

Gravitational Astronomy

Are we there yet?

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What are Gravitational Waves?

- Newton's gravity comes from **Poisson equation**:

$$\nabla^2 \Phi(t, \mathbf{X}) = 4\pi G \rho(t, \mathbf{X})$$

- In general relativity for weak gravitational fields, i.e.

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}, \quad |h_{\alpha\beta}| \ll 1$$

- Einstein's equations reduce to **wave equations**:

$$\square h_{\alpha\beta} = 8\pi G T_{\alpha\beta}$$

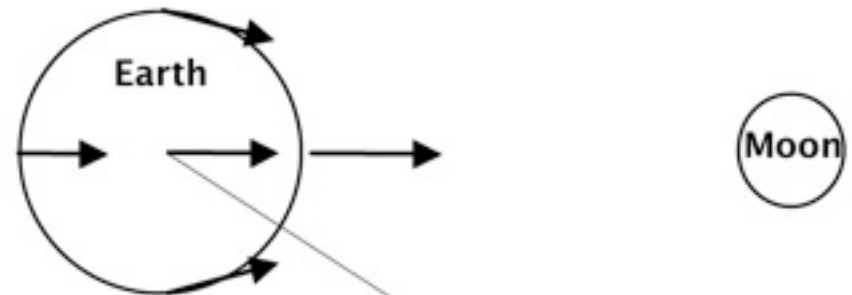
- **Non-axisymmetric motion** of mass-energy generates GW

- GWs are **ripples in the curvature of space-time**

Tidal Gravitational Forces of GW

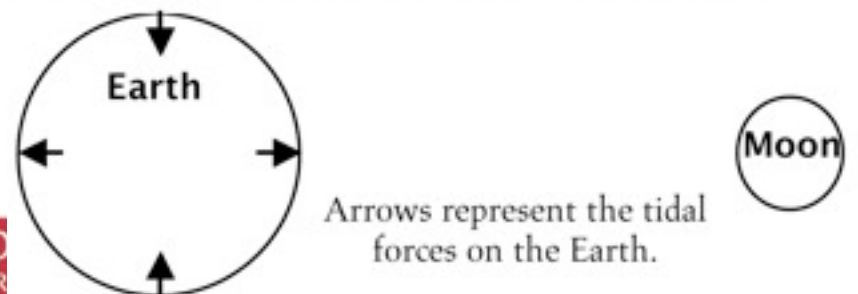
- Gravitational effect of a distant source can only be felt through its **tidal forces**
- Gravitational waves are traveling, **time-dependent tidal forces**.
- Tidal forces scale with size, typically produce elliptical deformations.

Acceleration of the Moon's gravity on Earth.
Length of arrow indicates size of acceleration.



The acceleration at the **center** is the mean acceleration with which the solid Earth will fall. The acceleration of gravity due to the Moon is larger near the Moon and smaller further away.

Residual acceleration of the Moon's gravity, after subtracting the mean acceleration of the Earth.



Arrows represent the tidal forces on the Earth.

Gravitational Wave Observables

- GW Luminosity $\mathcal{L} = (\text{Asymmetry factor}) v^{10}$
 - A strong function of velocity: During merger, a binary black hole in gravitational waves outshines the entire Universe in light
- GW Amplitude of a source of size r at a distance R
$$h = (\text{Asymmetry factor}) (M/R) (M/r)$$
 - Amplitude gives strain in space $h = \Delta L/L$
- GW frequency is the dynamical frequency $f \sim \sqrt{\rho}$
 - For binaries dominant the gravitational-wave frequency is twice the orbital frequency
- GW Polarization
 - In Einstein's theory two polarizations - plus and cross

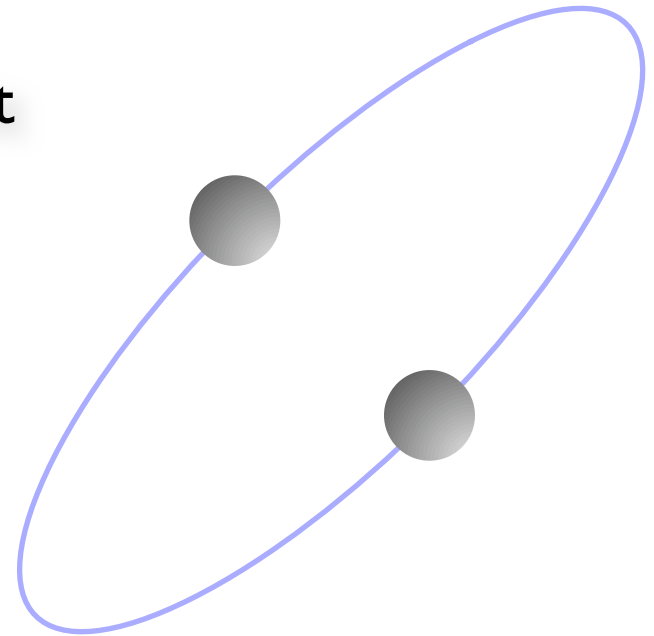
Gravitational Vs EM Waves

- Production: electronic transitions in atoms and accelerated charges – physics of small things
 - Incoherent superposition of many, many waves
 - Detectors sensitive to the intensity of radiation
 - Directional telescopes
- Production: coherent motion stellar and super-massive black holes, supernovae, big bang, ...
 - Often, a single coherent wave, but stochastic background expected
 - GW detectors are sensitive to the amplitude of the radiation
 - Sensitive to wide areas over the sky

A persistent source of GW

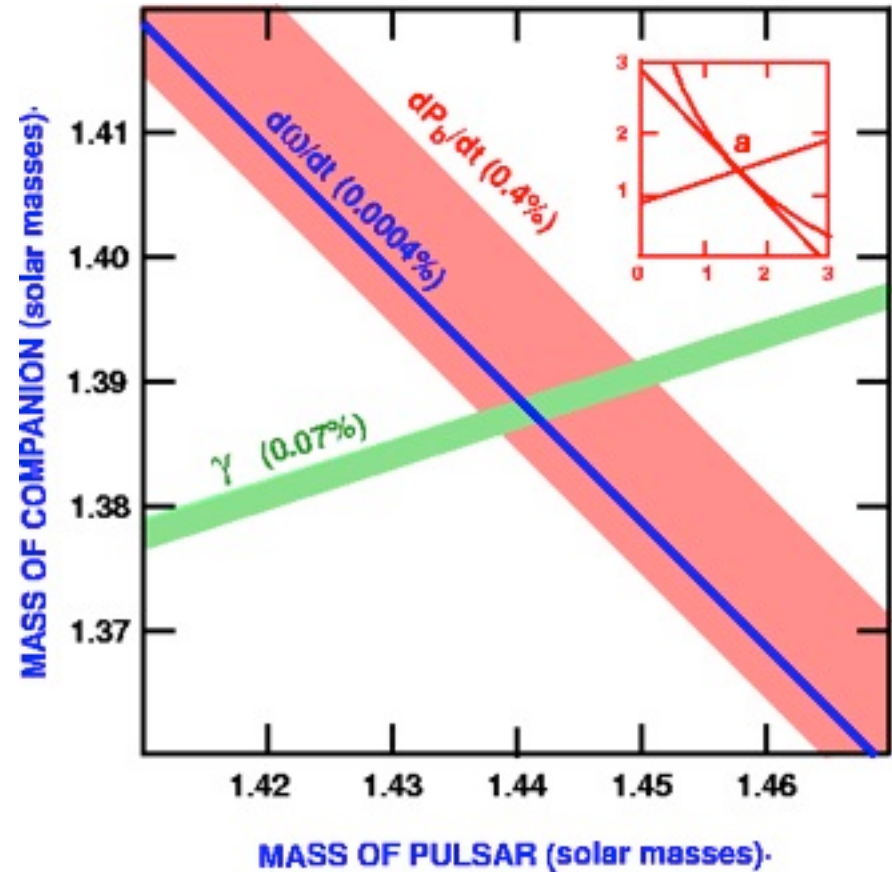
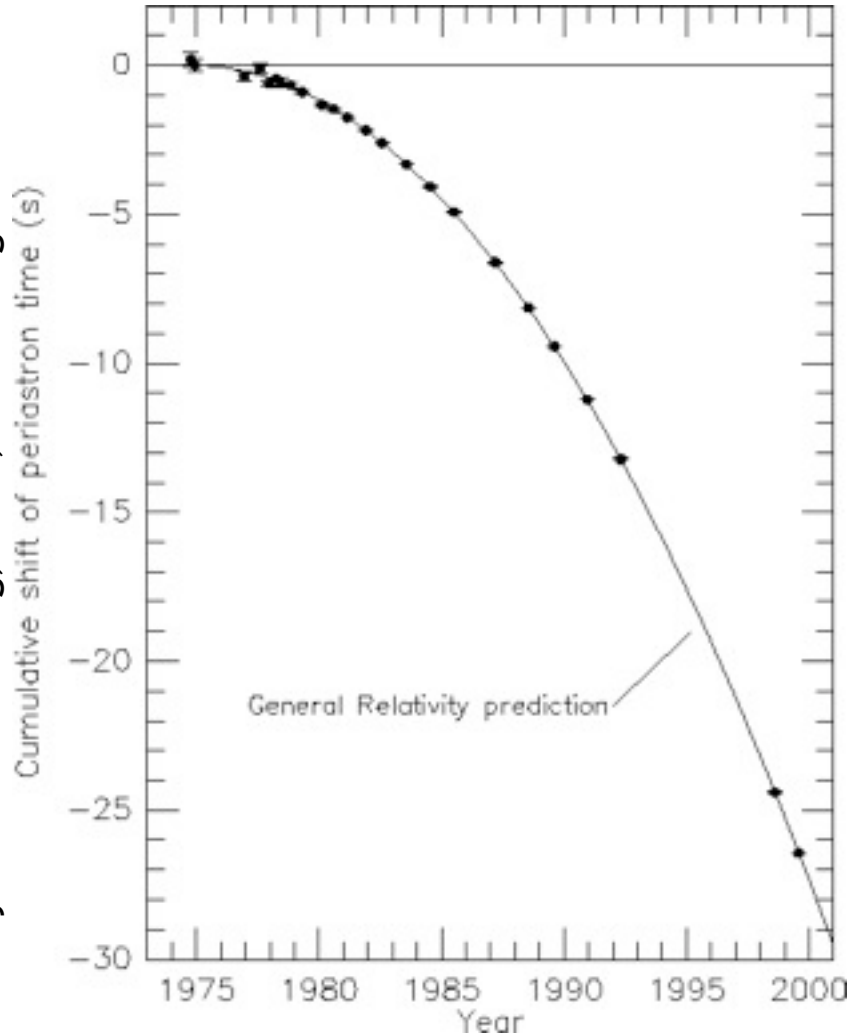
PSR 1913+16

