sources of Gravitational Waves

Summary of Sources



Burst Sources

- Gravitational wave bursts
  - Black hole collisions
  - Supernovae
  - gamma-ray bursts (GRBs)
- Short-hard GRBs
  - could be the result of merger of a neutron star with another NS or a BH
- Long-hard GRBs
  - could be triggered by supernovae



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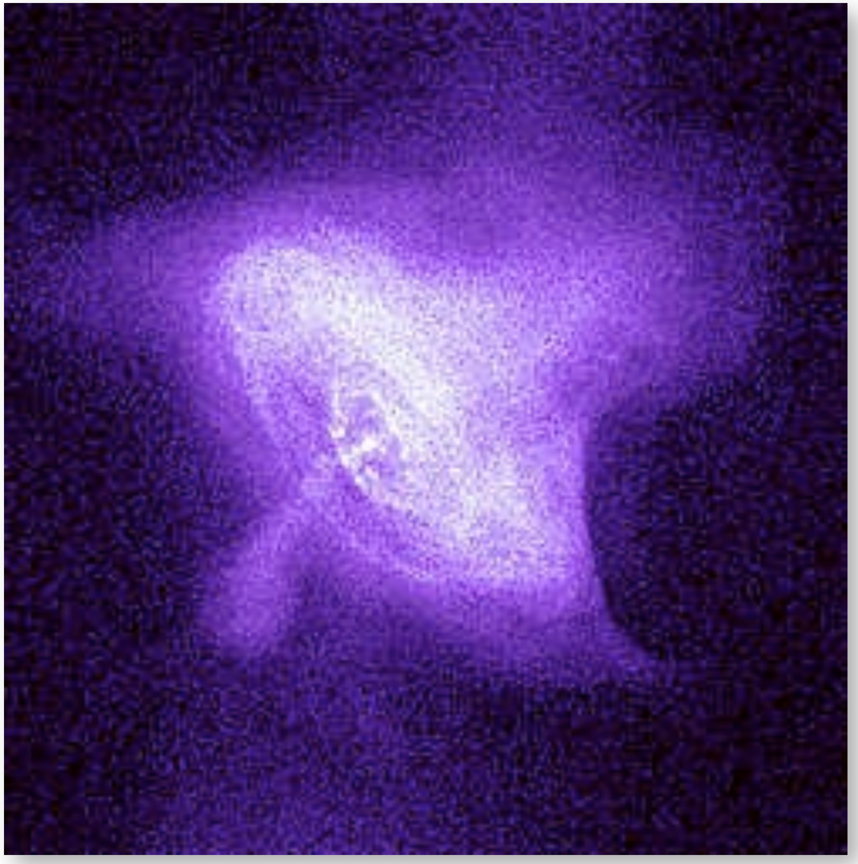
Continuous Wave Sources

- Rapidly spinning neutron stars or other objects
  - Mountains on neutron stars
- Low mass X-ray binaries
  - Accretion induced asymmetry
- Magnetars and other compact objects
  - Magnetic field induced asymmetries
- Relativistic instabilities
  - r-modes, etc.



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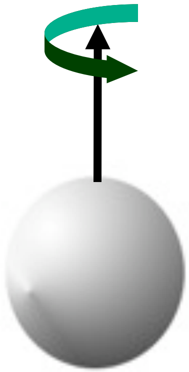
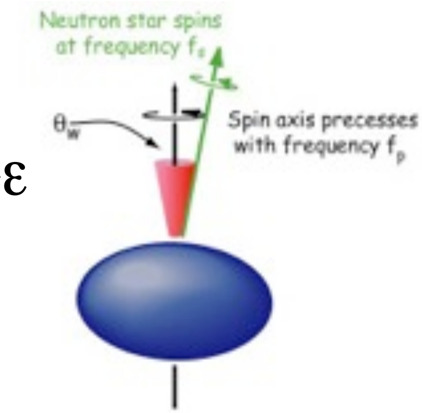
Radiation from Rotating Neutron Stars

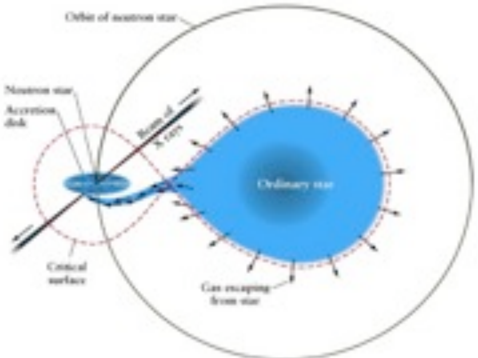


$$h_0 = \frac{16\pi^2 G}{c^4}\frac{I_{zz} f_0^2}{R}\varepsilon$$

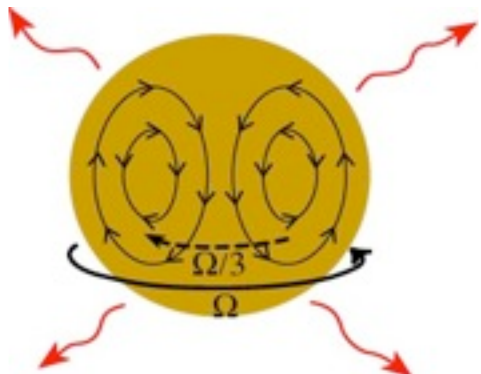
“Mountain” on neutron star



Wobbling neutron star



Accreting neutron star



R-modes

Stochastic Backgrounds

- Primordial background
  - Quantum fluctuations produce a background GW that is amplified by the background gravitational field
- Phase transitions in the Early Universe
  - Cosmic strings - kinks can form and "break" producing a burst of gravitational waves
- Astrophysical background
  - A population of Galactic white-dwarf binaries produces a background above instrumental noise in LISA

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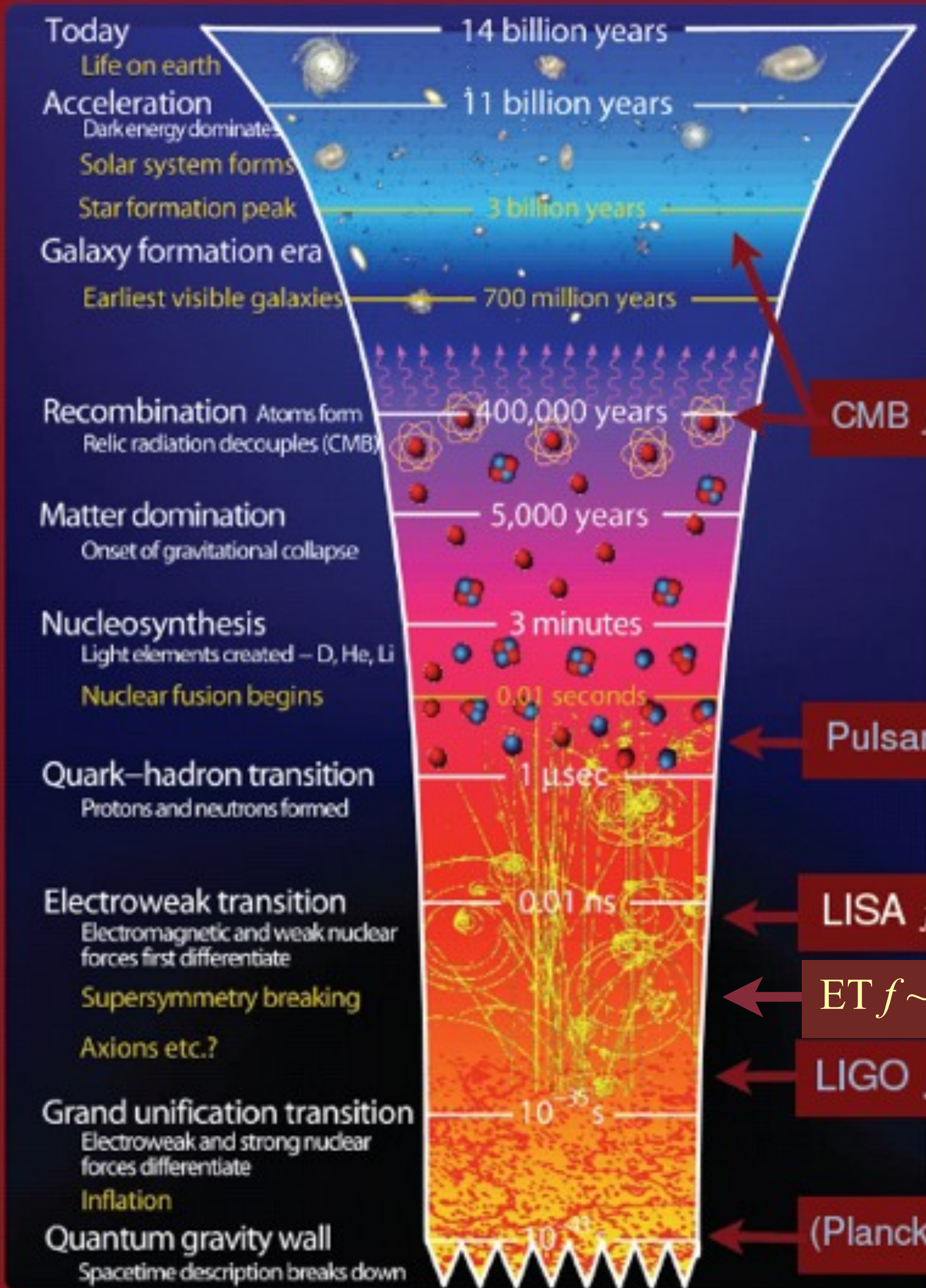
Slide from Shellard

A brief history of the Universe



CMB $f < 3 \times 10^{-17} h$Hz probes $300,000$yrs $< t_e < 14$ Gyrs

Pulsars $f \sim 10^{-8}$Hz probe $t_e \sim 10^{-4}$s ($T \sim 50$MeV)

LISA $f \sim 10^{-3}$Hz probes $t_e \sim 10^{-14}$s ($T \sim 10$TeV)

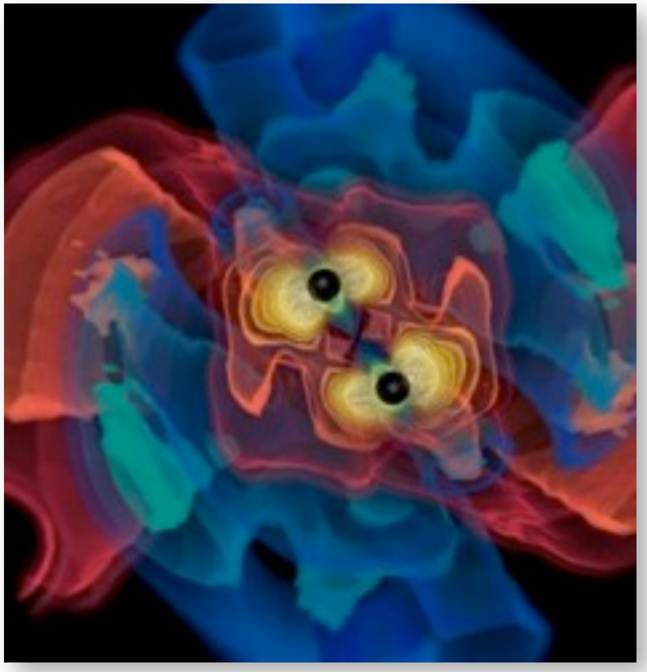
ET $f \sim 10$ Hz probes $t_e \sim 10^{-20}$ s ($T \sim 10^6$ GeV)

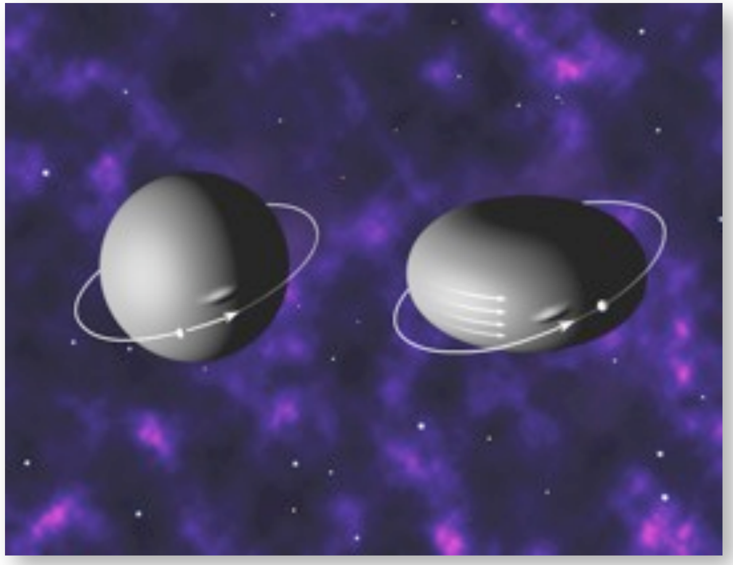
LIGO $f \sim 100$ Hz probes $t_e \sim 10^{-24}$s ($T \sim 10^8$GeV)

(Planck scale $f \sim 10^{11}$Hz has $t_e \sim 10^{-43}$s ($T \sim 10^{19}$GeV)

Compact Binary Mergers

- Binary neutron stars
- Binary black holes
- Neutron star–black hole binaries





- Loss of energy leads to steady inspiral whose waveform has been calculated to order v^7 in post-Newtonian theory
- Knowledge of the waveforms allows matched filtering

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Binary coalescence time

$$E = \frac{1}{2}\mu\, v^2 - \frac{G\mu\, M}{r} = -\frac{G\mu\, M}{2r} \quad \Rightarrow \quad r = -\frac{G\mu\, M}{2E}$$

$$\dot{r} = \frac{dr}{dE}\frac{dE}{dt} = -\frac{64}{5}\frac{G\mu\, M^2}{r^3} \quad \text{integrating} \quad \Rightarrow \quad r(t) = \left(r_0^4 - \frac{256}{5}\, G\mu\, M^2\, \Delta\tau_{\rm coal}\right)^{1/4}$$

If $\quad r(t_f) \ll r_0 \quad \Rightarrow \quad \Delta\tau_{\rm coal} = \frac{5}{256}\frac{r_0^4}{G\mu\, M^2}$

Examples:

- **LIGO/VIRGO/GEO/TAMA source:** $M = (10+10)M_\odot$ **at** $r_0 \sim 500$ **km,**

  $f_{\rm GW} \sim 40$**Hz,** $T_0 \sim 0.05$**sec** $\Rightarrow$ $\Delta\tau_{\rm coal} \sim 1\,\text{sec}$

- **LISA source:** $M = (10^6 + 10^6)M_\odot$ **at** $r_0 \sim 200 \times 10^6\,\text{km}$**,**

  $f_{\rm GW} \sim 4.5 \times 10^{-5}$ **Hz,** $T_0 \sim 11$ **hours** $\Rightarrow$ $\Delta\tau_{\rm coal} \sim 1\,\text{year}$

Expected Annual Coalescence Rates

- Rates quoted are mean of the distribution; In a 95% confidence interval, rates uncertain by 3 orders of magnitude
- Rates are quoted for
  - Binary Neutron Stars (BNS)
  - Binary Black Boles (BBH)
  - Neutron Star-Black Hole binaries (NS-BH)

	BNS	NS-BH	BBH
Initial LIGO (2002-06)	0.02	0.006	0.009
Advanced LIGO x12 sensitivity (2014)	40	10	20
Einstein Telescope x 100 sensitivity (2025)	Million	100,000	Millions

Post-Newtonian Evolution

$$\underbrace{G^{\mu\nu}\,[g, \partial g, \partial^2 g]}_{\text{Einstein's tensor}} = \frac{8\pi G}{c^4}\,\underbrace{T^{\mu\nu}\,[g, \phi]}_{\text{stress-energy tensor of the matter fields }(\phi)}$$

Einstein's tensor $G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R$

$$\nabla_\nu G^{\mu\nu} \equiv 0 \implies \nabla_\nu T^{\mu\nu} = 0$$

$$\Box h^{\alpha\beta} = \frac{16\pi G}{c^4}\tau^{\alpha\beta}, \quad \tau^{\alpha\beta} = \underbrace{|g|\; T^{\alpha\beta}}_{\text{matter term}} + \underbrace{\frac{c^4}{16\pi G}\Lambda^{\alpha\beta}[h, \partial h, \partial^2 h]}_{\text{gravitational source term}}$$

Blanchet, Damour, Iyer, Jaranowski, Schaefer, Thorne, Will, Wiseman

Andrade, Arun, Buonanno, Gopakumar, Joguet, Esposito-Farase, Faye, Kidder, Nissanke, Ohashi, Owen, Ponsot, Qusaillah, Tagoshi
...