- In 1974 Hulse and Taylor observed the first binary pulsar
- Two neutron stars in relativistic orbit
 - Masses, each $\sim 1.4 M_{\odot}$
 - Period ~ 7.5 Hrs
 - Eccentricity ~ 0.62
- Einstein's theory predicts the binary should emit gravitational radiation
- The stars spiral in toward each other, causing a decrease in the period
- Observed decrease in period - about 10 micro seconds per year - is in agreement with Einstein's theory to fraction of a percent



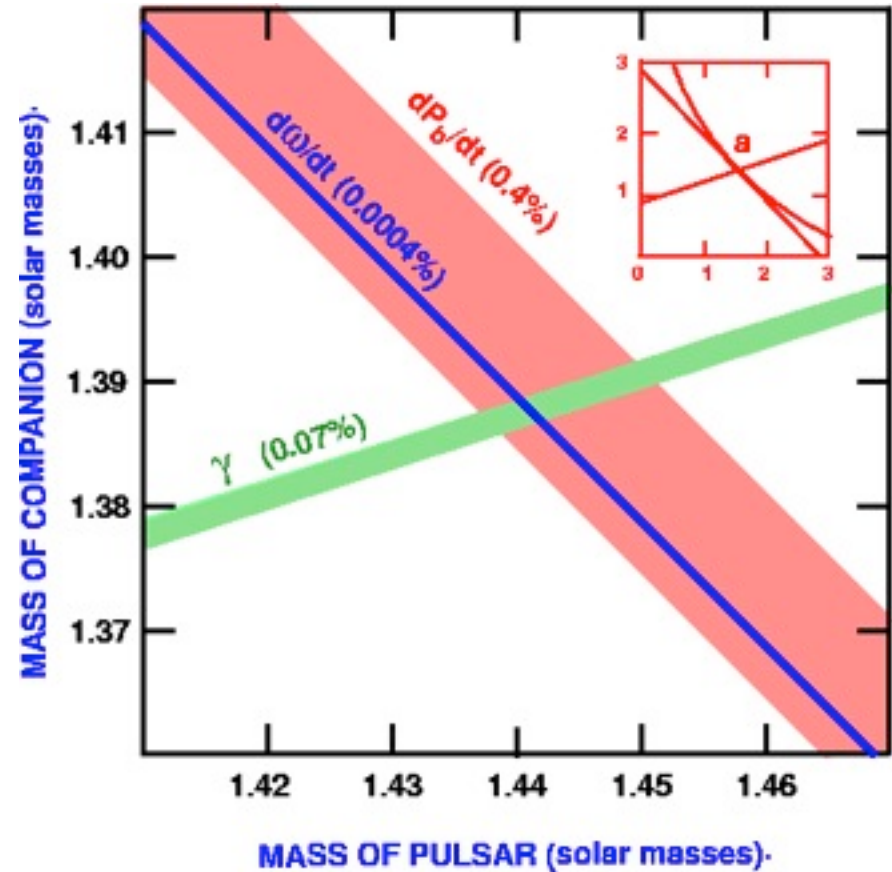
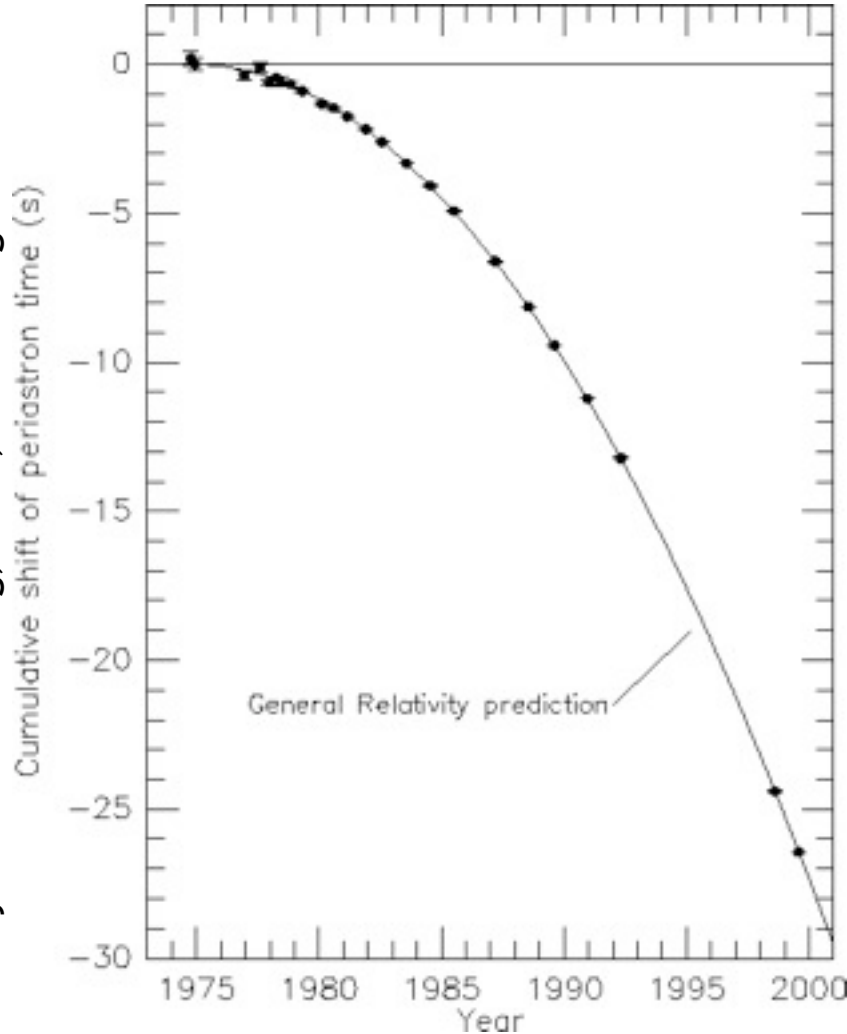
Accumulated orbital phase shift in PSR 1913+16

Taylor and Weisberg, 2000, Will Living Review



Accumulated orbital phase shift in PSR 1913+16

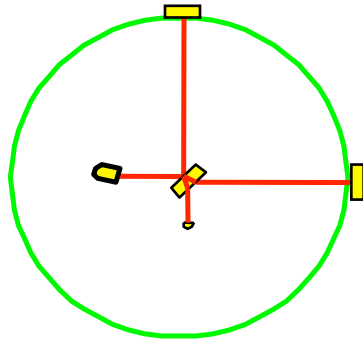
Taylor and Weisberg, 2000, Will Living Review



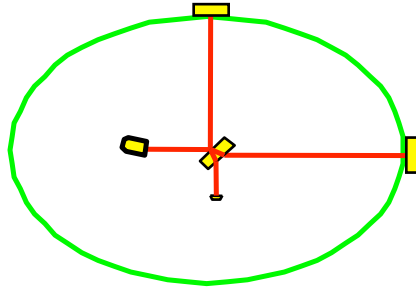
Eventually the two stars will coalesce, but that will take another 100 million years

Gravitational Wave Detectors - Now and in the Future

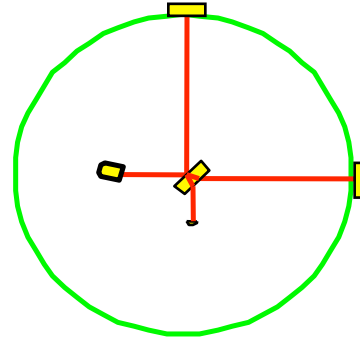
Interferometric gravitational-wave detectors



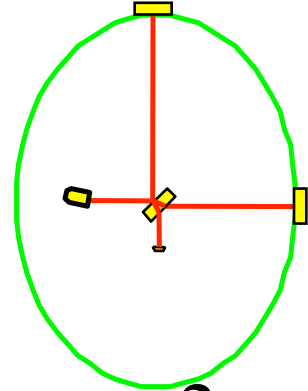
$$t = 0$$



$$t = \frac{\tau}{4}$$



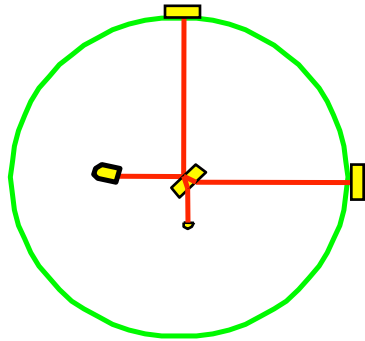
$$t = \frac{\tau}{2}$$



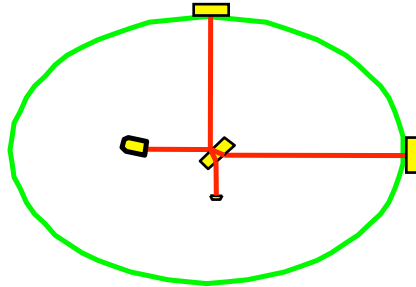
$$t = \frac{3\tau}{4}$$

$$\delta l = \frac{h}{2} l$$

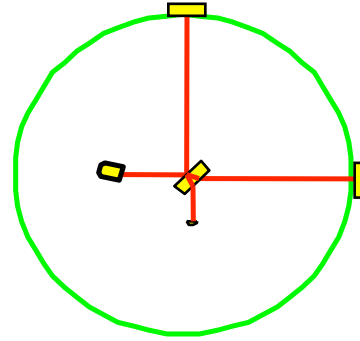
Interferometric gravitational-wave detectors