Evolution of Compact Binaries

- Binding energy and GW flux are given by

$$E = -\frac{\nu M v^2}{2}\left\{1 + \left(-\frac{9+\nu}{12}\right)v^2 + \left(\frac{-81+57\nu-\nu^2}{24}\right)v^4 + \left(-\frac{675}{64} + \left[\frac{34445}{576} - \frac{205\pi^2}{96}\right]\nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3\right)v^6 + \mathcal{O}(v^8)\right\},$$

$$\mathcal{F} = \frac{32\nu^2 v^{10}}{5}\left\{1 - \left(\frac{1247}{336} + \frac{35}{12}\nu\right)v^2 + 4\pi v^3 + \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right)v^5 - \left(\frac{8191}{672} + \frac{583}{24}\right)\pi v^5 + \left[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}(\gamma + \ln(4v)) + \left(-\frac{4709005}{272160} + \frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3\right]v^6 + \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right)\pi v^7 + \mathcal{O}(v^8)\right\},$$

- The energy balance: GW flux must result in a loss of energy from the system

$$\mathcal{F} = -\frac{dE}{dt} \qquad \frac{d\varphi(t)}{dt} = \frac{v^3}{M}, \quad \frac{dv}{dt} = \frac{-\mathcal{F}(v)}{E'(v)}.$$

Phasing formula for Binary Systems

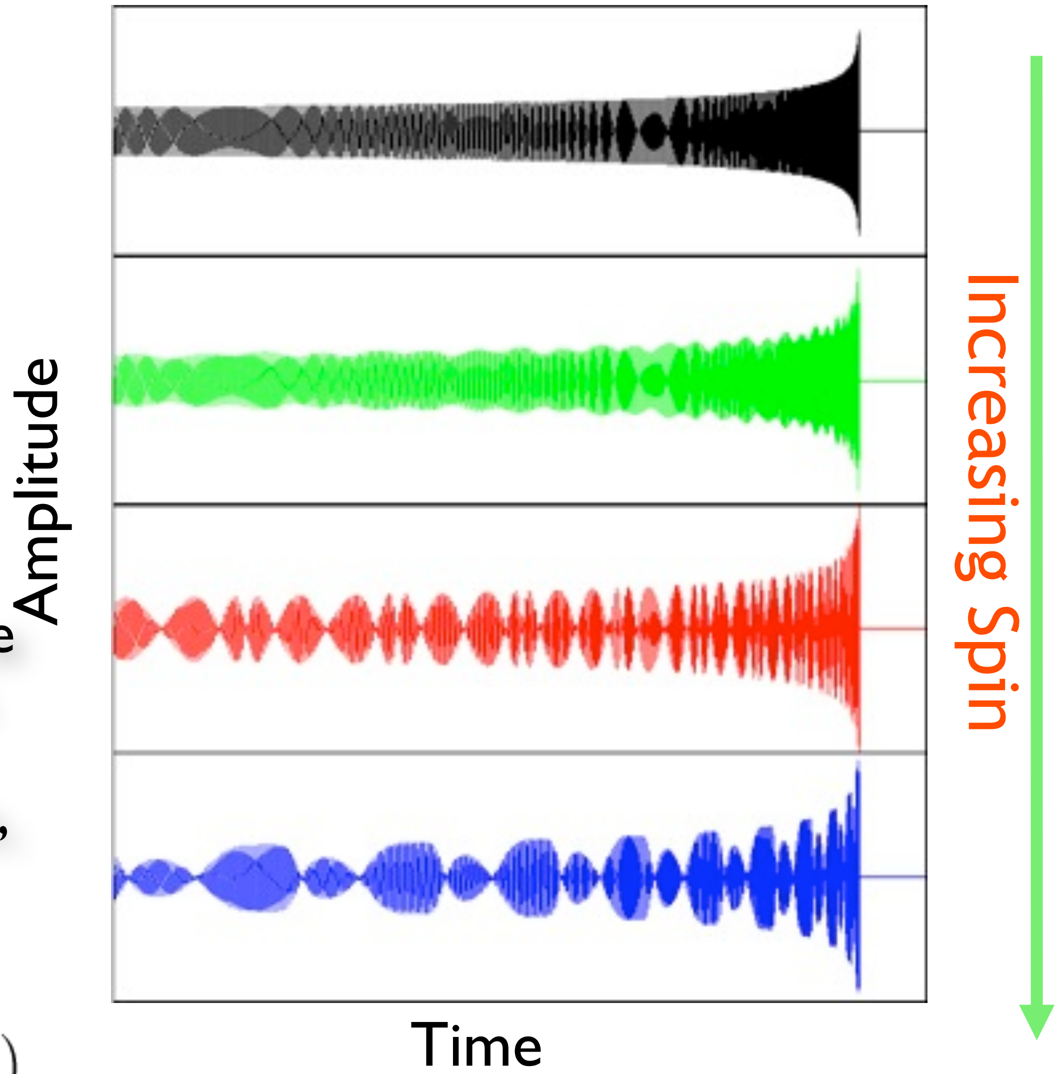
$$\varphi(t) = \frac{-1}{\nu\tau^5}\left\{1 + \left(\frac{3715}{8064} + \frac{55}{96}\nu\right)\tau^2 - \frac{3\pi}{4}\tau^3 + \left(\frac{9275495}{14450688} + \frac{284875}{258048}\nu + \frac{1855}{2048}\nu^2\right)\tau^4\right.$$
$$+ \left(-\frac{38645}{172032} + \frac{65}{2048}\nu\right)\pi\tau^5 \ln\tau + \left[\frac{831032450749357}{57682522275840} - \frac{53}{40}\pi^2 - \frac{107}{56}\left(\gamma + \ln(2\tau)\right)\right.$$
$$\left. + \left(-\frac{126510089885}{4161798144} + \frac{2255}{2048}\pi^2\right)\nu + \frac{154565}{1835008}\nu^2 - \frac{1179625}{1769472}\nu^3\right]\tau^6$$
$$\left. + \left(\frac{188516689}{173408256} + \frac{488825}{516096}\nu - \frac{141769}{516096}\nu^2\right)\pi\tau^7\right\},$$

$$\tau = [\nu(t_C - t)/(5\,M)]^{-1/8}$$

Black hole binary waveforms

- Late-time dynamics of compact binaries is highly relativistic, dictated by non-linear general relativistic effects
- Post-Newtonian theory, which is used to model the evolution, is now known to $O(v^7)$
- The shape and strength of the emitted radiation depend on many parameters of the binary: masses, spins, distance, orientation, sky location, ...

$$h(t) = 4\eta \frac{M}{D} \frac{M}{r(t)} \cos 2\varphi(t)$$



Structure of the waveform

- Radiation is emitted not just at twice the orbital frequency but at all other harmonics too

$$h(t) = \frac{2M\eta}{D_{\rm L}} \sum_{k=1}^{7} \sum_{n=0}^{5} A_{(k,n/2)} \cos\left[k\Psi(t) + \phi_{(k,\, n/2)}\right] x^{\frac{n}{2}+1}(t)$$

- These amplitude corrections have a lot of additional structure
  - Increased mass reach of detectors
  - Greatly improved parameter estimation accuracies

Blanchet, Damour, Iyer, Jaranowski, Schaefer, Will, Wiseman

Andrade, Arun, Buonanno, Gopakumar, Joguet, Esposito-Farase, Faye, Kidder, Nissanke, Ohashi, Owen, Ponsot, Qusaillah, Tagoshi …