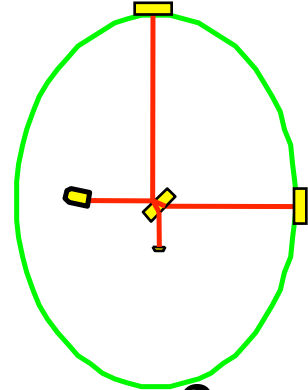
$$t = 0$$



$$t = \frac{\tau}{4}$$



$$t = \frac{\tau}{2}$$



$$t = \frac{3\tau}{4}$$

$$\delta l = \frac{h}{2} l$$

For Typical Astronomical sources

$$h = \frac{2\delta l}{l} \leq 10^{-22}$$

American Laser Interferometer Gravitational-Wave Observatory (LIGO) at Hanford



Monday, 7 March 2011

LIGO at Livingstone, Louisiana

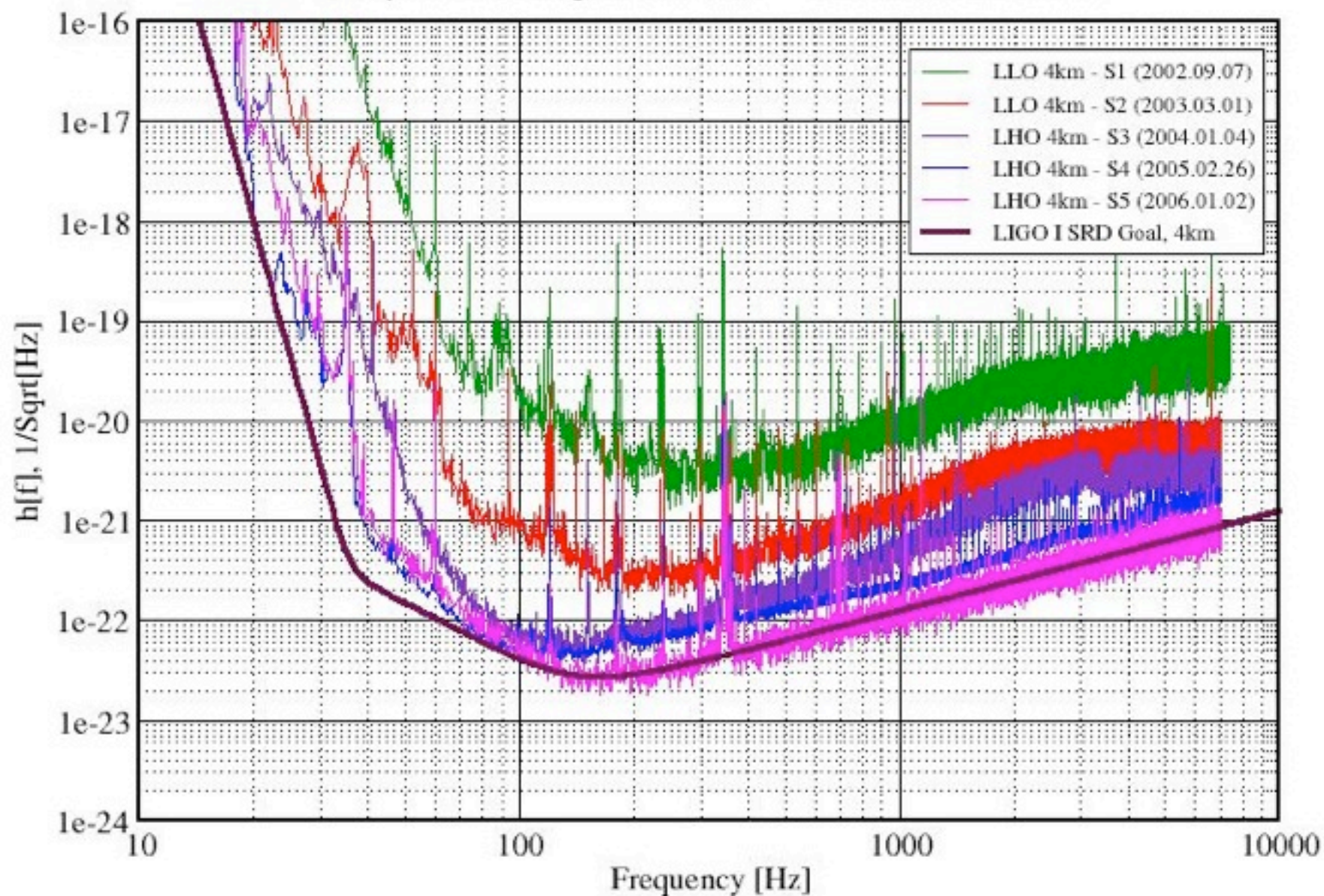


Monday, 7 March 2011

Best Strain Sensivities for the LIGO Interferometers

Comparisons among S1 - S5 Runs

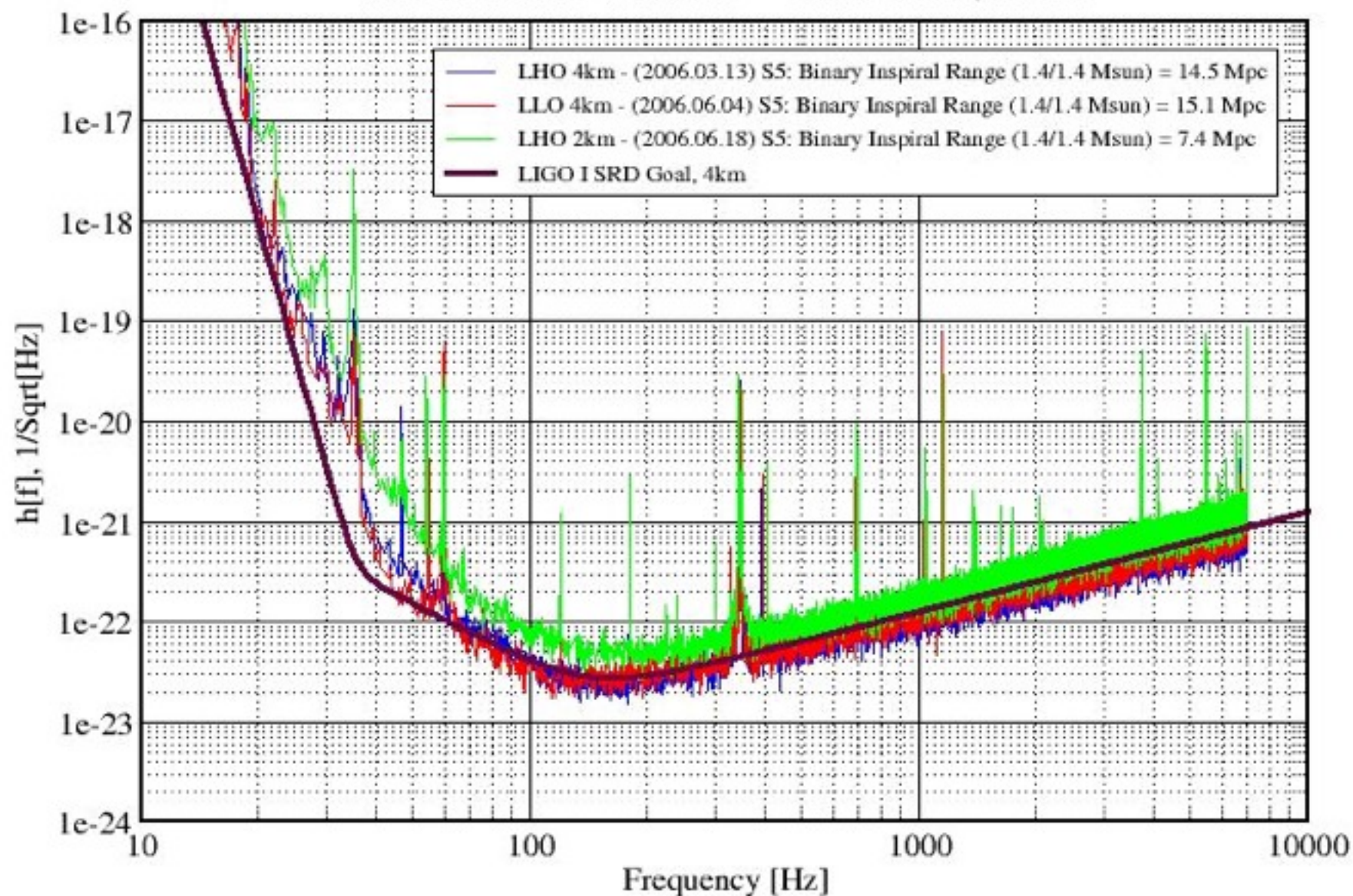
LIGO-G060009-01-Z



Strain Sensitivity for the LIGO 4km Interferometers

S5 Performance - June 2006

LIGO-G060293-01-Z



German-British GEO600, Hanover, Germany



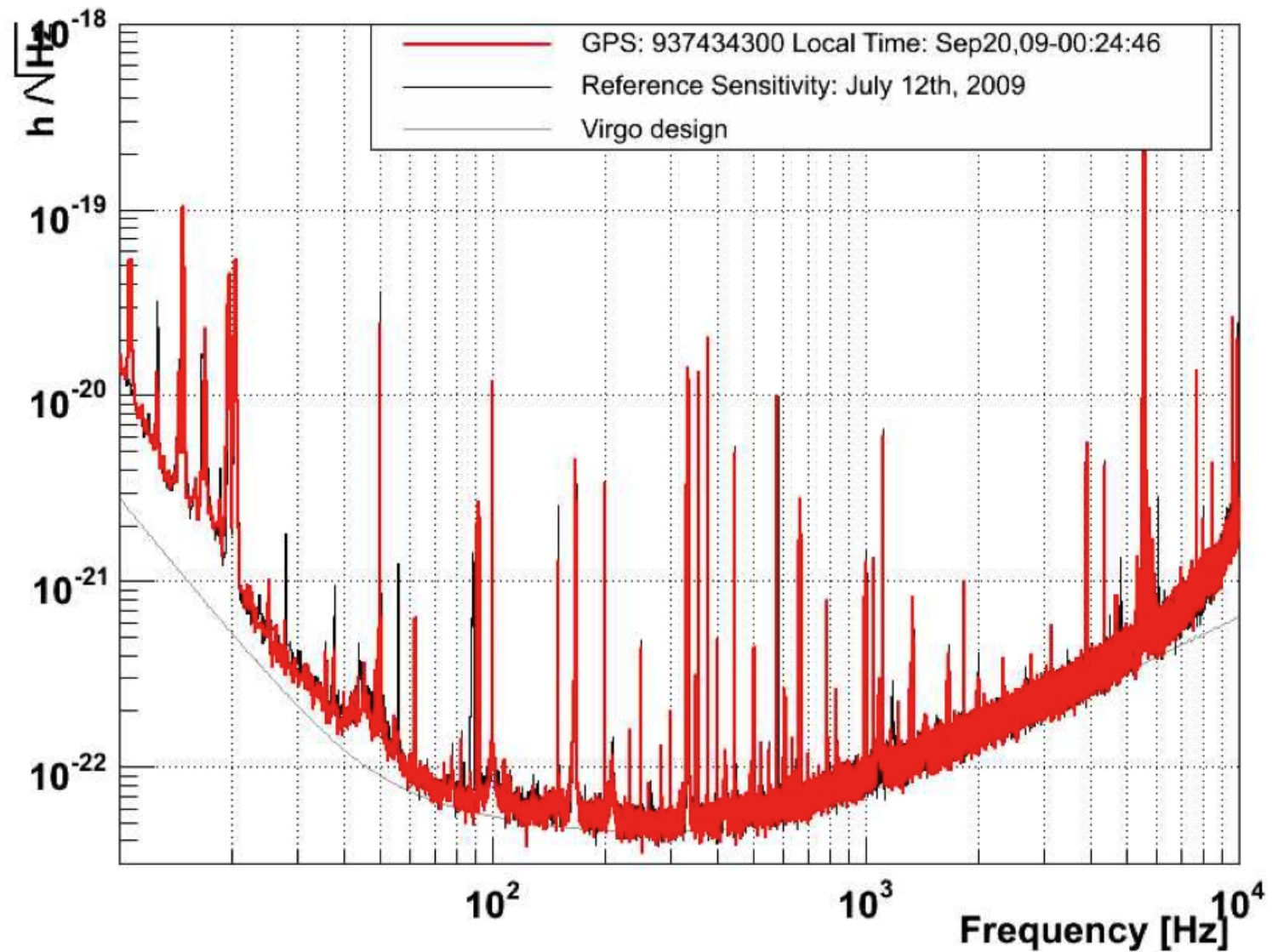
Monday, 7 March 2011

French Italian VIRGO near PISA



Monday, 7 March 2011

Virgo Science Run-2



G070221-00-Z

Monday, 7 March 2011

Future Improvements

- Enhanced Detectors (2009-11)
 - 2 x increase in sensitivity
 - 8 x increase in rate
- Advanced Detectors, LIGO and Virgo (2015- ...)
 - 12 x increase in sensitivity
 - Over 1000 x increase in rate
- 3G Detectors: **Einstein Telescope** (2025-)
 - 100 x increase in sensitivity
 - 10^6 increase in rate

Einstein Telescope



- ET is a conceptual design study supported, for about 3 years (2008-2011), by the European Commission under the Framework Programme 7
- Aim of the project is the delivery of a conceptual design of a 3rd generation GW observatory
- Sensitivity of the detector ~ 10 better than advanced detectors