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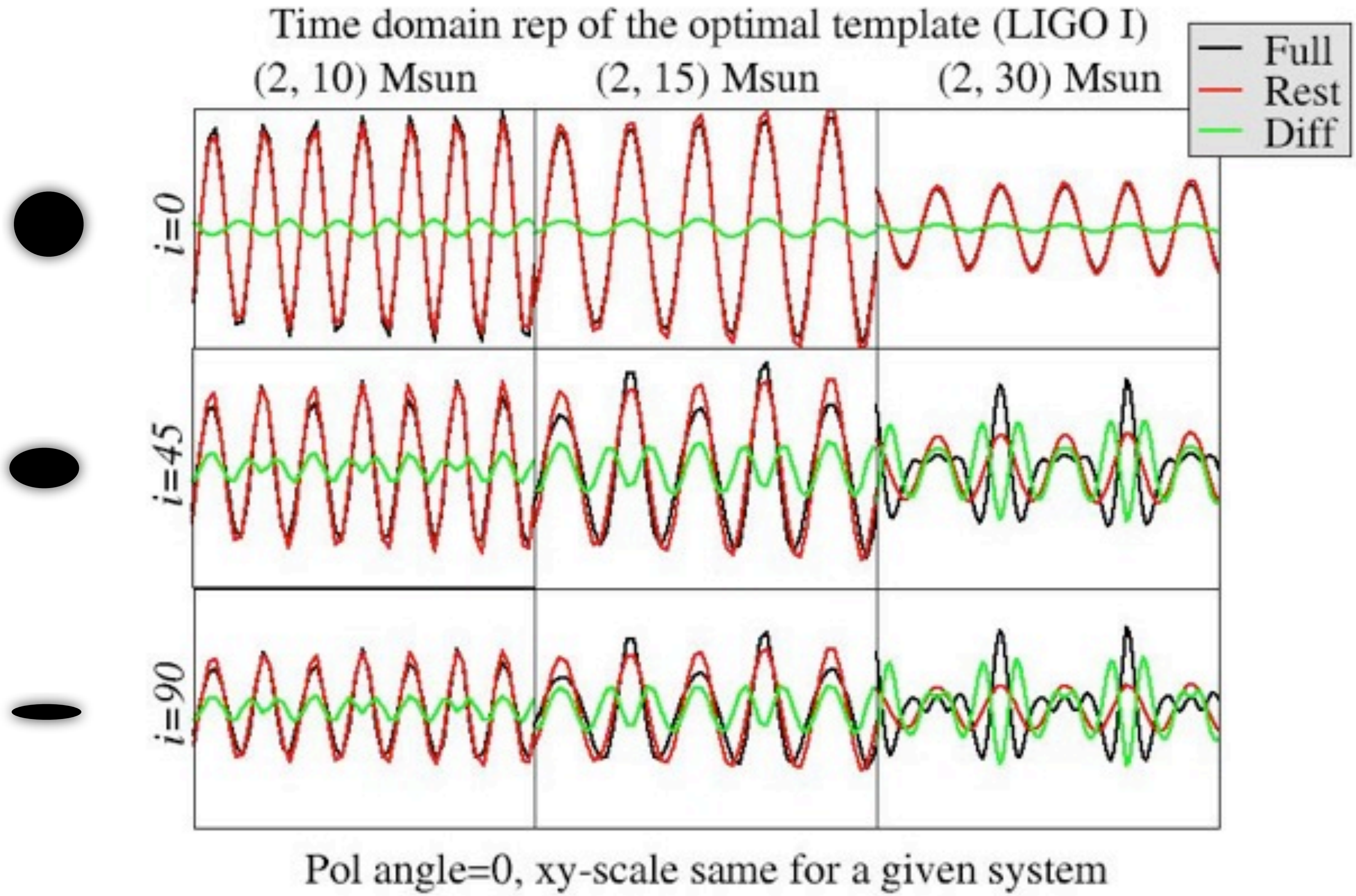
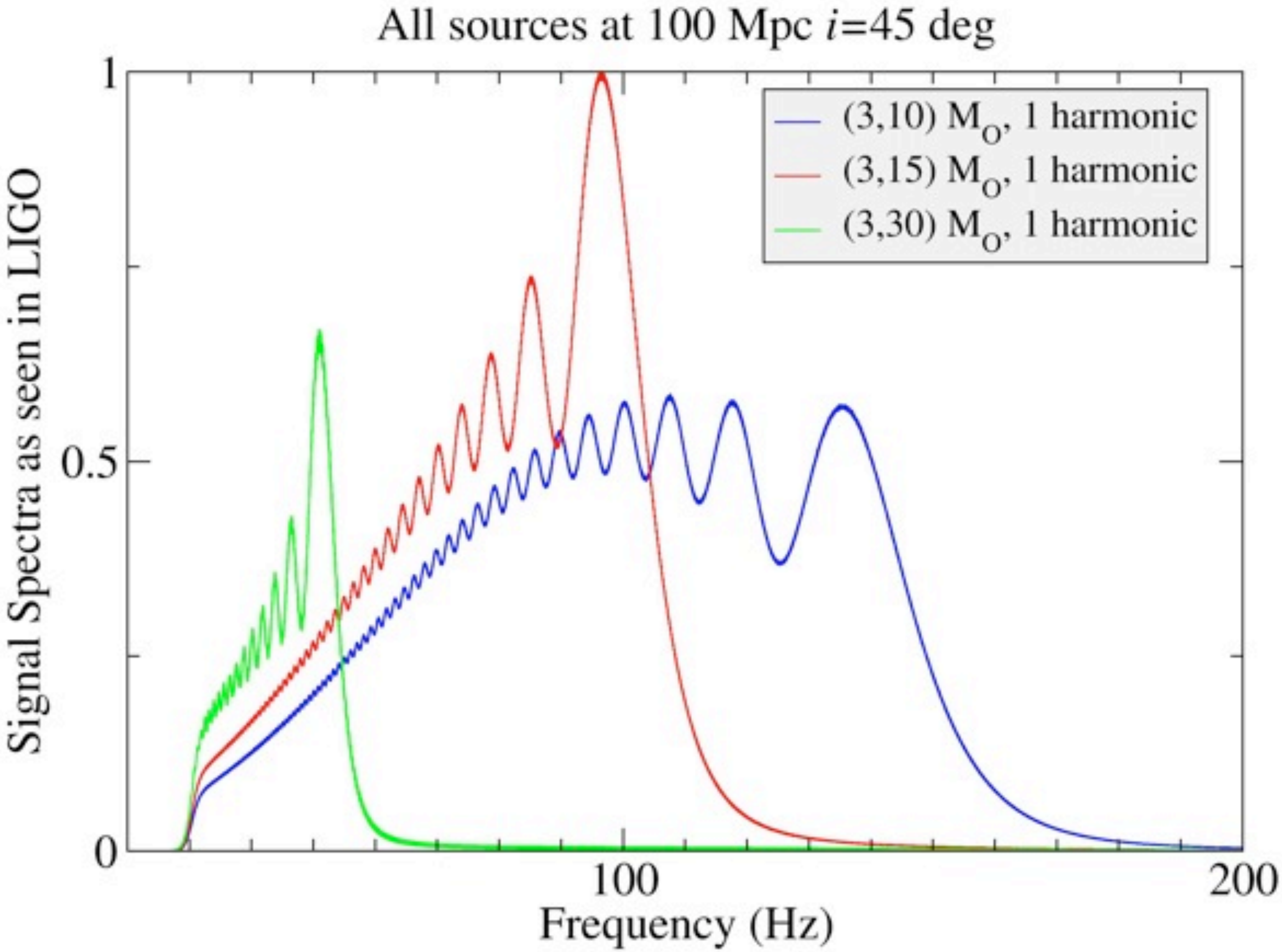
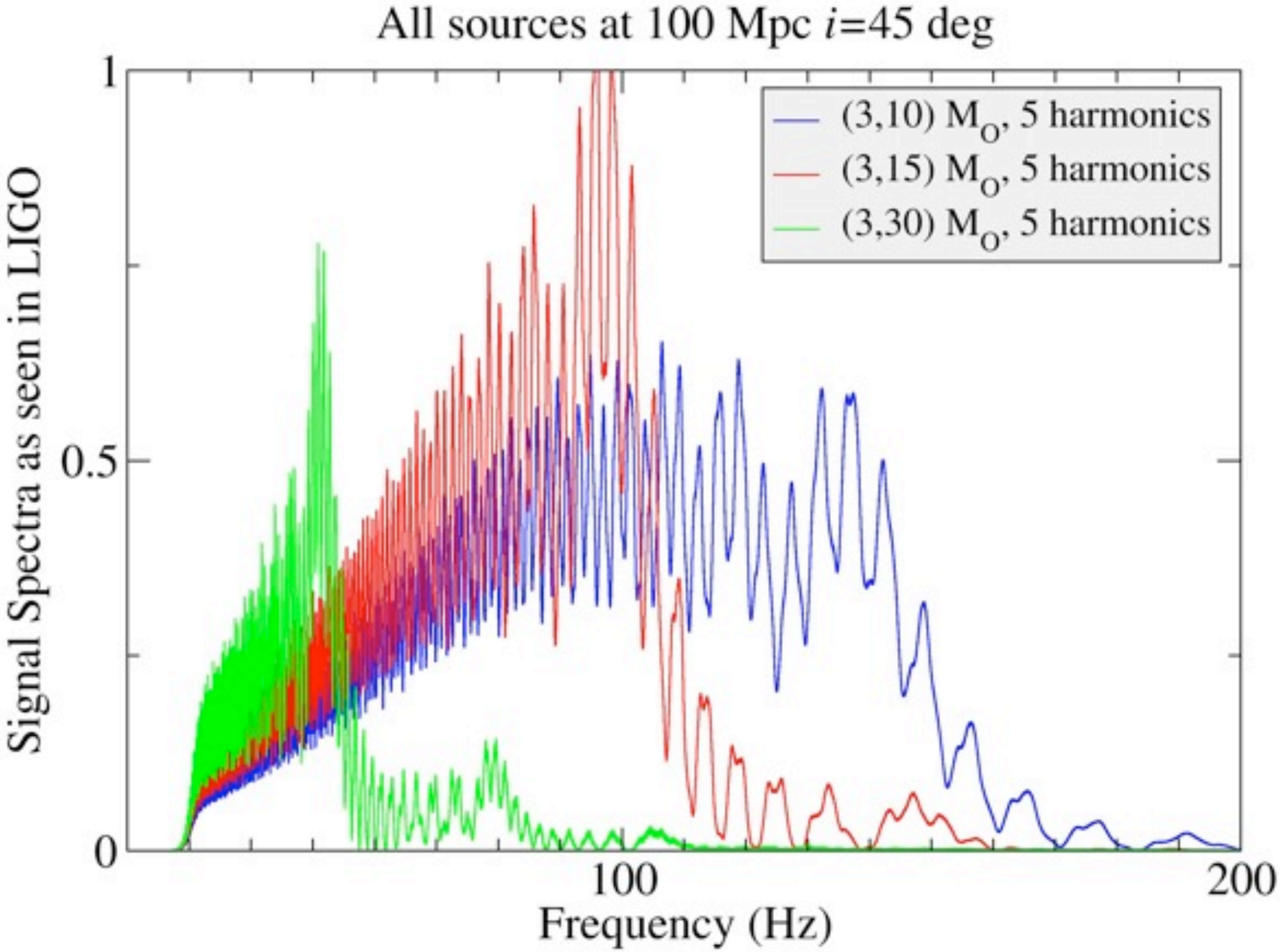
Edge-on vs face-on binaries



McKechan et al (2009)



McKechan et al (2009)



Black Hole Mergers from Numerical Relativity

- After several decades NR is now able to compute accurate waveforms for use in extracting signals and science
  - New physics - e.g. super-kick velocities
  - Analytical understanding of merger dynamics
- We should be able to see further and more massive objects

A Big Industry: Golm, Jena (Germany), Maryland, Princeton, Rochester, Baton Rouge, Georgia Tech, Caltech, Cornell (USA), Canada, Mexico, Spain, Austria

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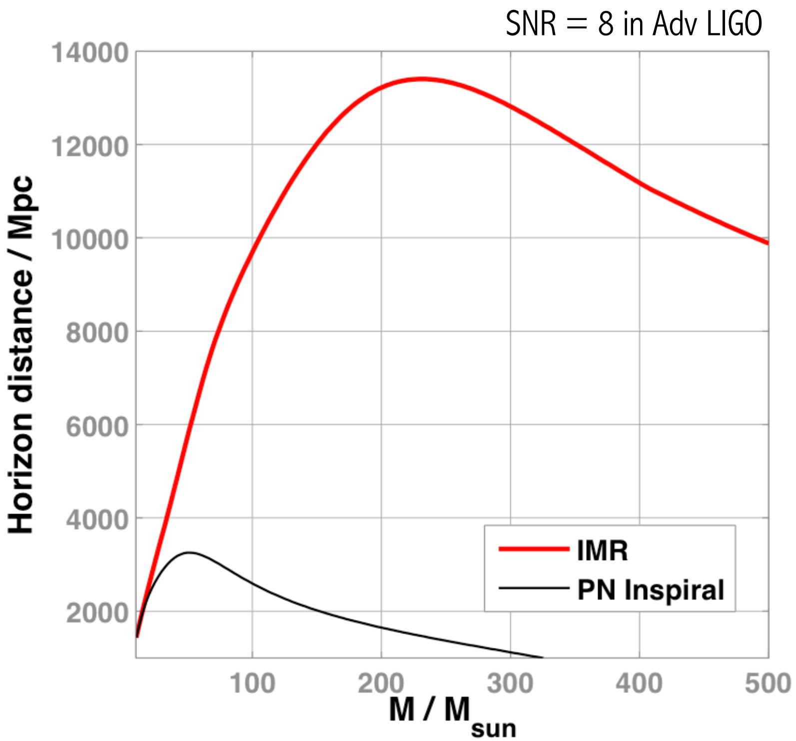
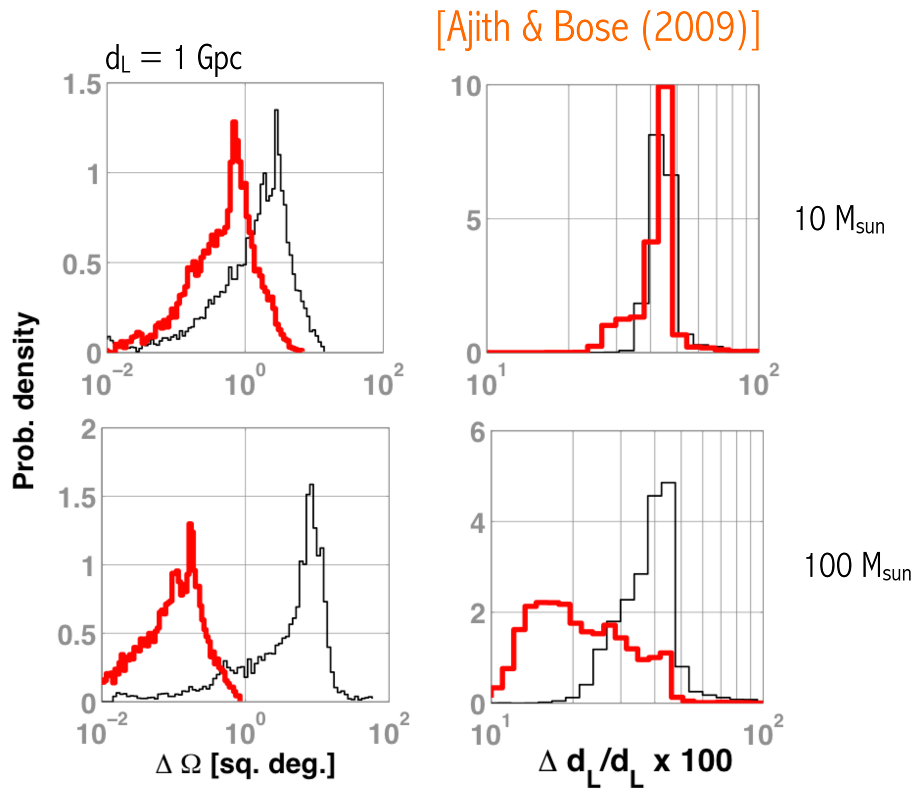
Comparison of Inspiral and Inspiral-Merger-Ringdown waveforms: Distance Reach (left) Parameter Estimation (right)