EINSTEIN TELESCOPE

gravitational wave observatory

CENTRAL FACILITY

COMPUTING CENTRE

DETECTOR STATION

END STATION

Length ~10 km

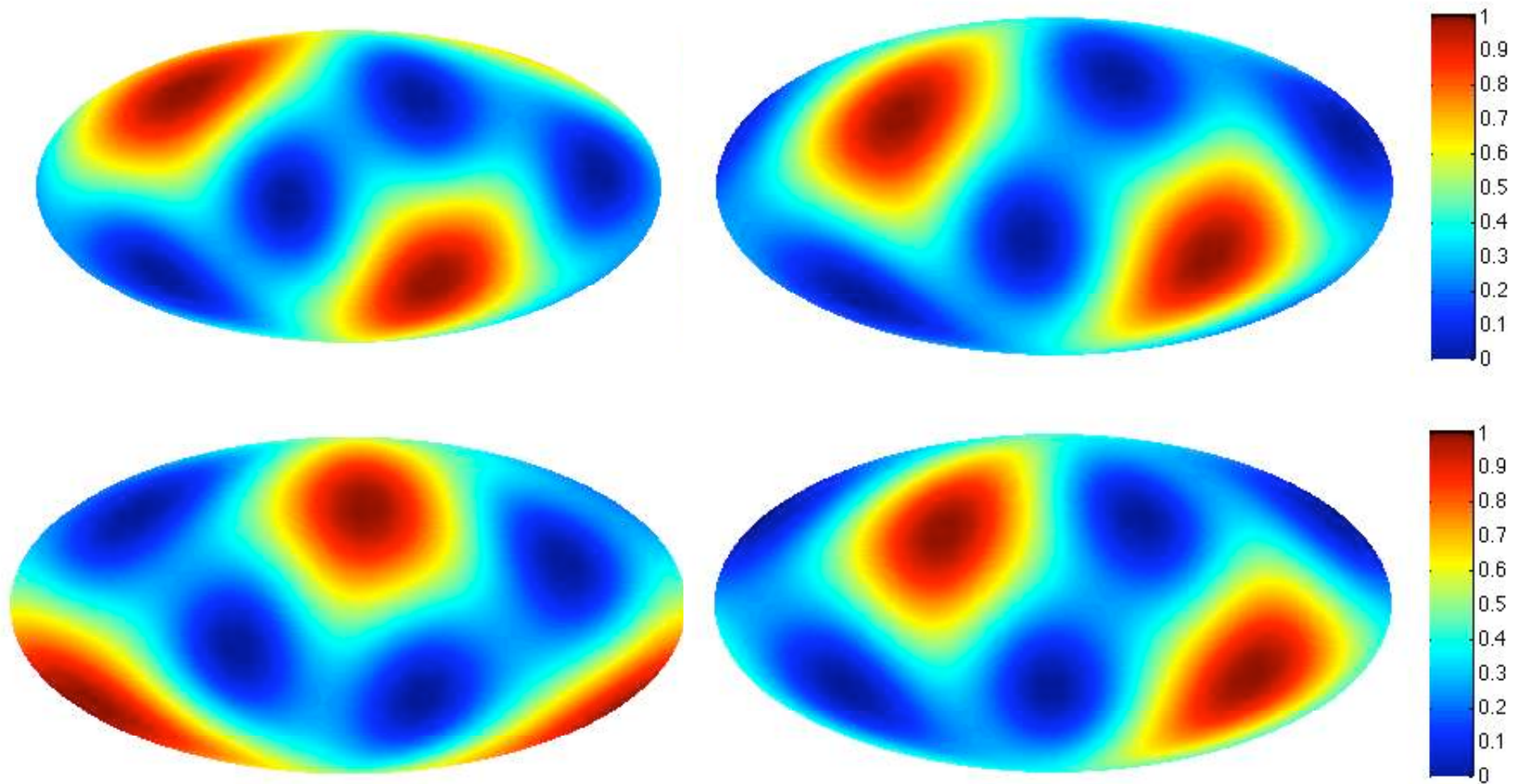
TUNNEL \varnothing ~5 m



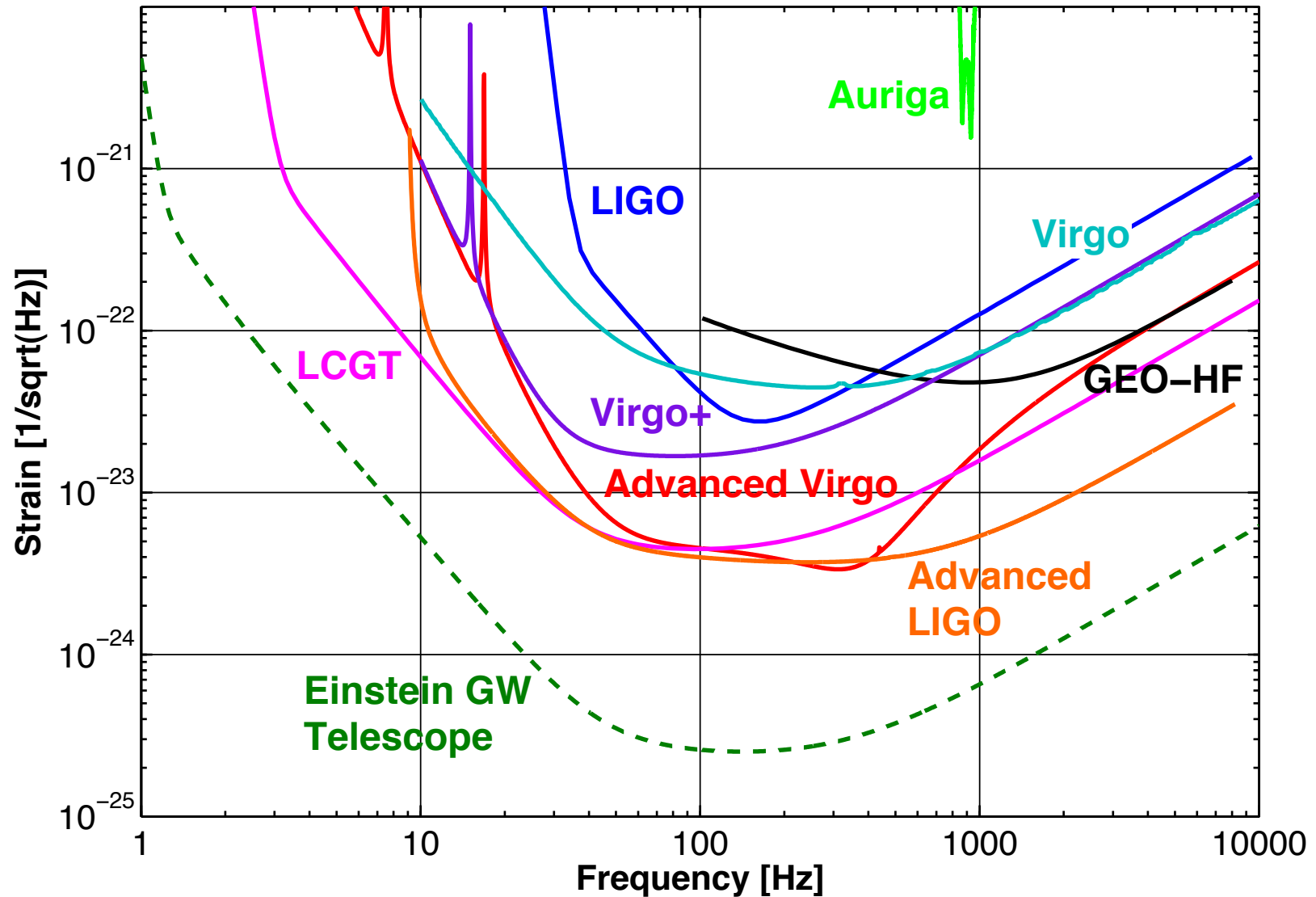
Why a global network

- Improved sky coverage
 - Non-overlapping antenna pattern => sky coverage
- Improved angular resolution/localization
 - Longer baselines lead to greater time-delays and therefore improved angular resolution
- Improved distance reach
 - For detectors in overlapping antenna patterns, great improvement in distance reach
- Improved 3-way duty cycle
 - If each detector has 80% duty cycle