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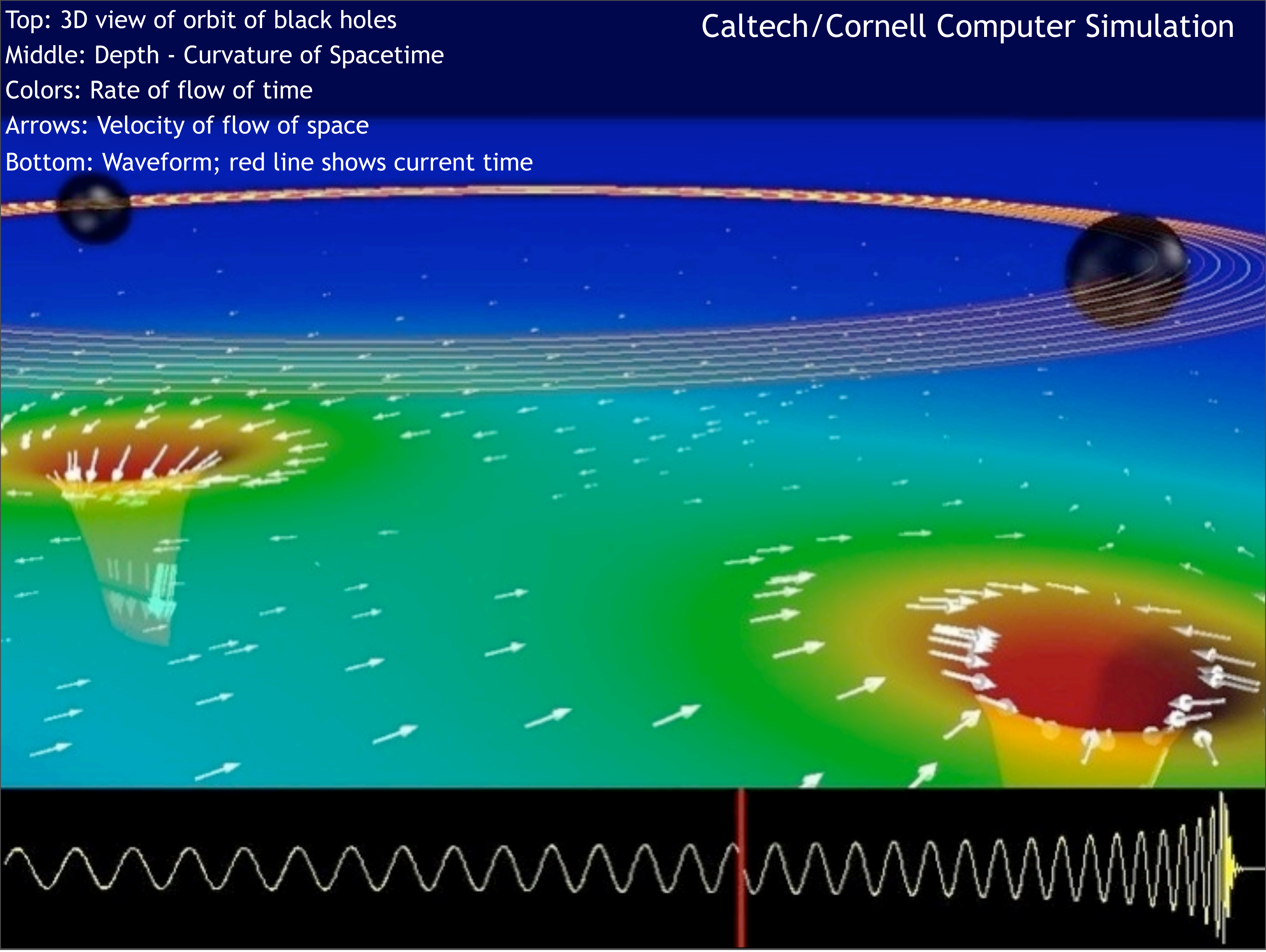
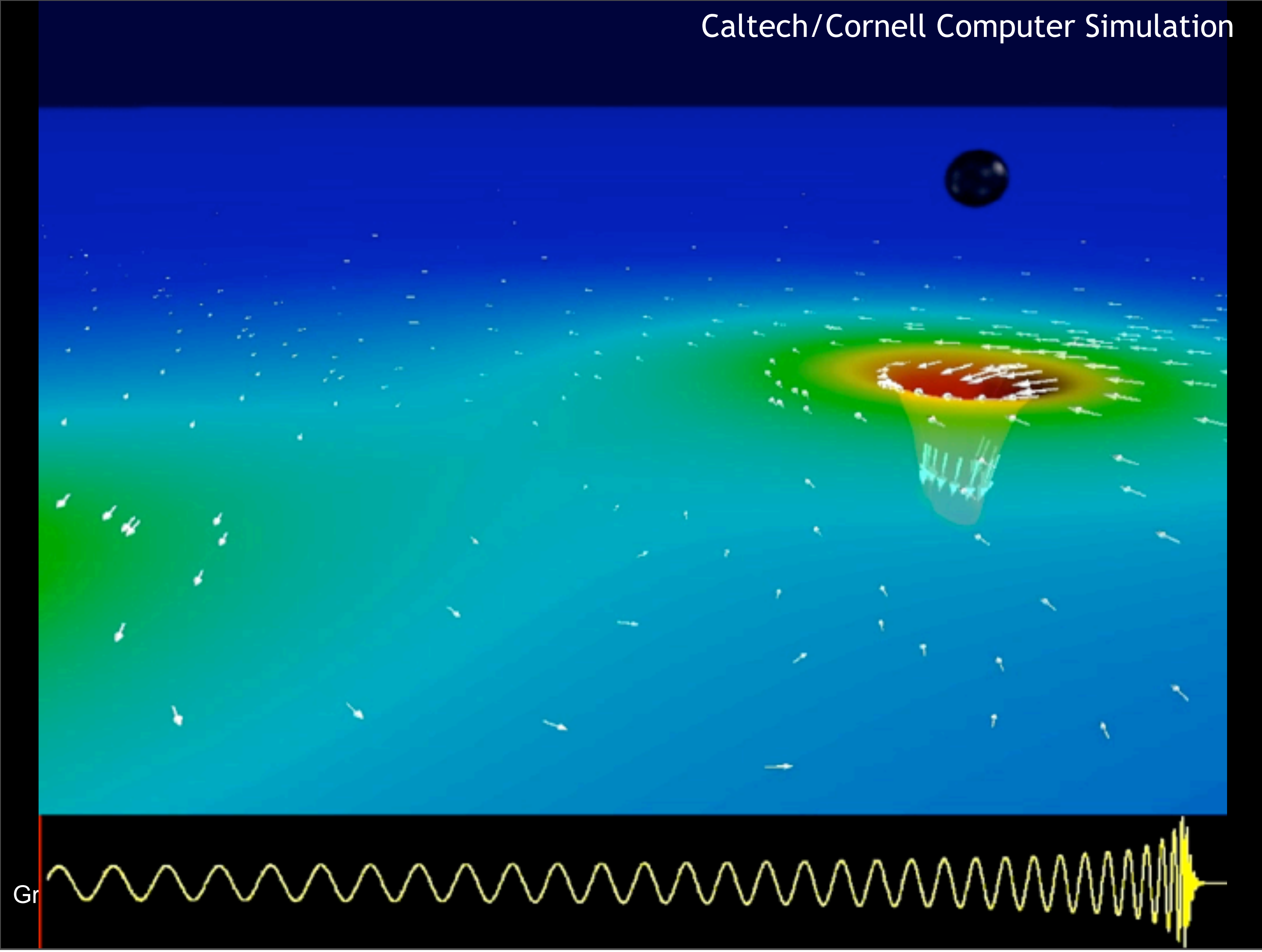
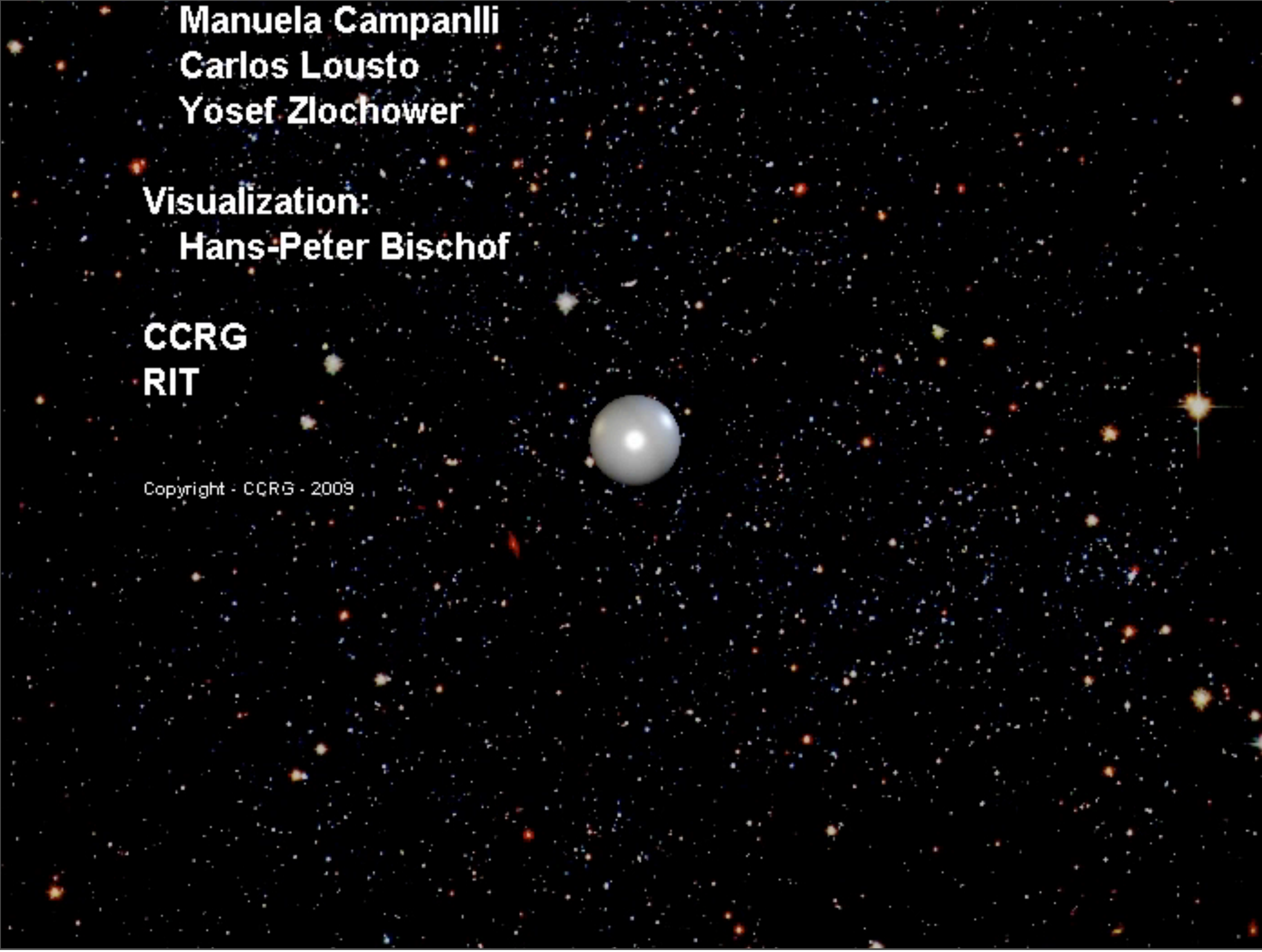
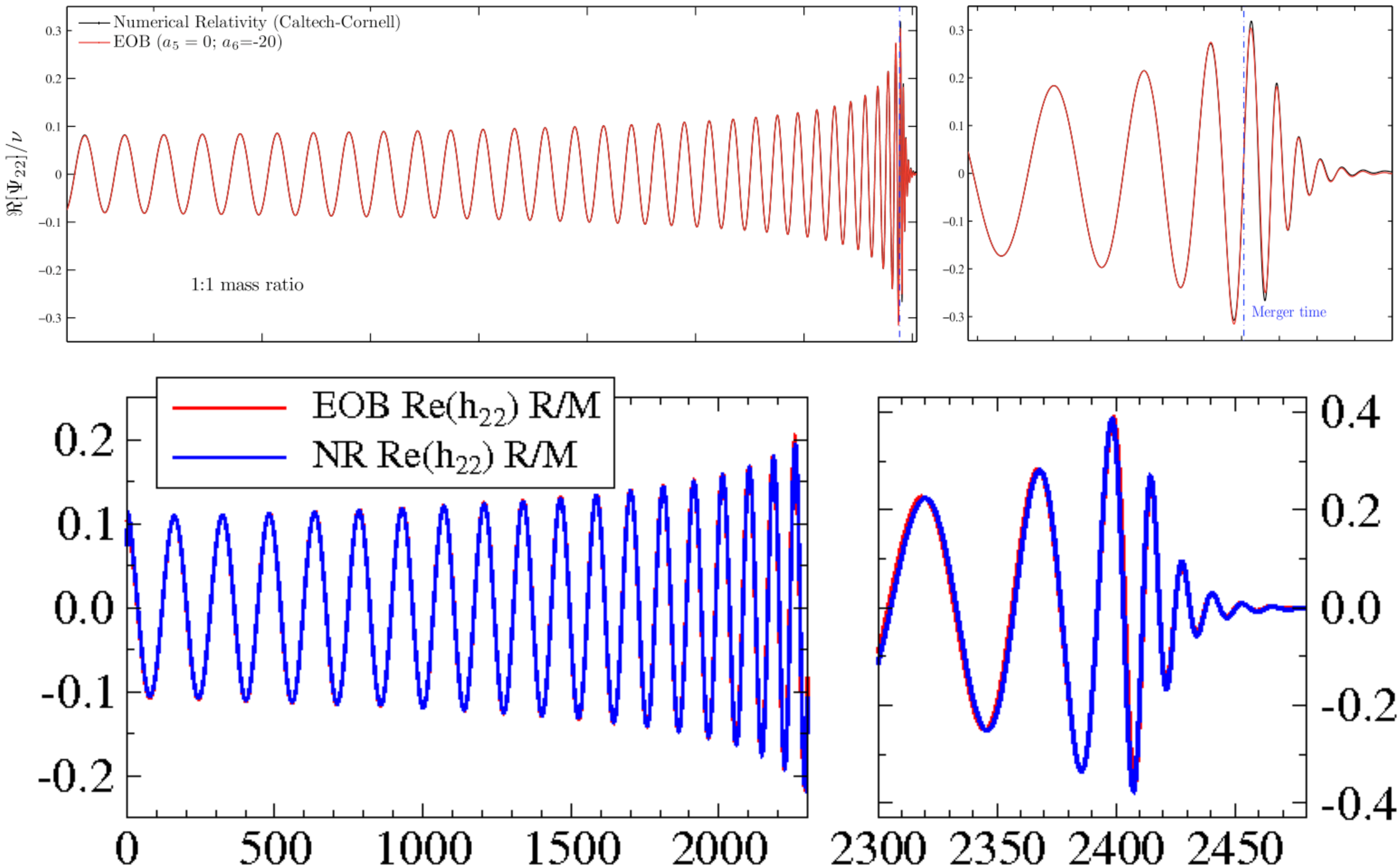

Top: 3D view of orbit of black holes
Middle: Depth - Curvature of Spacetime
Colors: Rate of flow of time
Arrows: Velocity of flow of space
Bottom: Waveform; red line shows current time
Caltech/Cornell Computer Simulation


Caltech/Cornell Computer Simulation
Gr


Manuela Campanlli
Carlos Lousto
Yosef Zlochower
Visualization:
Hans-Peter Bischof
CCRG
RIT
Copyright - CCRG - 2009

Effective-One-Body Formalism for Inspiral-Merger-Ringdown Dynamics



Conclusions from Lecture 1

- Gravitational waves are a well-understood phenomena
  - Well confirmed by binary pulsars
- Analytical and numerical relativity have progressed well
  - Today we understand the dynamics of binary black holes pretty well
- Challenges remain when "matter" is included
  - In particular we do not have a good understanding of binary neutron star mergers, relativistic instabilities, etc.