 - With three detectors, 3-way duty cycle is about 50%
 - With four detectors, \geq 3-way duty cycle improves to 82%
 - With five detectors, \geq 3-way duty cycle improves to 94%

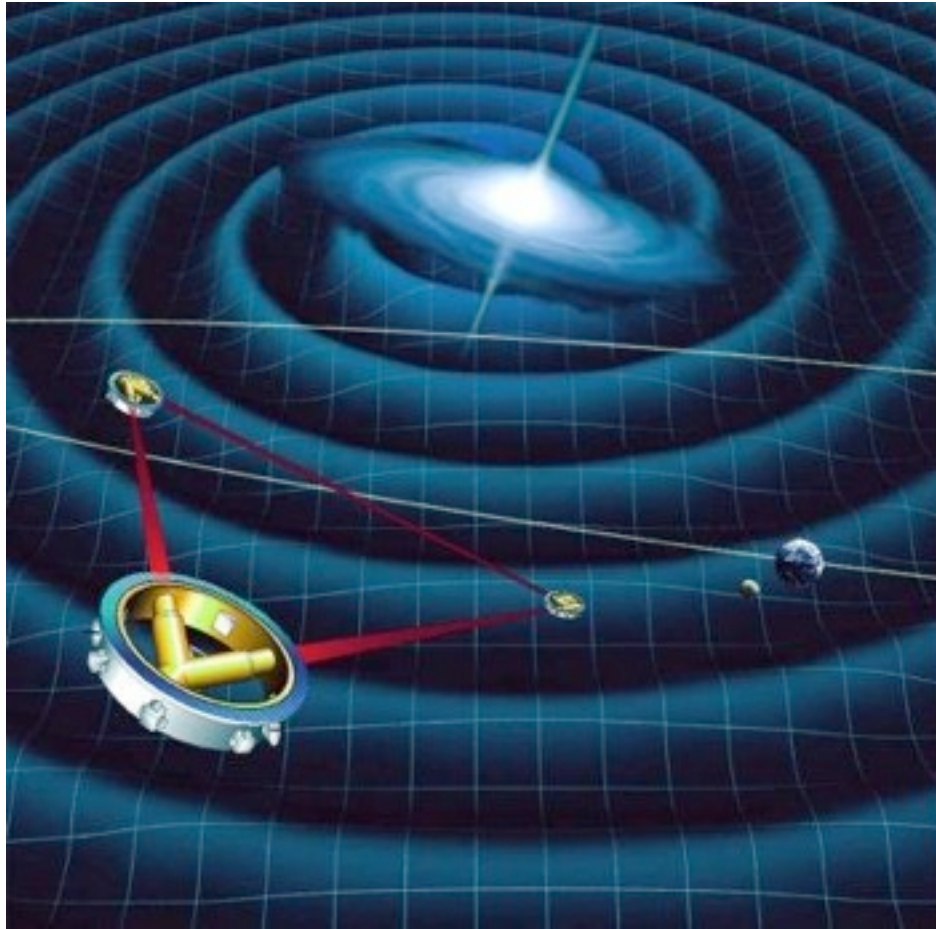
Antenna Patterns of Hanford, Livingston, Virgo and Gingin detectors



Expected Future Sensitivities



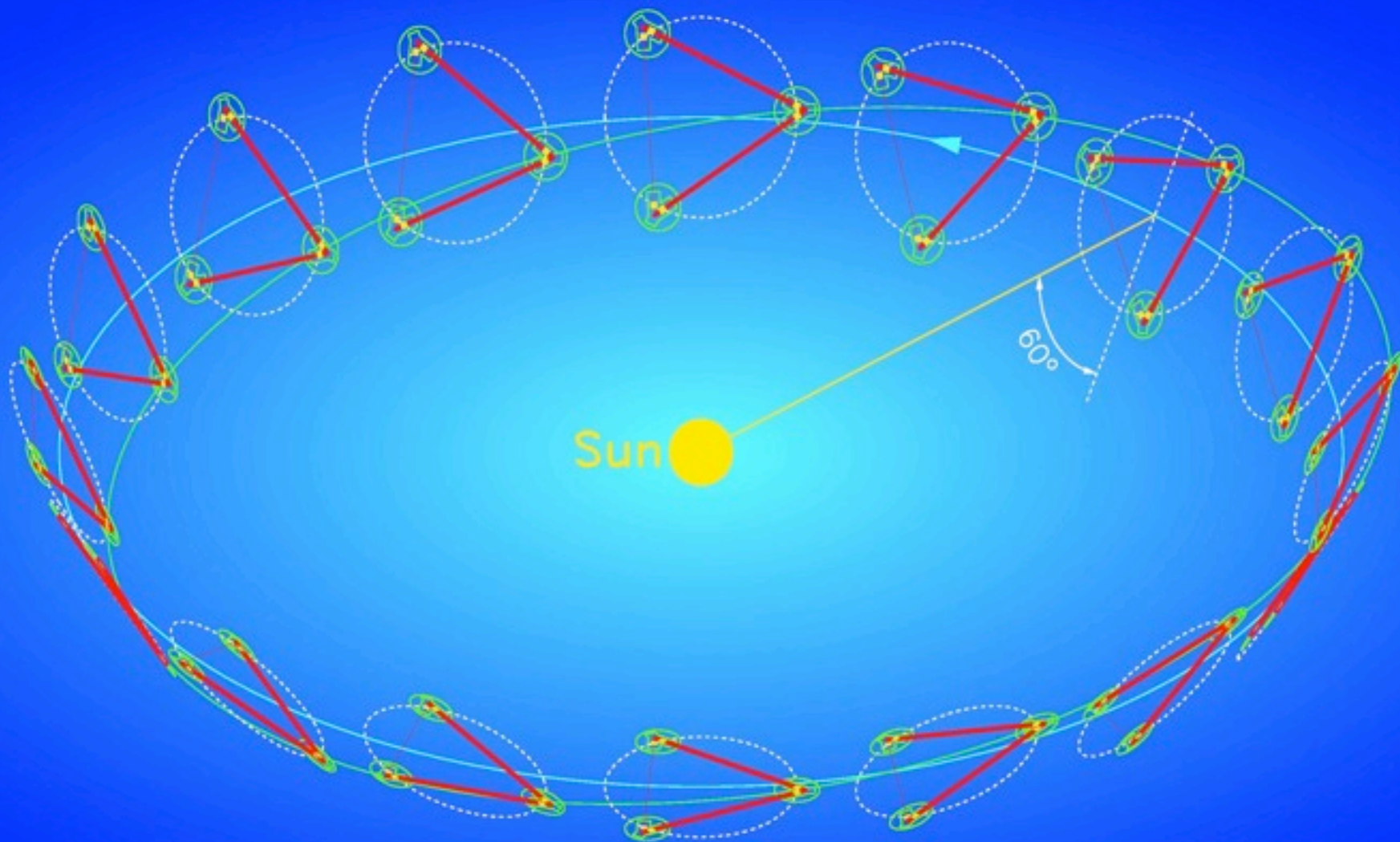
Laser Interferometer Space Antenna



- ESA-NASA collaboration
 - [Intended for launch in 2020](#)
- 3 space craft, 5 million km apart, in heliocentric orbit
- Test masses are passive mirrors shielded from solar radiation
- Crafts orbit out of the ecliptic always retaining their formation

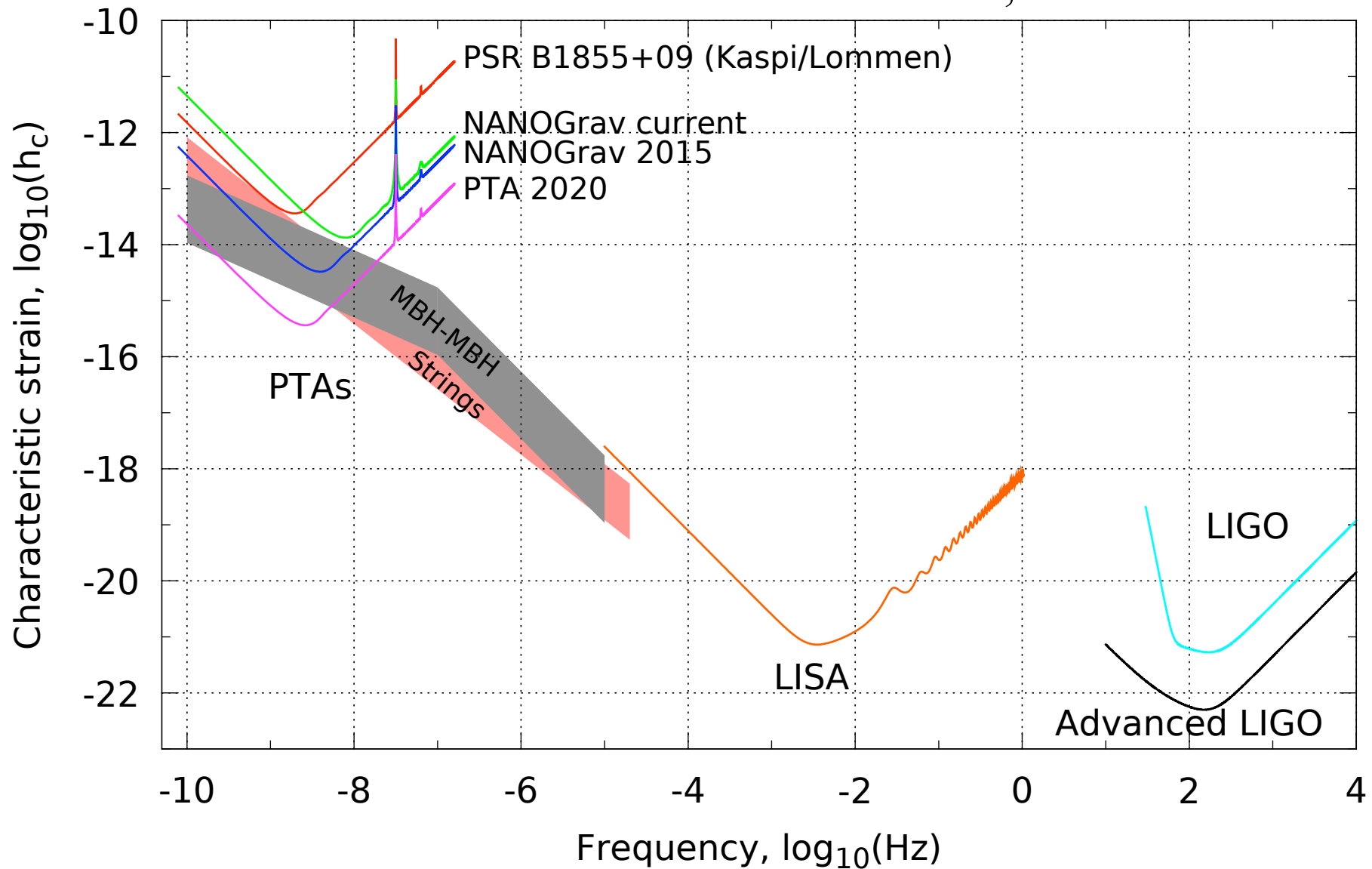


Monday, 7 March 2011



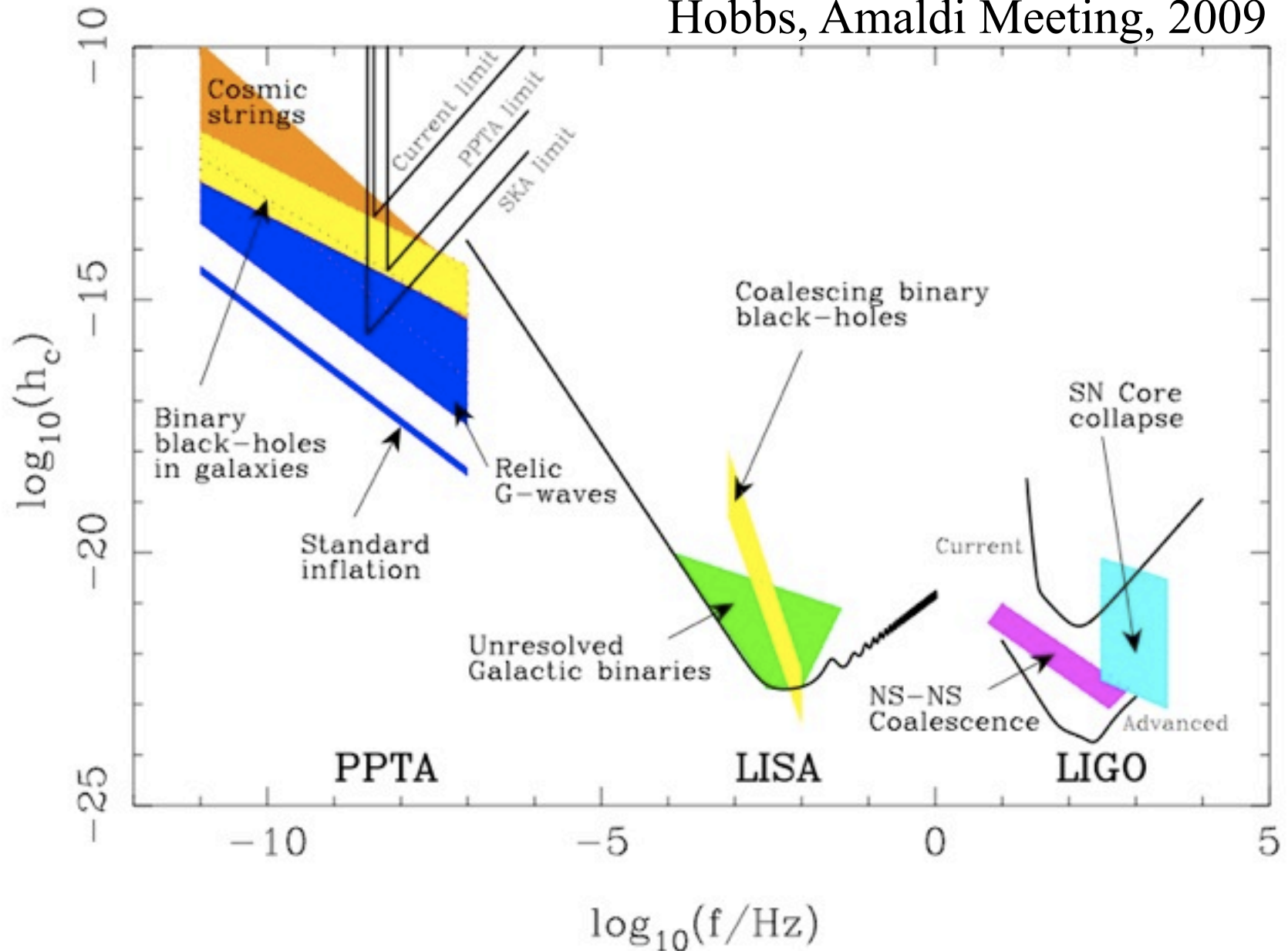
Pulsar Timing Arrays

Demorest, arXiv:0902.2968



Pulsar Timing Arrays and SKA

Hobbs, Amaldi Meeting, 2009

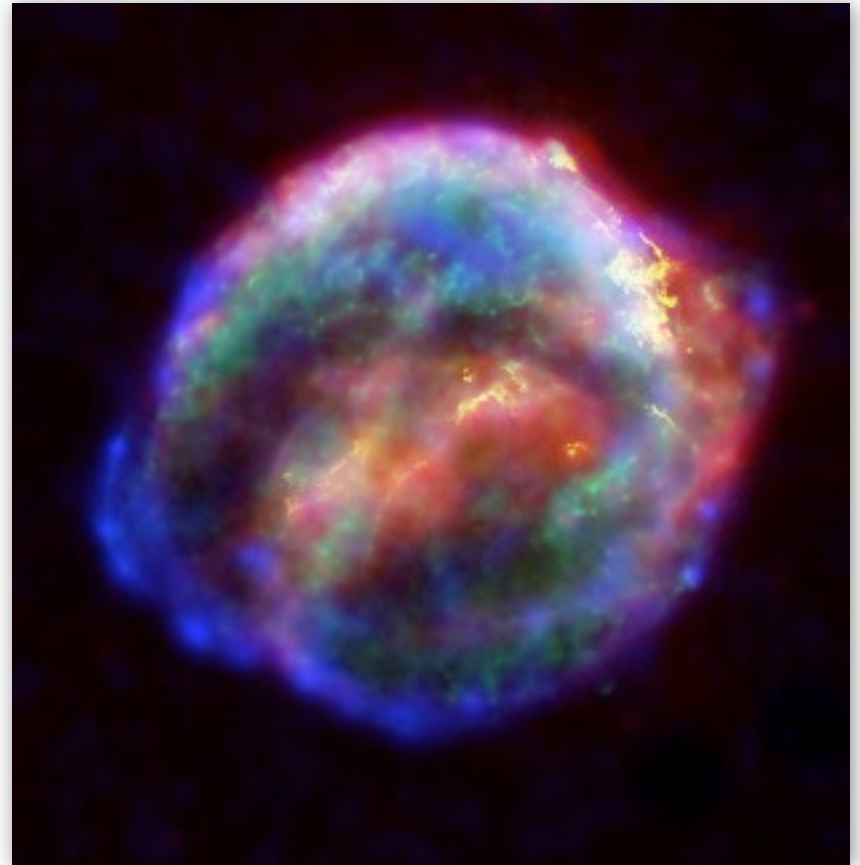


Sources of Gravitational Waves



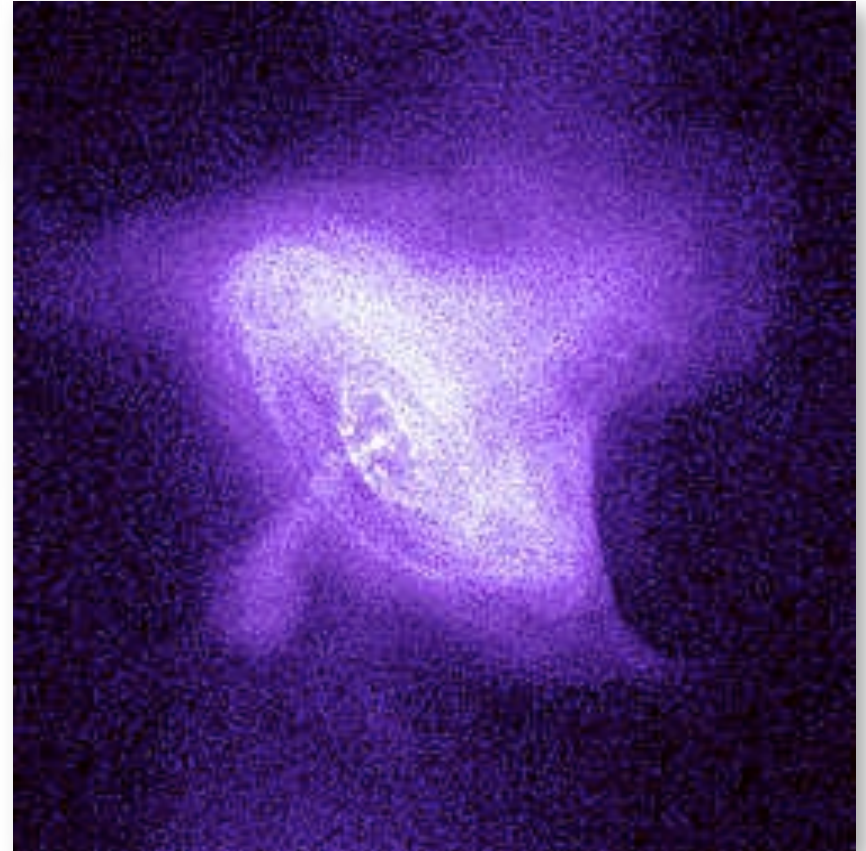
Burst Sources

- Gravitational wave bursts
 - Black hole collisions
 - Supernovae
 - gamma-ray bursts (GRBs)
- Short-hard GRBs
 - could be the result of merger of a neutron star with another NS or a BH
- Long-hard GRBs
 - could be triggered by supernovae