Gravitational Wave Detectors

B.S. Sathyaprakash
Cardiff University, Cardiff, United Kingdom
ISAPP School, Pisa, Italy, September 27-29, 2010

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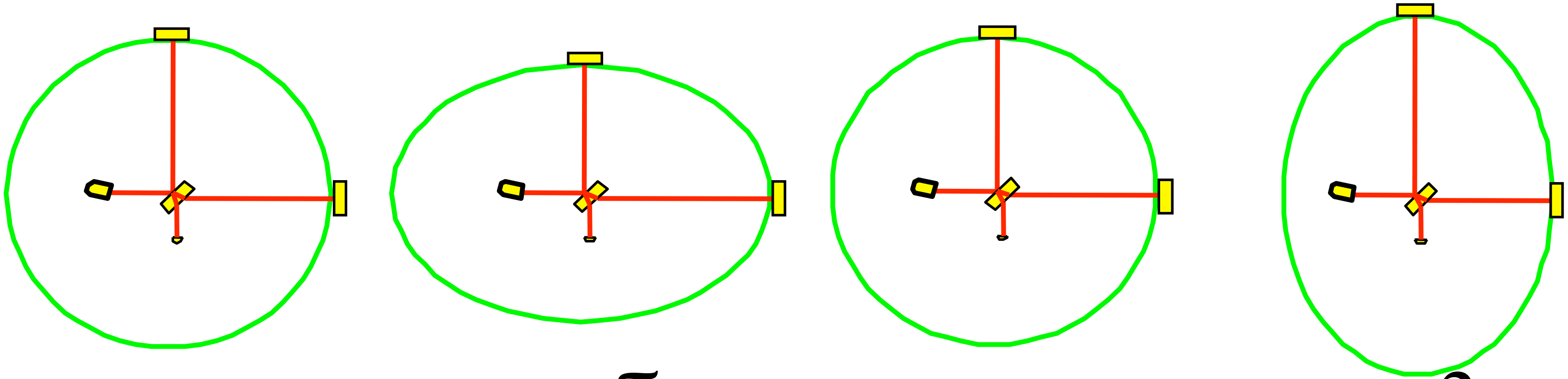
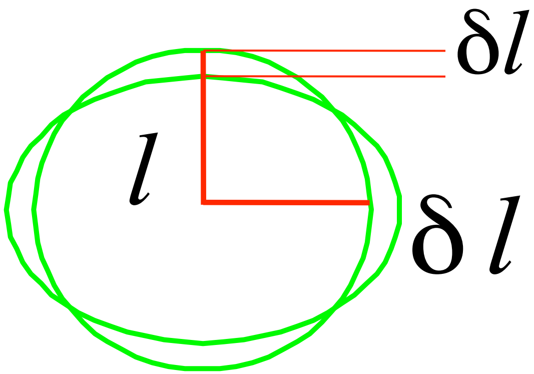
Gravitational Wave Detectors - Now and in the Future

Interferometric gravitational-wave detectors

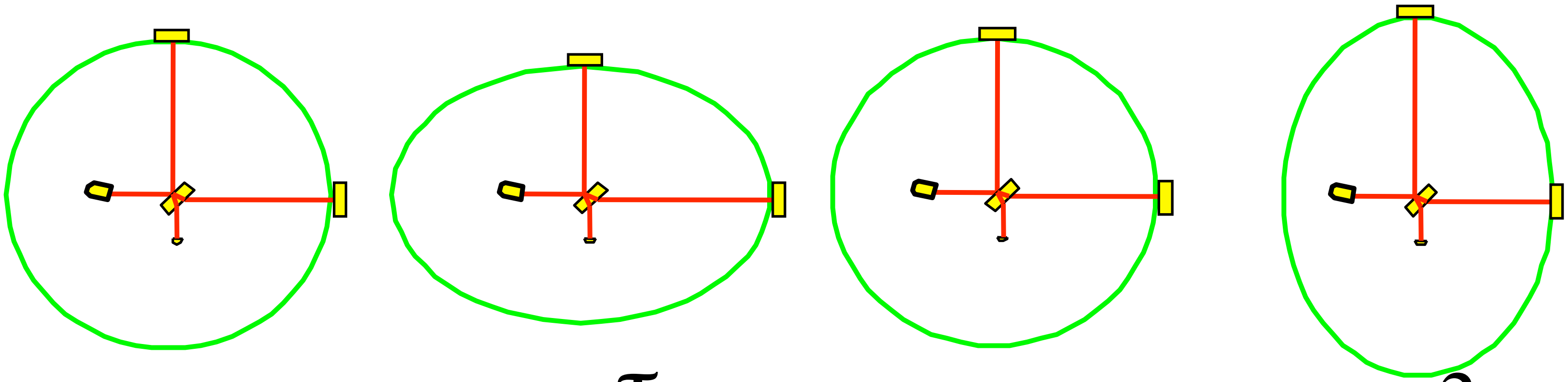


$t = 0$ $\quad t = \frac{\tau}{4}$ $\quad t = \frac{\tau}{2}$ $\quad t = \frac{3\tau}{4}$

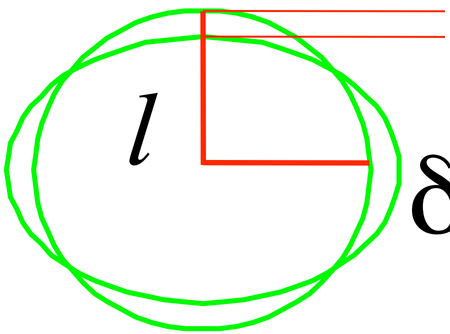


$$\delta l = \frac{h}{2} l$$

Interferometric gravitational-wave detectors



$t = 0$ $\quad t = \frac{\tau}{4}$ $\quad t = \frac{\tau}{2}$ $\quad t = \frac{3\tau}{4}$



$$\delta l = \frac{h}{2} l$$

For Typical Astronomical sources

$$h = \frac{2\delta l}{l} \leq 10^{-22}$$

American Laser Interferometer Gravitational-Wave Observatory (LIGO) at Hanford

LIGO at Livingstone, Louisiana

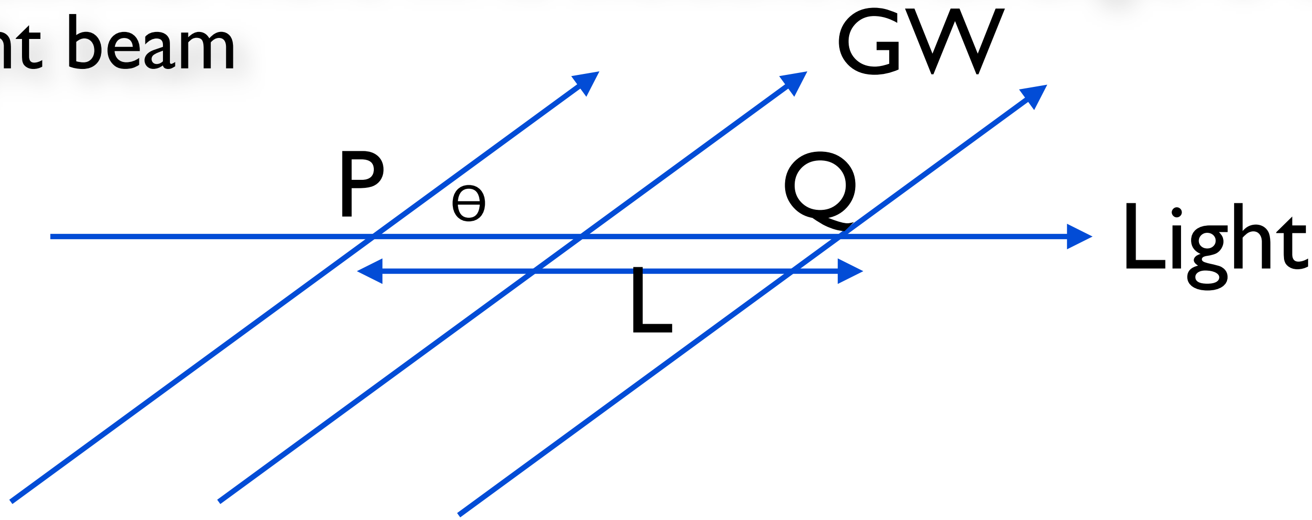

German-British GEO600

French Italian VIRGO near PISA

Large Cryogenic Gravitational Telescope

A Simple Experiment

- A light beam starts at a point P and reaches a point Q a distance L away.
  - Clocks at P and Q have proper times t and t_f.
- Gravitational wave h_+ is incident at an angle Θ to the light beam



$$\frac{dt_f}{dt} = 1 + \frac{1}{2}(1+\cos\theta)\left\{h_+[t+(1-\cos\theta)L] - h_+(t)\right\}$$

Formula for return time

- If we consider the round trip of a light beam from P to Q and back to Q then the time of return varies as:

$$\begin{aligned}\frac{dt_{\text{return}}}{dt} &= 1 + \frac{1}{2}\left\{(1-\cos\theta)\, h_+(t+2L) - (1+\cos\theta)\, h_+(t)\right.\\ &+ \left. 2\,\cos\theta\, h_+[t + L(1-\cos\theta)]\right\}.\end{aligned}$$

- In the long wavelength approximation this becomes

$$\frac{dt_{\text{return}}}{dt} = 1 + \sin^2\theta L \dot{h}_+(t).$$

Timing formula for an interferometer

- In an interferometer we have two arms, say x-arm and y-arm.
  - What interferometers measure is the differential change in the return time

$$\left(\frac{d\delta t_{\rm return}}{dt}\right) = \left(\frac{dt_{\rm return}}{dt}\right)_{\rm x-arm} - \left(\frac{dt_{\rm return}}{dt}\right)_{\rm y-arm}$$

Response of a detector to an incident wave

$$\mathbf{h}(t) = h_+(t)\mathbf{e}_+ + h_\times(t)\mathbf{e}_\times,$$

$$\mathbf{e}_+ = (\hat{e}^R_x \otimes \hat{e}^R_x - \hat{e}^R_y \otimes \hat{e}^R_y)$$

$$\mathbf{e}_\times = (\hat{e}^R_x \otimes \hat{e}^R_y + \hat{e}^R_y \otimes \hat{e}^R_x).$$

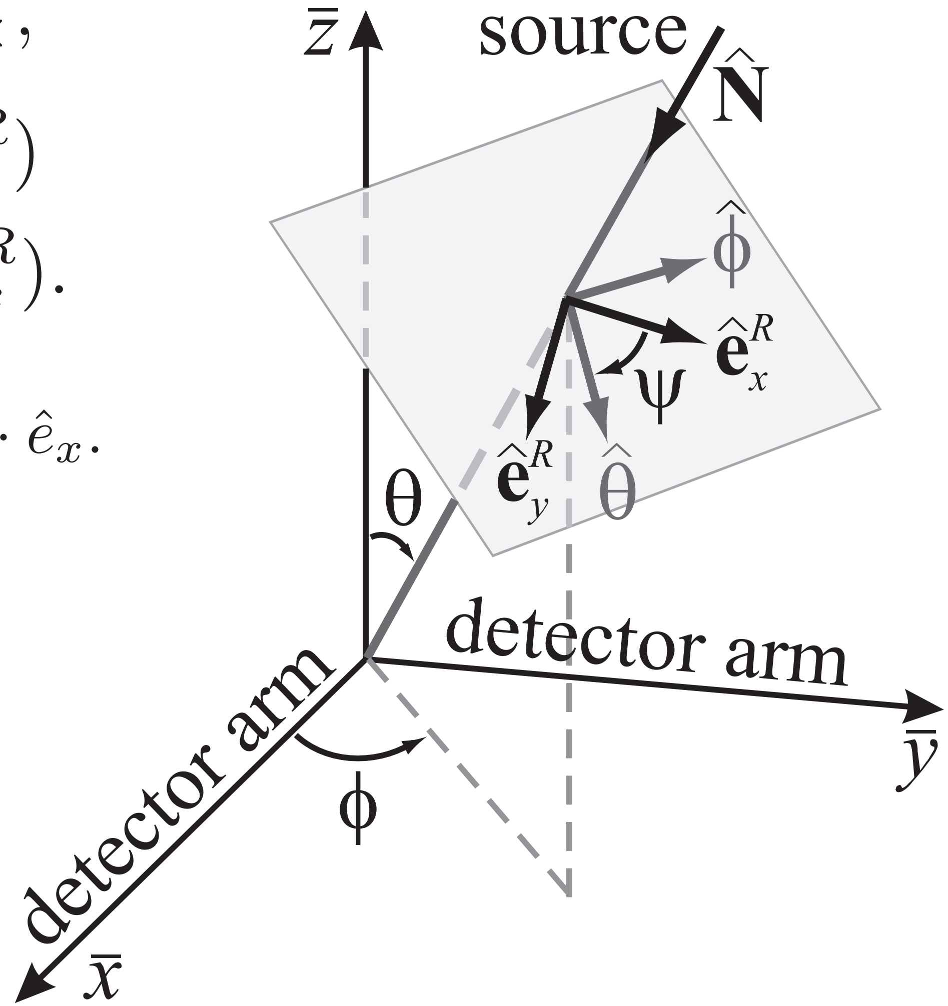
$$\left(\frac{dt_{\mathrm{return}}}{dt}\right)_{\mathrm{x-arm}} = 1 + L\hat{e}_x \cdot \dot{\mathbf{h}} \cdot \hat{e}_x.$$

$$\left(\frac{d\delta t_{\mathrm{return}}}{dt}\right) = \mathbf{d} : \dot{\mathbf{h}},$$

$$\delta t_{\mathrm{return}}(t) = \mathbf{d} : \mathbf{h}.$$

$$\mathbf{d} = L(\hat{e}_x \otimes \hat{e}_x - \hat{e}_y \otimes \hat{e}_y).$$

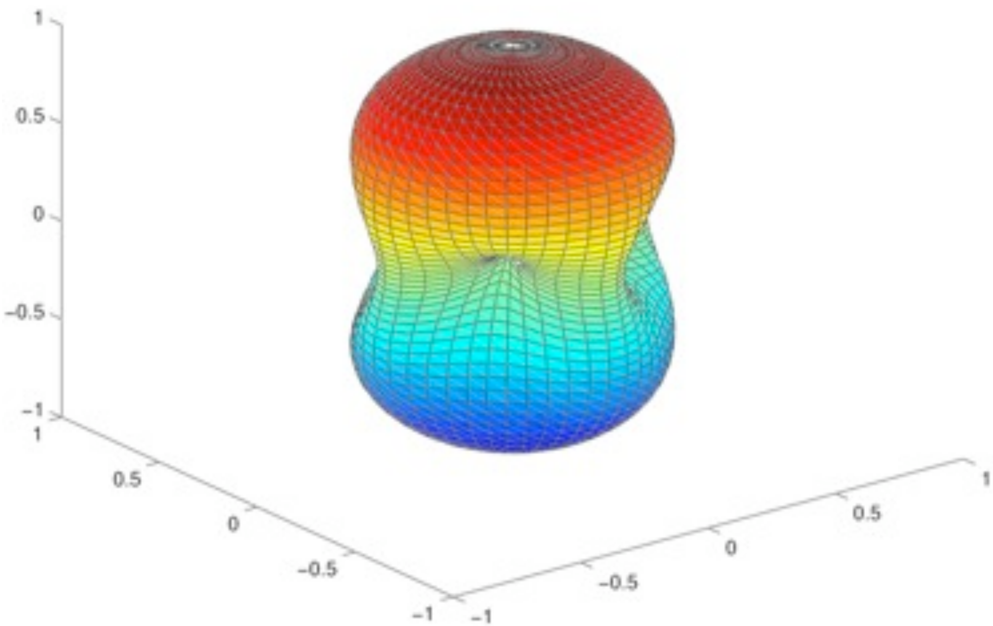
$$\delta L(t) = \frac{1}{2}\mathbf{d} : \mathbf{h}.$$



Antenna Pattern Functions

$$F_+ \equiv \mathbf{d} : \mathbf{e}_+, \quad F_\times \equiv \mathbf{d} : \mathbf{e}_\times .$$

$$\begin{aligned} F_+ &= \frac{1}{2}\left(1+\cos^2\theta\right)\cos 2\phi\cos 2\psi - \cos\theta\sin 2\phi\sin 2\psi, \\ F_\times &= \frac{1}{2}\left(1+\cos^2\theta\right)\cos 2\phi\sin 2\psi + \cos\theta\sin 2\phi\cos 2\psi. \end{aligned}$$

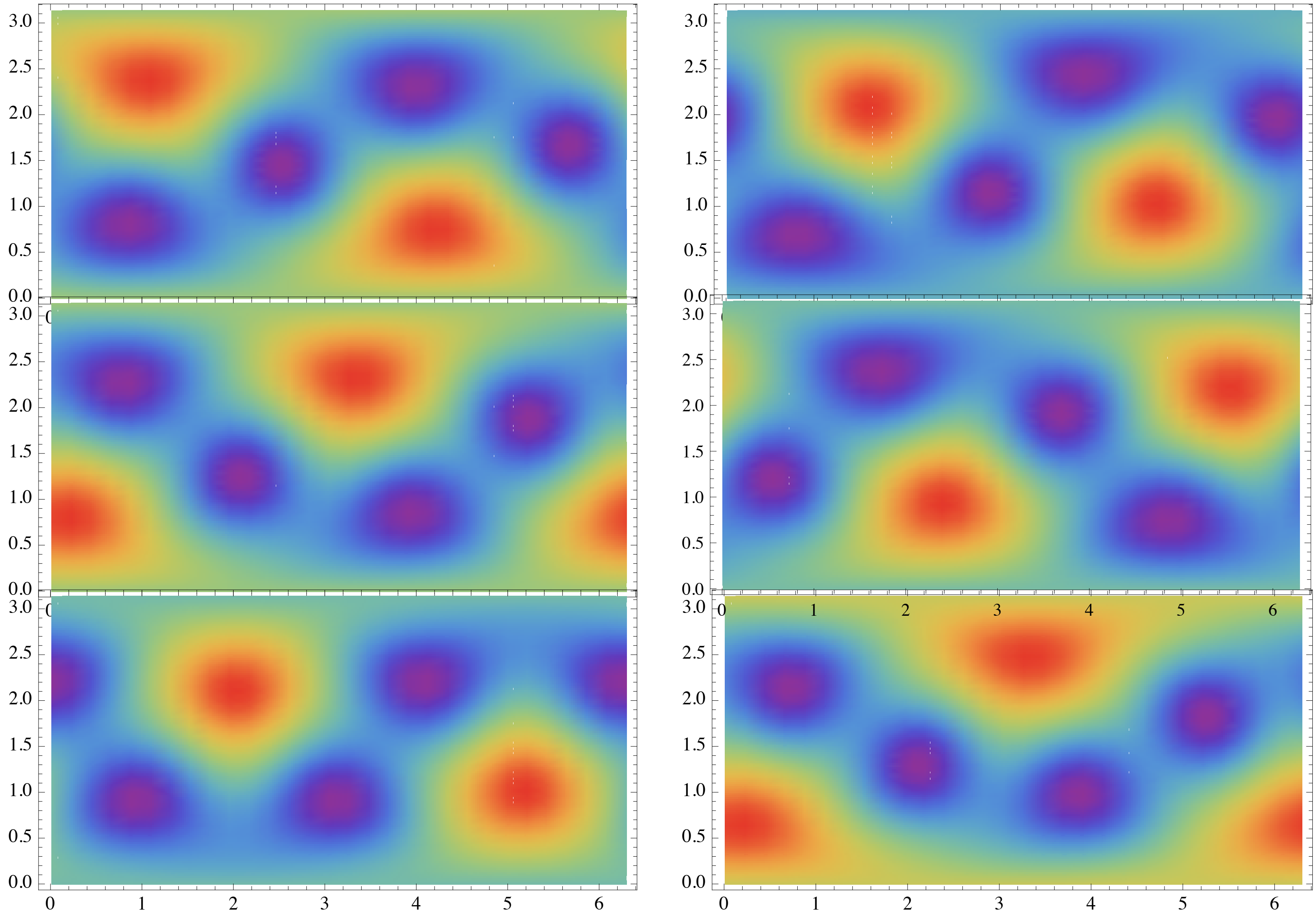


Capabilities of Advanced GW Detector Networks

Schutz, 2010

Network	Maximum Range	Detection Volume	Capture Rate (at 80%)	Capture Rate (at 95%)	Sky Coverage	Network Accuracy
L	1.00	1.23	-	-	33.6%	-
HLV	1.43	5.76	2.95	4.94	71.8%	0.98
HHLV	1.74	8.98	4.86	7.81	47.3%	1.15
HLVA	1.69	8.93	6.06	8.28	53.5%	5.09
HHLVJ	1.82	12.1	8.37	11.25	73.5%	4.65
HHLVI	1.81	12.3	8.49	11.42	71.8%	3.93
HLVJA	1.76	12.1	8.71	11.25	85.0%	7.48
HHLVJI	1.85	15.8	11.43	14.72	91.4%	6.01
HLVJAI	1.85	15.8	11.50	14.69	94.5%	9.01

Antenna pattern of GW detector network



Source Localization

- A single detector cannot localize the source on the sky
  - The antenna pattern is too wide: good for sky coverage but bad for source localization
- A network of three or more detectors needed to reconstruct the source
  - Alternatively, if the source lasts long enough, the detector motion can mimic multiple detectors and triangular a source

Timing and Triangulation

- The time at which the signal passes through detector i is given by

$$T_i = T_o + \mathbf{R} \cdot \mathbf{d}_i \ ,$$

- Here T_0 is the arrival time at geo-centre; **R** is the true location of the source and $\mathbf{d}_i$ is a vector giving the location of the source in geocentric frame.
- The probability distribution for *measured* arrival times t_i in **each** detector can be assumed to be Gaussian. In a network of N detectors the **joint** distribution will be

$$p(t_i|T_i) = \prod_i \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left[\frac{-(t_i - T_i)^2}{2\sigma_i^2}\right] .$$

- Using Bayes' theorem one can compute the posterior probability distribution for true times.

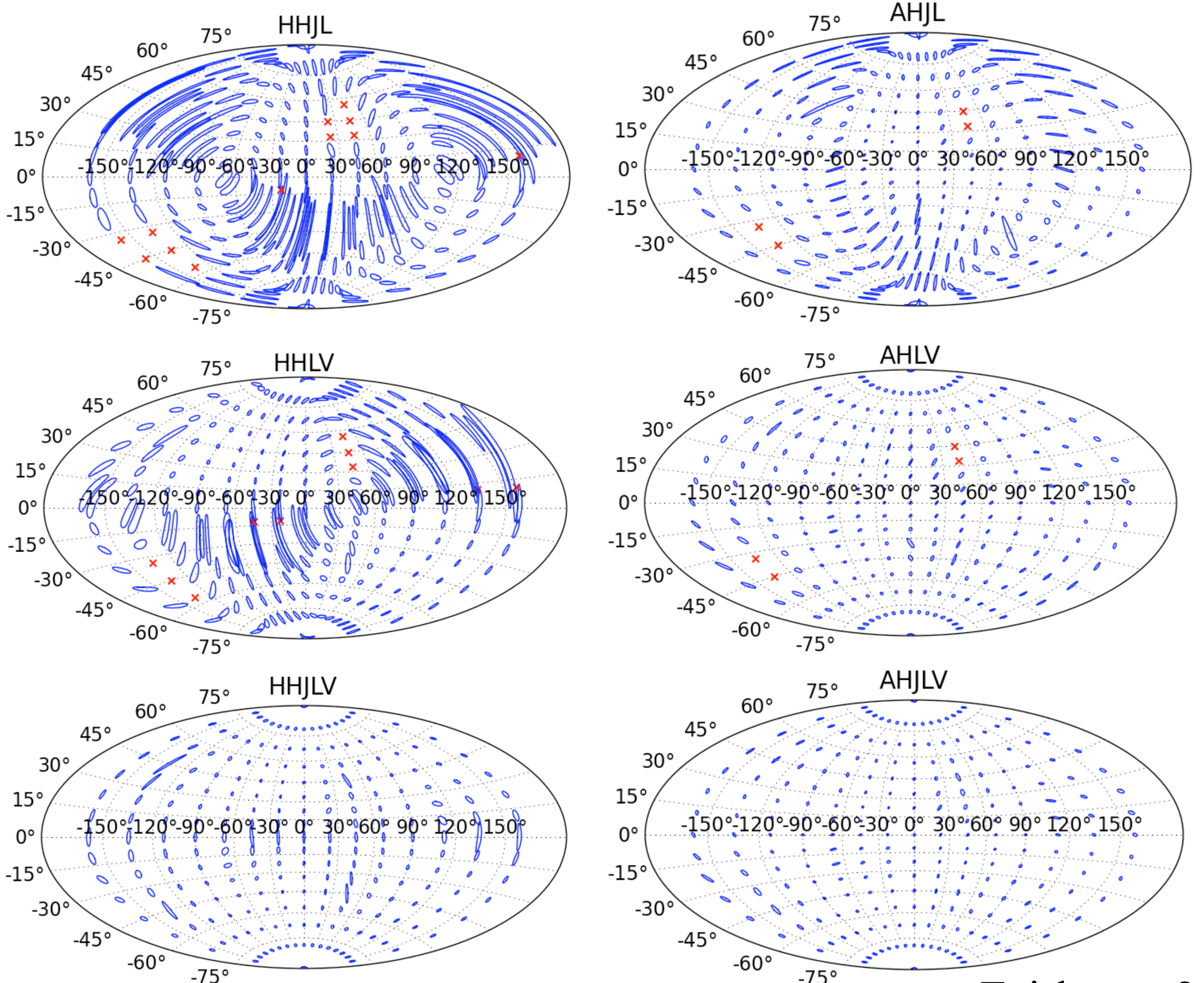
$$p(T_i|t_i) \propto p(T_i) \exp\left[\sum_i \frac{-(t_i - T_i)^2}{2\sigma_i^2}\right] . \qquad t_i = t_o + \mathbf{r} \cdot \mathbf{d}_i \ .$$

Sky localization Error Matrix

- Eliminating the true and measured times in favour of the source location one can get (after marginalizing over T_0)

$$p(\mathbf{R}|\mathbf{r}) \propto p(\mathbf{R}) \exp\left[-\frac{1}{2}(\mathbf{r}-\mathbf{R})^T \mathbf{M}(\mathbf{r}-\mathbf{R})\right] .$$

$$\mathbf{M} = \frac{1}{\sum_i \sigma_i^{-2}} \sum_{i,j} \frac{\mathbf{D}_{ij}\mathbf{D}_{ij}^T}{2\sigma_i^2\sigma_j^2} , \qquad \mathbf{D}_{ij} = \mathbf{d}_i - \mathbf{d}_j .$$