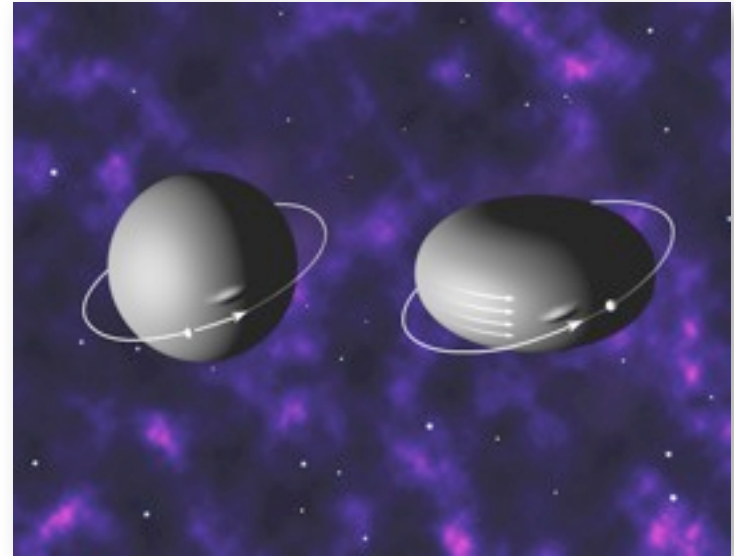
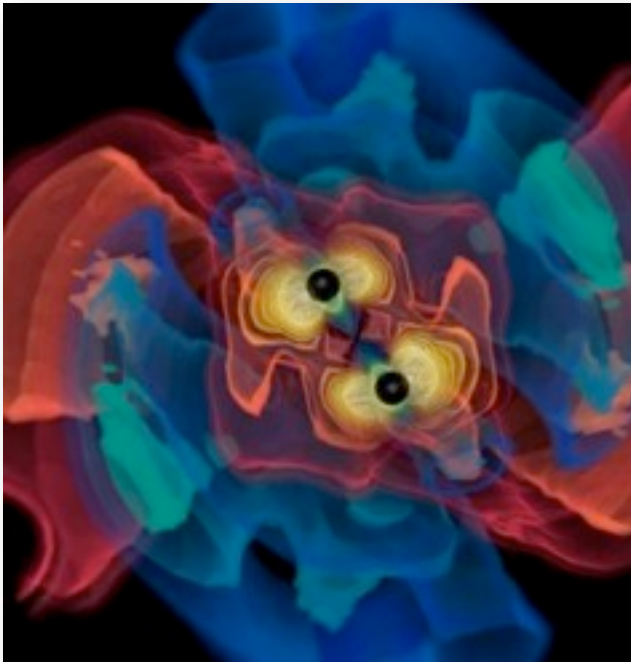
Continuous Wave Sources

- Rapidly spinning neutron stars or other objects
 - Mountains on neutron stars
- Low mass X-ray binaries
 - Accretion induced asymmetry
- Magnetars and other compact objects
 - Magnetic field induced asymmetries
- Relativistic instabilities
 - r-modes, etc.



Compact Binary Mergers

- Binary neutron stars
- Binary black holes
- Neutron star–black hole binaries

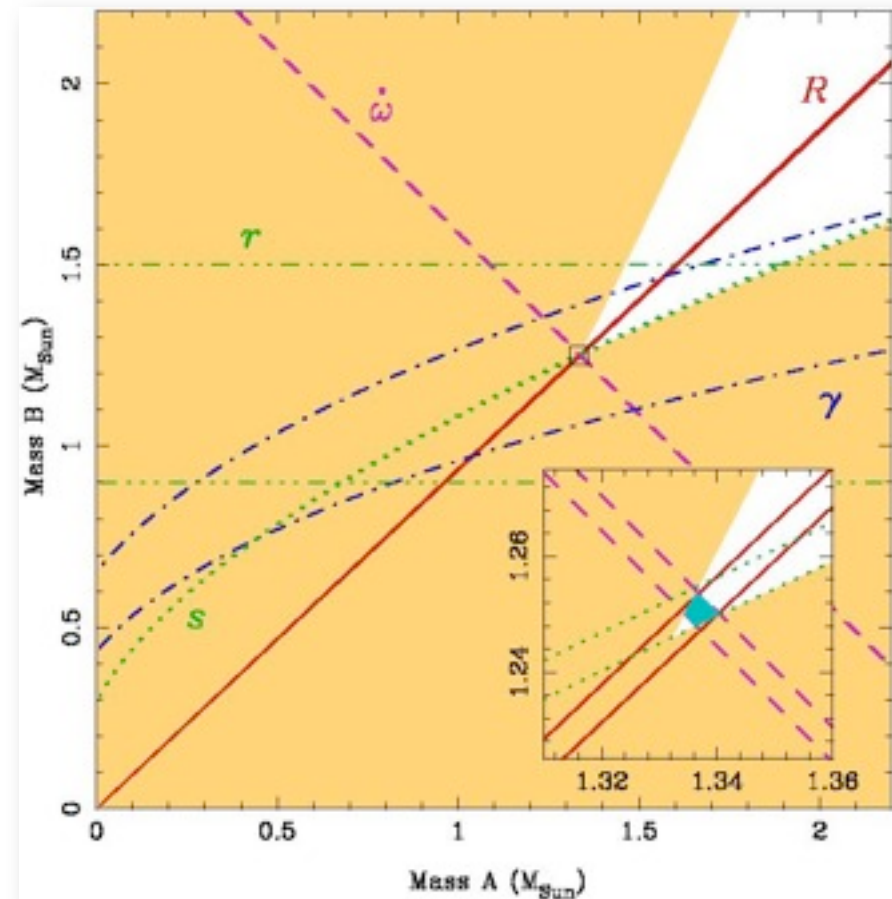


- Loss of energy leads to steady inspiral whose waveform has been calculated to order v^7 in post-Newtonian theory
- Knowledge of the waveforms allows matched filtering