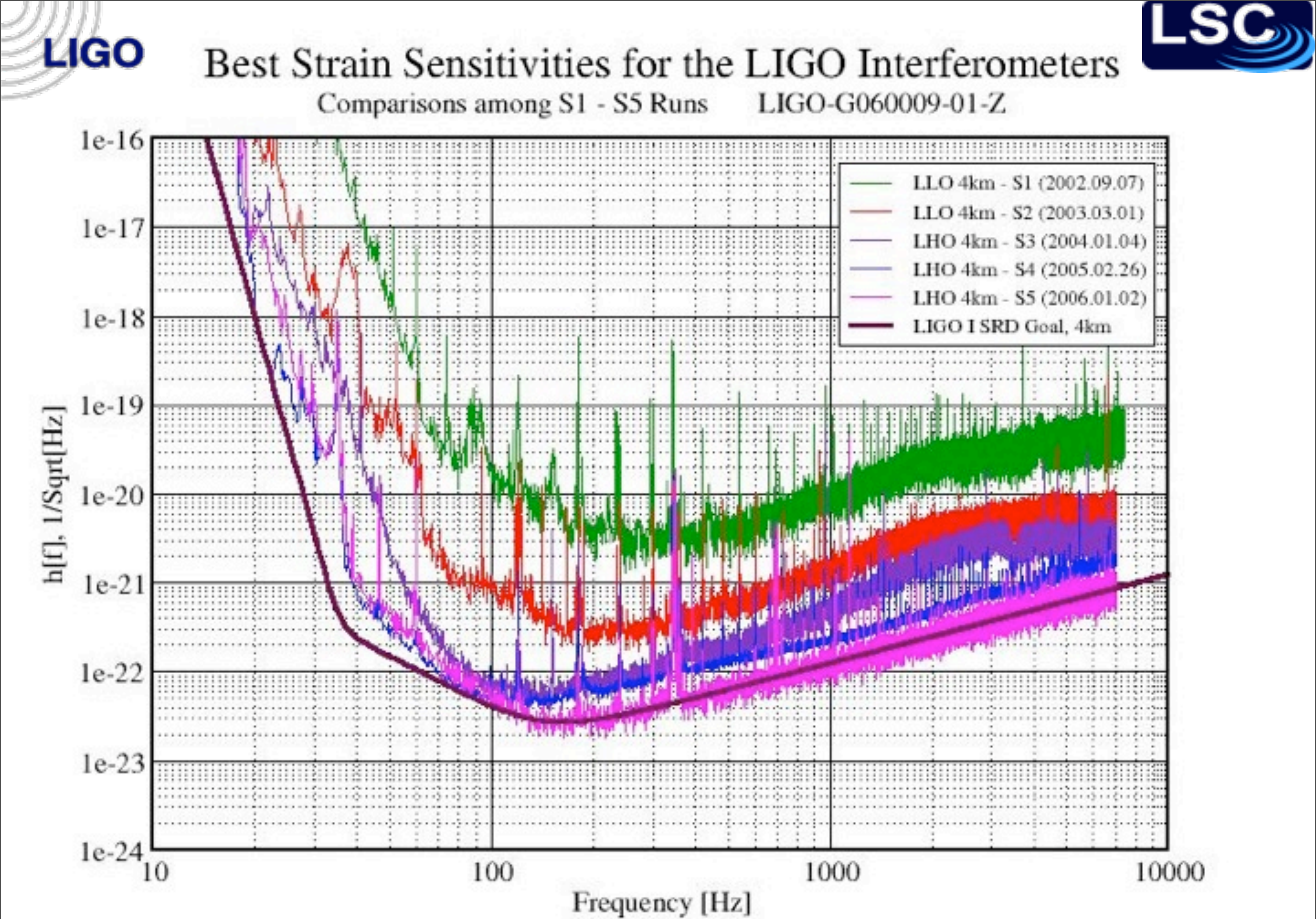
Fairhurst, 2010

Source Localization with Advanced Detector Network

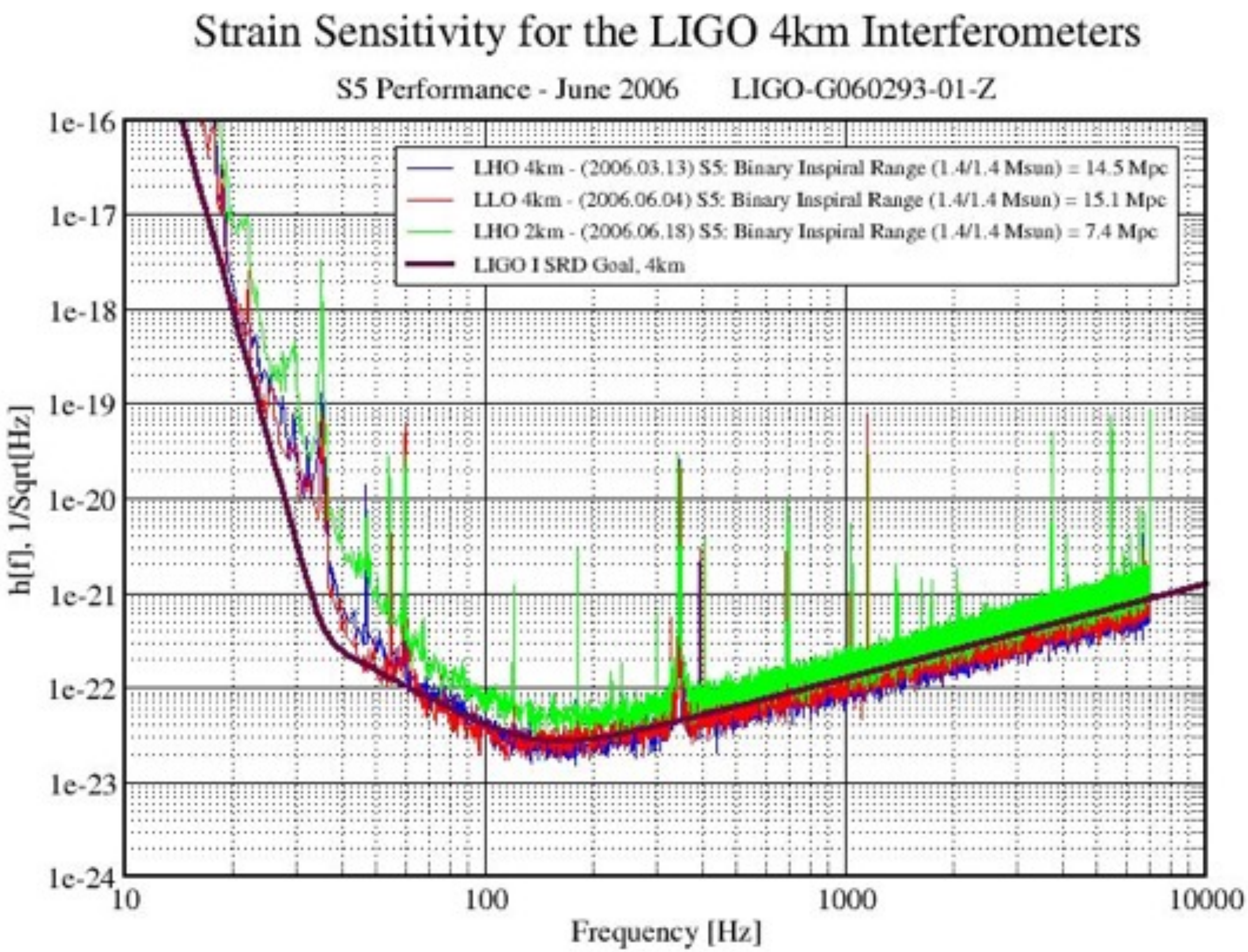
Network	Detectable Sources	Sources Localized within			
		$1\,\mathrm{deg}^2$	$5\,\mathrm{deg}^2$	$10\,\mathrm{deg}^2$	$20\,\mathrm{deg}^2$
HHL	59	0	0	0	0
AHL	59	0.4	5	13	30
HHJL	85	0.2	2	5	14
AHJL	85	1	14	36	59
HHLV	83	0.4	5	13	35
AHLV	84	2	21	48	76
HHJLV	112	2	19	47	77
AHJLV	114	3	34	84	111

Fairhurst, 2010

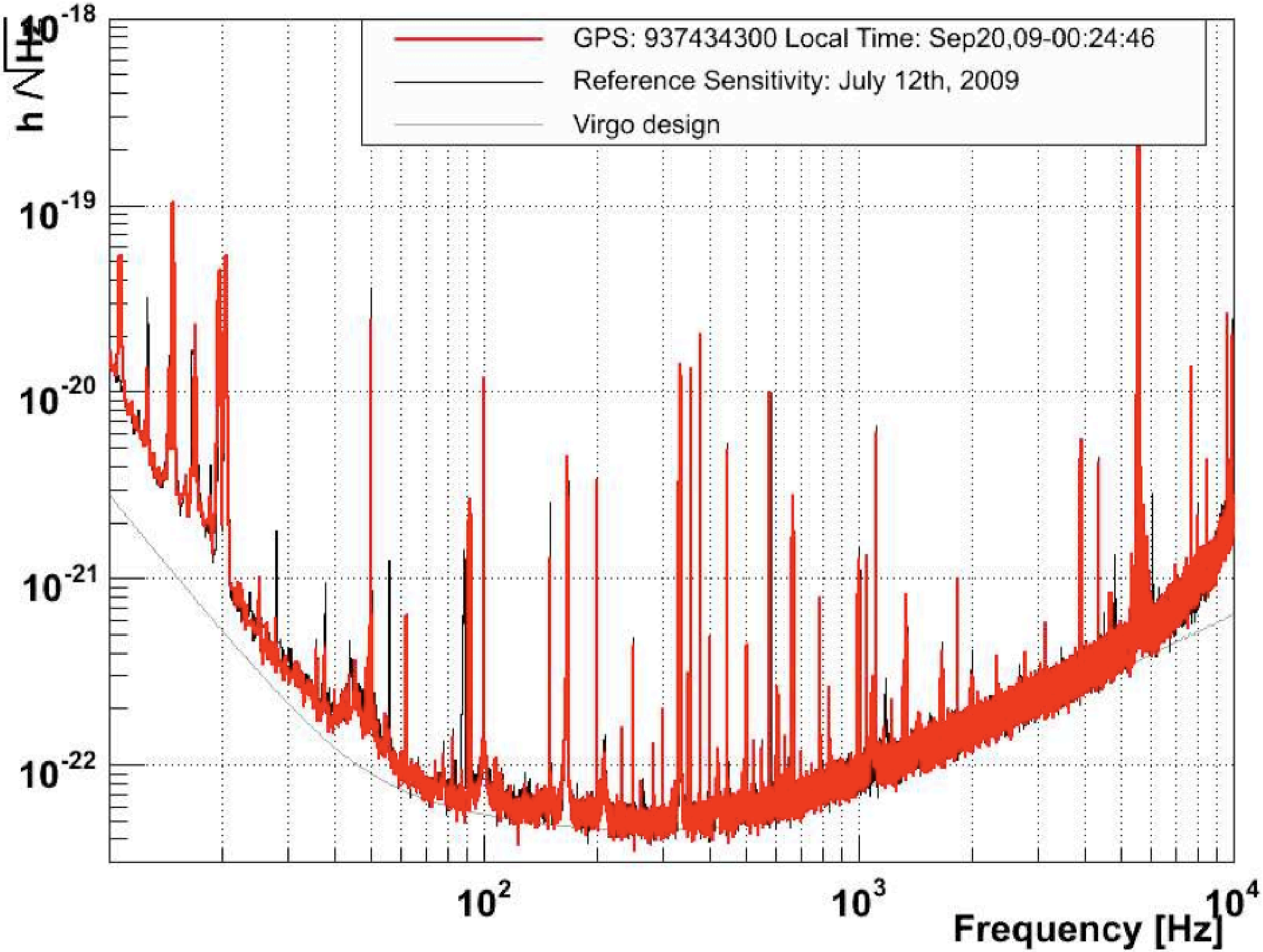
Detector Sensitivities


LIGO
LSC
Best Strain Sensitivities for the LIGO Interferometers
Comparisons among S1 - S5 Runs LIGO-G060009-01-Z
LLO 4km - S1 (2002.09.07)
LLO 4km - S2 (2003.03.01)
LHO 4km - S3 (2004.01.04)
LHO 4km - S4 (2005.02.26)
LHO 4km - S5 (2006.01.02)
LIGO I SRD Goal, 4km
h[f], 1/Sqrt[Hz]
1e-16
1e-17
1e-18
1e-19
1e-20
1e-21
1e-22
1e-23
1e-24
10
100
1000
10000
Frequency [Hz]

S5 Sensitivity



Virgo VSR-2



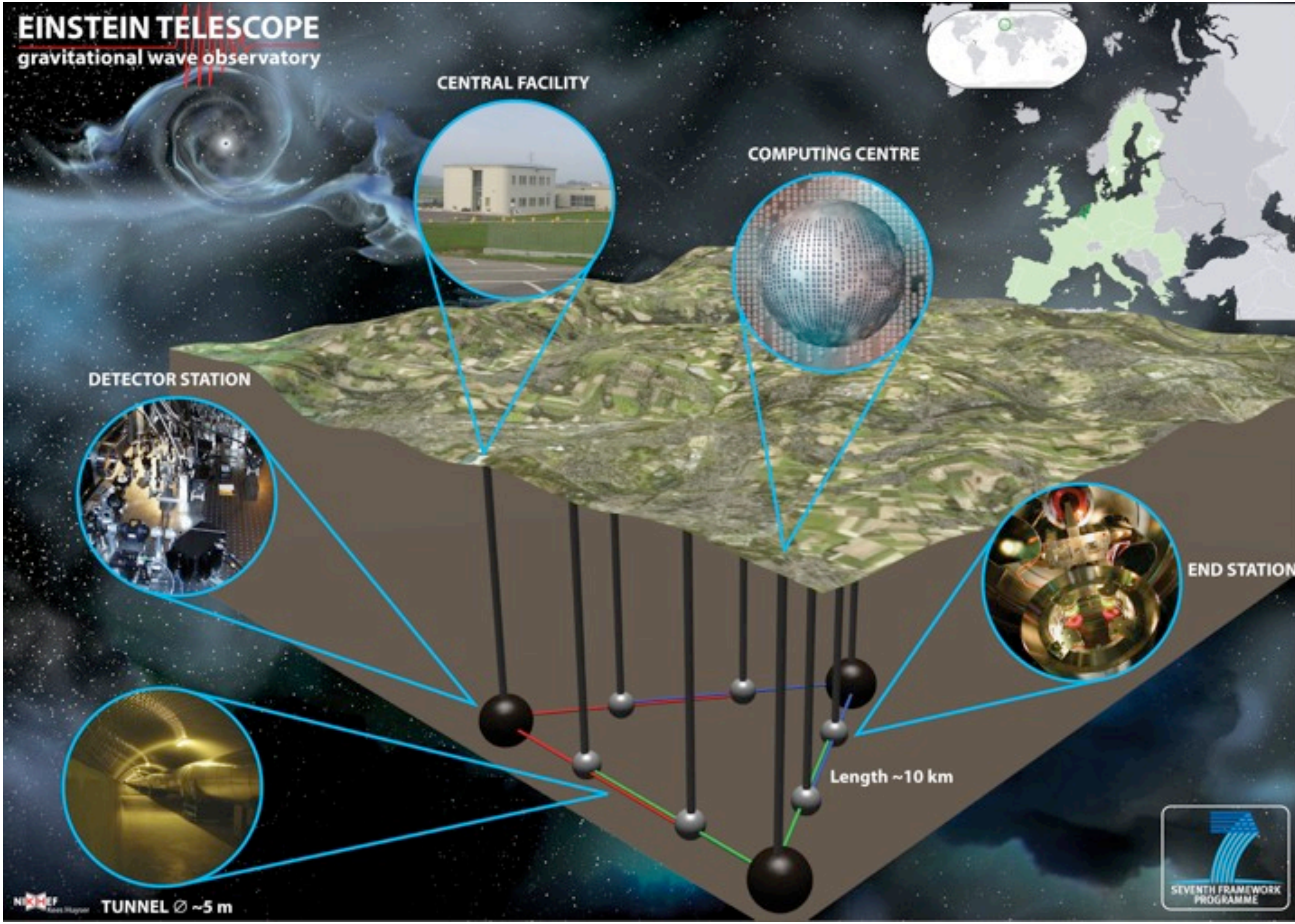
G070221-00-Z

Future Improvements

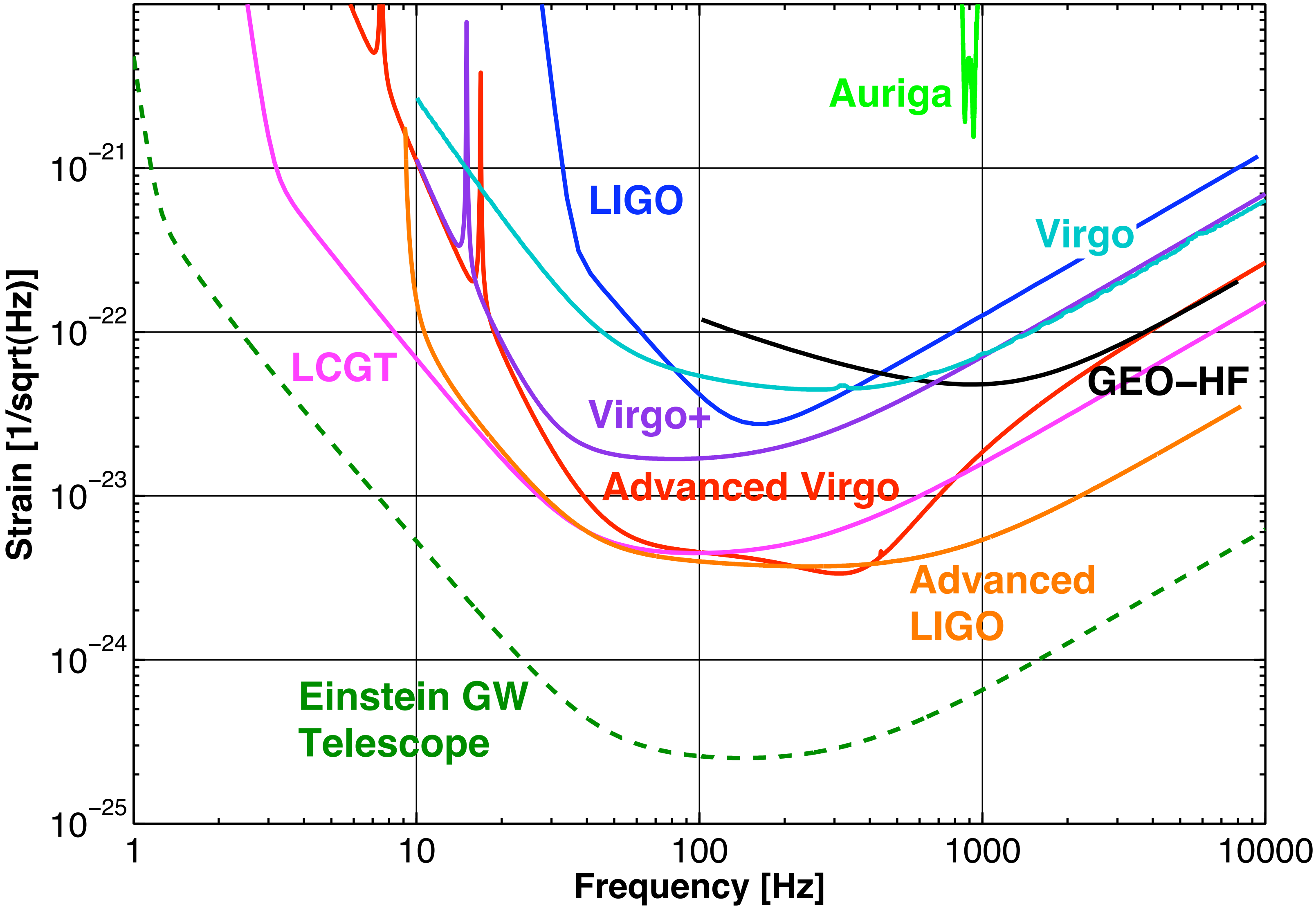
- Enhanced Detectors (2009-11)
  - 2 x increase in sensitivity
  - 8 x increase in rate
- Advanced Detectors, LIGO and Virgo (2015- …)
  - 12 x increase in sensitivity
  - Over 1000 x increase in rate
- 3G Detectors: Einstein Telescope (2027+)
  - 100 x increase in sensitivity
  - 10^6 increase in rate

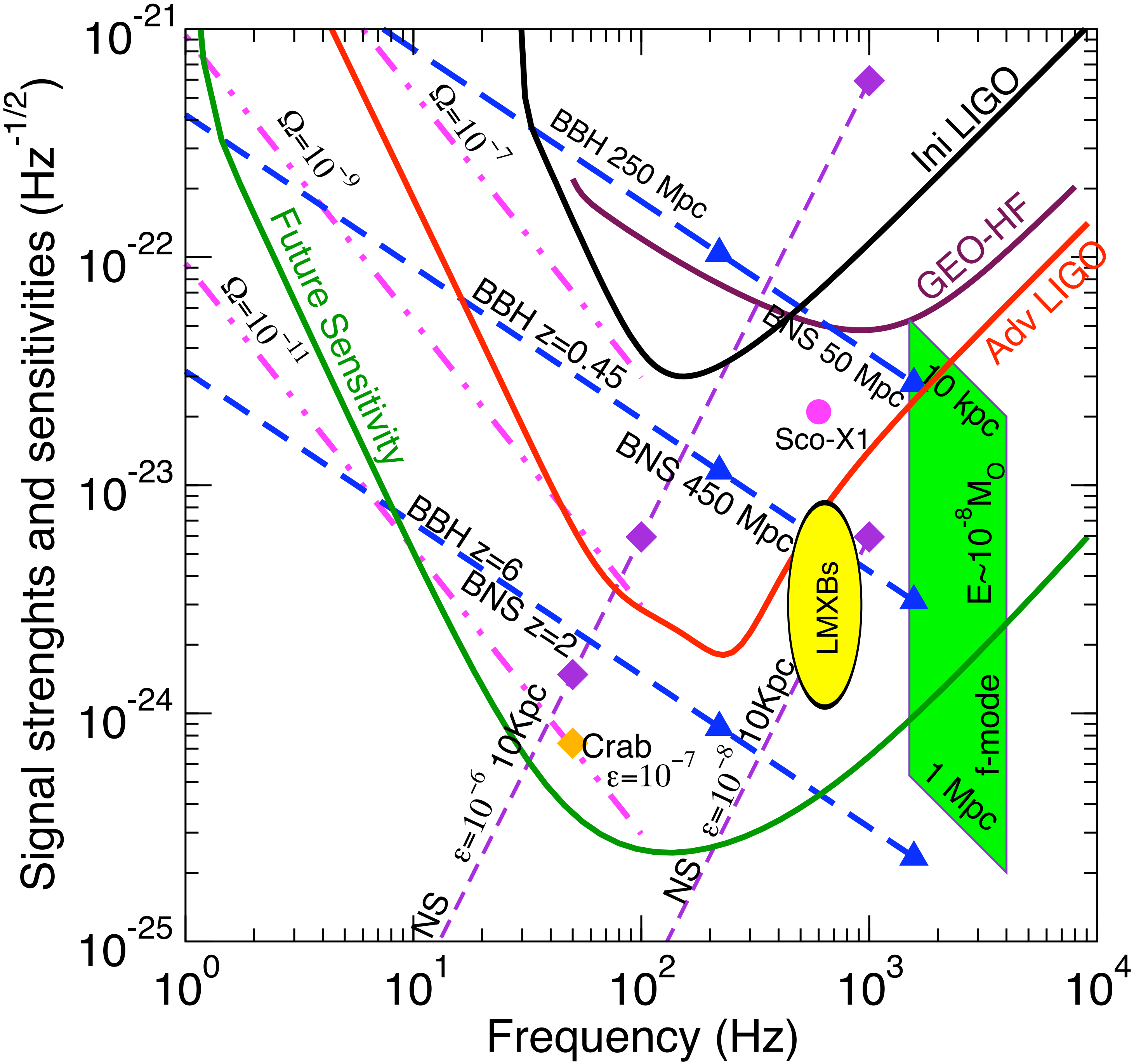
Einstein Telescope

- ET is a conceptual design study supported, for about 3 years (2008-2011), by the European Commission under the Framework Programme 7
- EU financial support ~ 3M€
- Aim of the project is the delivery of a conceptual design of a 3rd generation GW observatory
- Sensitivity of the apparatus~10 better than advanced detectors


EINSTEIN TELESCOPE
gravitational wave observatory
CENTRAL FACILITY
COMPUTING CENTRE
DETECTOR STATION
END STATION
Length ~10 km
TUNNEL ∅ ~5 m
SEVENTH FRAMEWORK PROGRAMME

Expected Future Sensitivities





Signal strenghts and sensitivities (Hz$^{-1/2}$)
Frequency (Hz)
Ini LIGO
GEO-HF
Adv LIGO
Future Sensitivity
BBH 250 Mpc
BBH z=0.45
BBH z=6
BNS 50 Mpc
BNS 450 Mpc
BNS z=2
Ω=10^{-7}
Ω=10^{-9}
Ω=10^{-11}
Sco-X1
LMXBs
10 kpc
E~10^{-8}M$_O$
f-mode
1 Mpc
Crab
ε=10^{-7}
ε=10^{-6} 10Kpc
ε=10^{-8} 10Kpc
NS
NS

LISA

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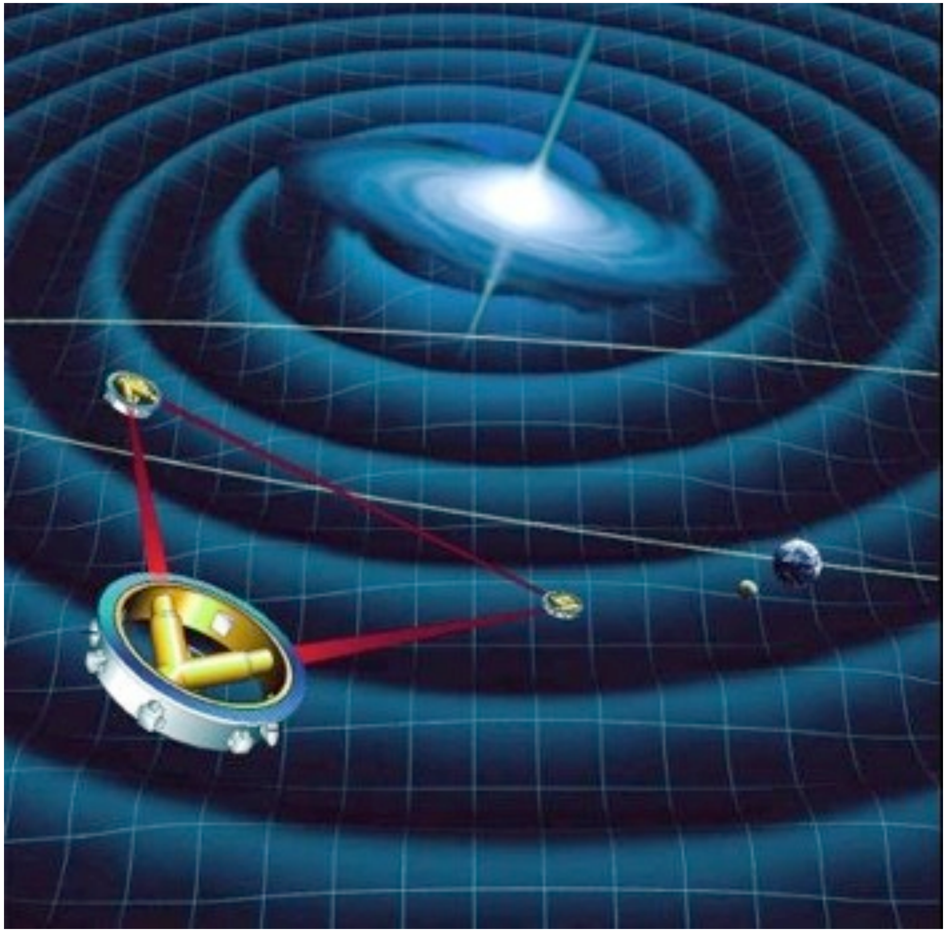
Laser Interferometer Space Antenna



- ESA-NASA collaboration
  - Intended for launch in 2020
- 3 space craft, 5 million km apart, in heliocentric orbit
- Test masses are passive mirrors shielded from solar radiation
- Crafts orbit out of the ecliptic always retaining their formation

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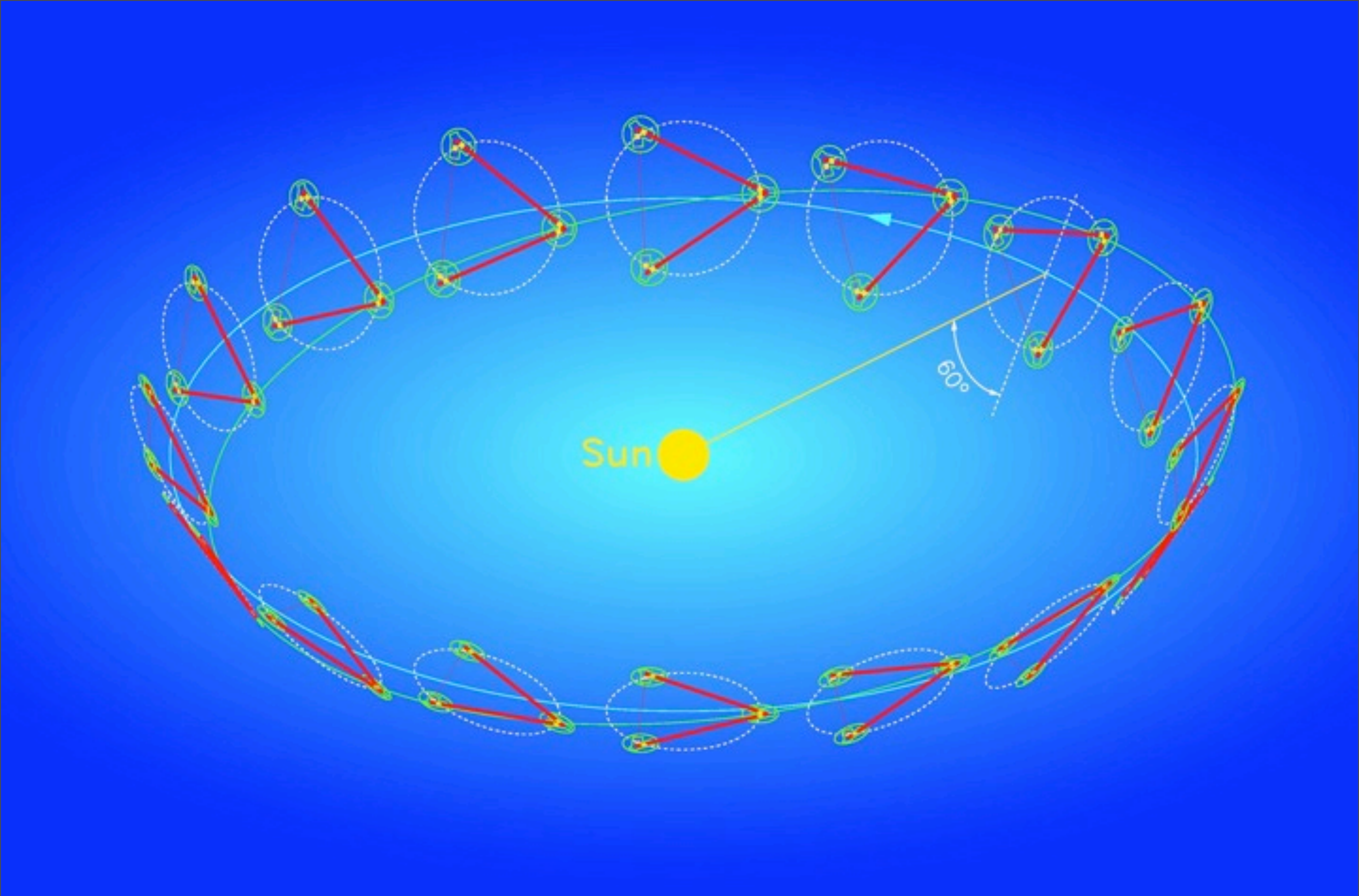

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Beyond LISA

- Big bang observer (NASA)
- DECIGO (Deci-hertz Gravitational Observatory)
  - Both detectors will operate in the 0.1-10 Hz band not covered by LISA or ground-based detectors
- Concepts under study for cosmography and to measure primordial background at the level of $\Omega_{GW} \sim 10^{-15}$

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