Examples of Merging Neutron Star Binaries

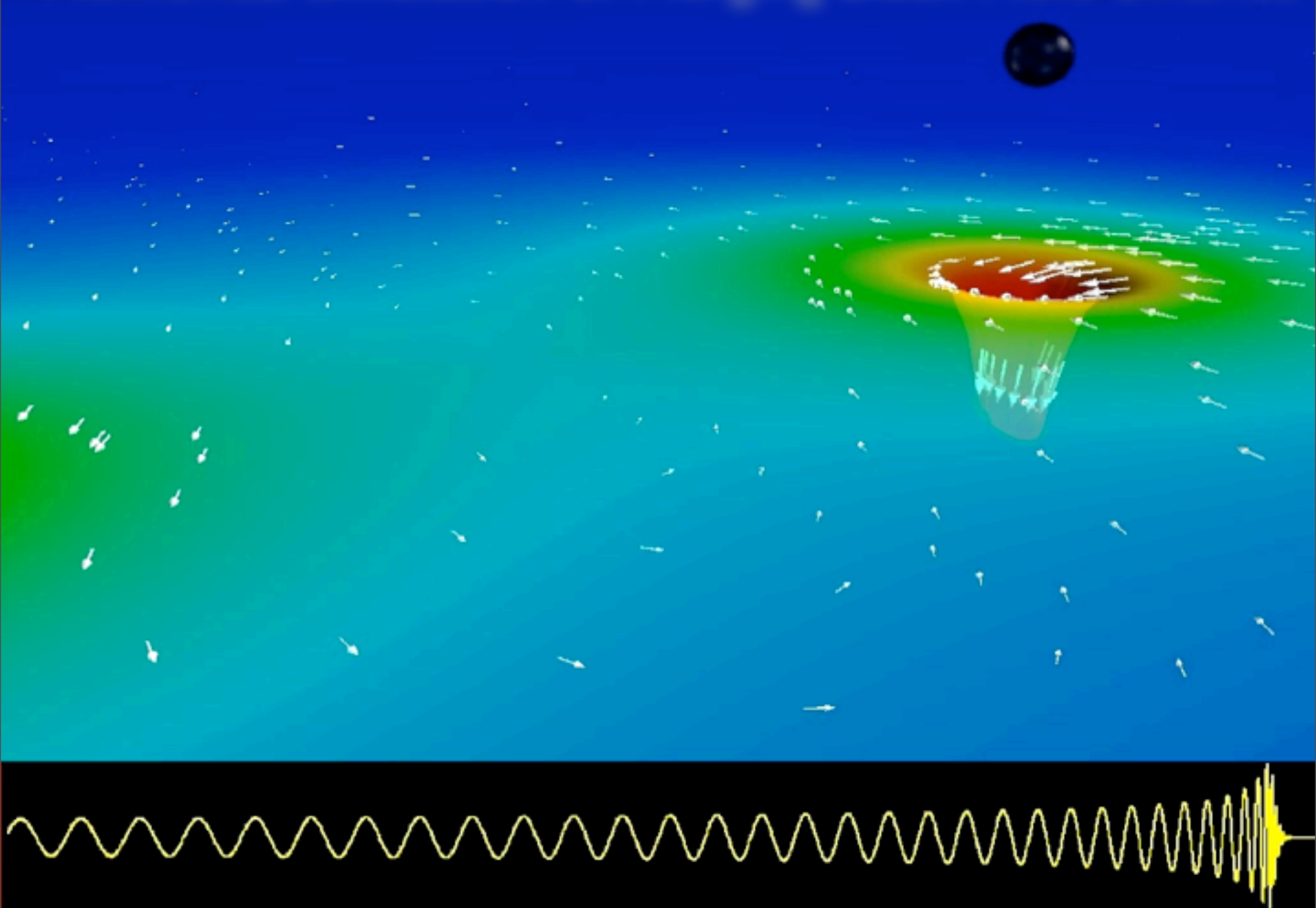
Burgay et al Nature 2003

- PSR 1913+16, J0737-3039
- J0737-3039 - the fastest
 - Strongly relativistic, $P_b = 2.5$ Hrs
 - Mildly eccentric, $e = 0.088$
 - Highly inclined ($i > 87$ deg)
- The most relativistic
 - Greatest periastron advance:
 $d\omega/dt$: 16.8 degrees per year
(almost entirely general relativistic effect), compared to relativistic part of Mercury's perihelion advance of 42 sec per century
 - Orbit is shrinking by a few millimeters each year due to gravitational radiation reaction



Numerical Simulation of Merging Black Hole Binaries

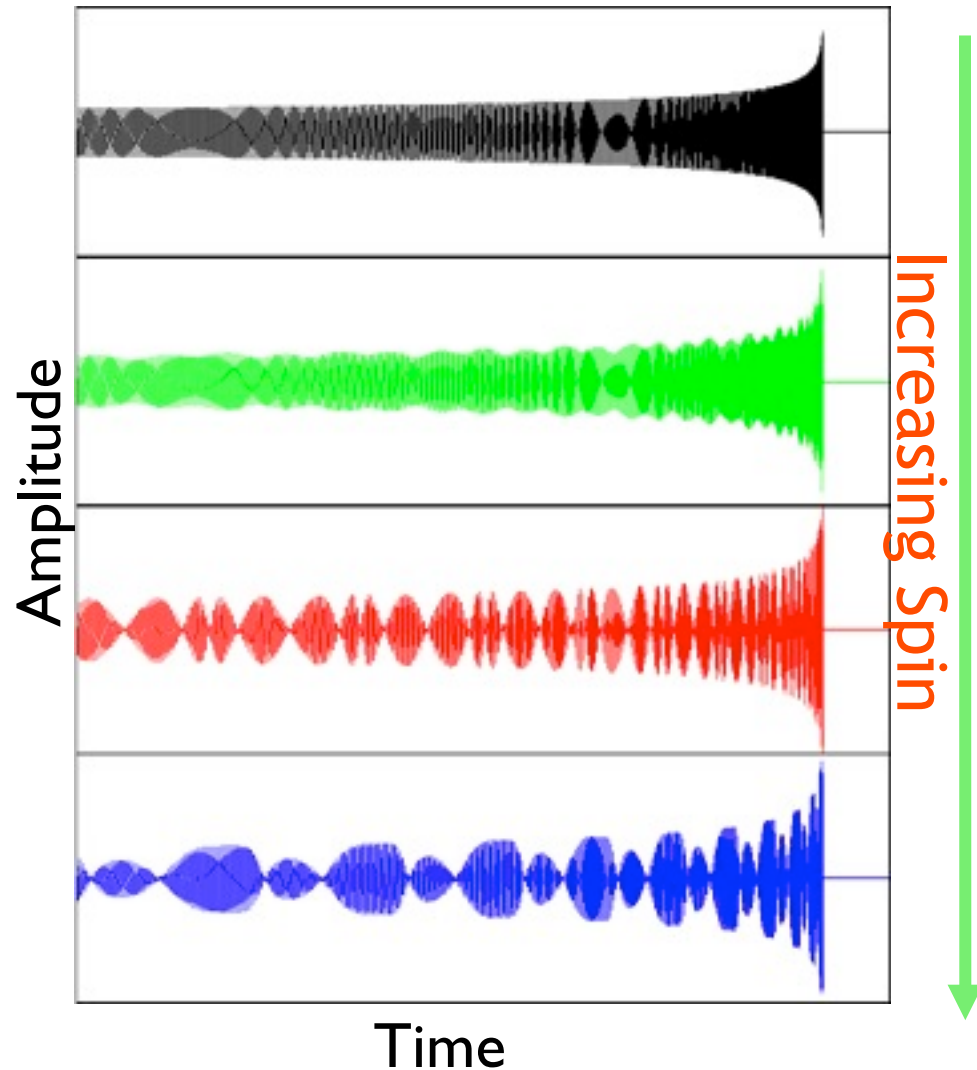
Numerical Simulation of Merging Black Hole Binaries



Waveforms from Inspiralling Binaries

- Late-time dynamics of compact binaries is highly relativistic, dictated by **non-linear general relativistic effects**
- Post-Newtonian theory, which is used to model the evolution, is now **known to $O(v^7)$**
- The shape and strength of the emitted radiation depend on many parameters of the binary: masses, spins, distance, orientation, sky location, ...

$$h(t) = 4\eta \frac{M}{D} \frac{M}{r(t)} \cos 2\varphi(t)$$



Astrophysics

Astrophysics

- Unveiling progenitors of short-hard GRBs
 - Short-hard GRBs believed to be merging NS-NS and NS-BH
- Understanding Supernovae
 - Astrophysics of gravitational collapse and supernova?
- Evolutionary paths of compact binaries
- Finding why pulsars glitch and magnetars flare
 - What causes sudden excursions in pulsar spin frequencies
 - What is behind ultra high-energy transients in magnetars
- Ellipticity of neutron stars
 - Mountains of what size can be supported on neutron stars?
- NS spin frequencies in LMXBs
 - Why are spin frequencies of neutron stars in low-mass X-ray binaries bounded, CFS instability and r-modes

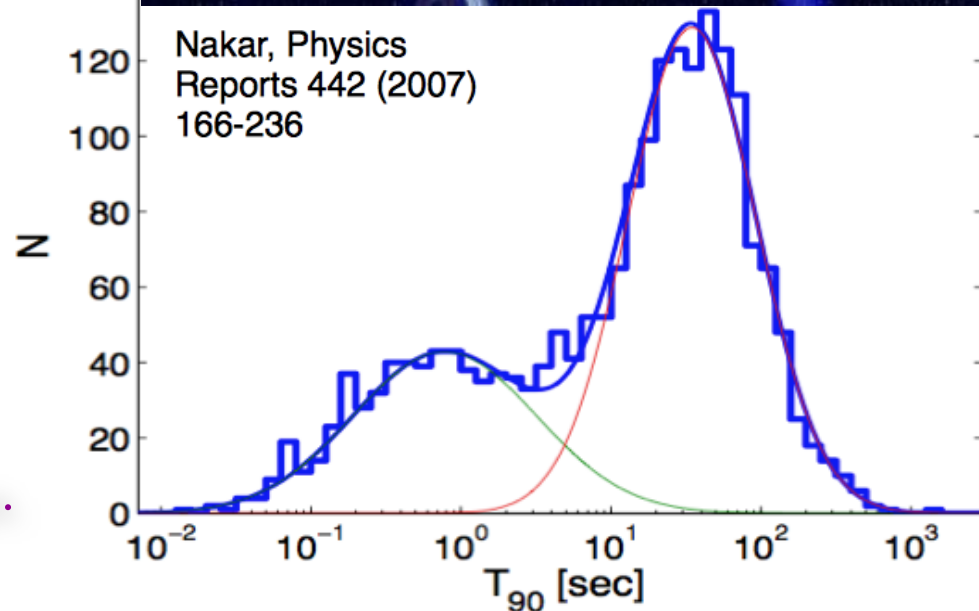
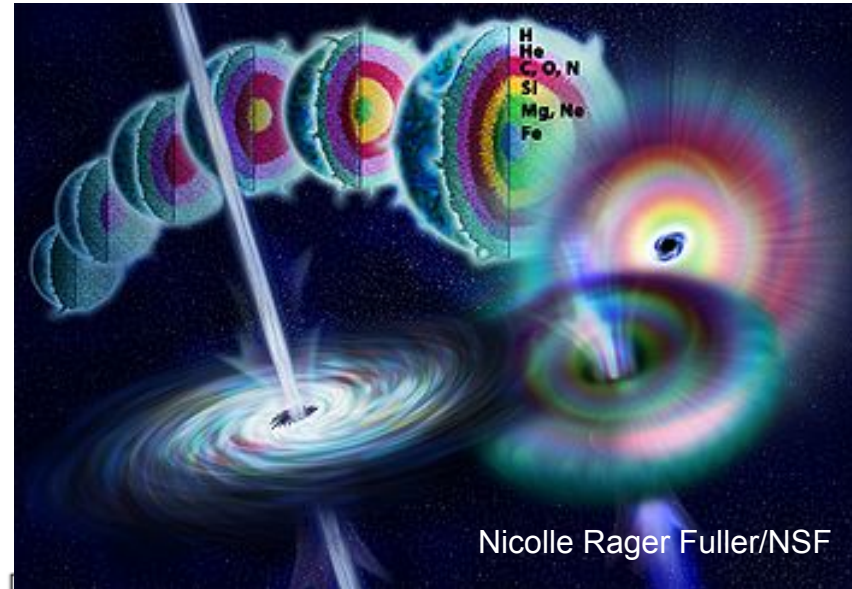
Expected Annual Coalescence Rates

- Rates are mean of the distribution; in a 95% confidence interval, rates uncertain by 3 orders of magnitude
- Rates are for Binary Neutron Stars (**BNS**) Binary Black Boles (**BBH**) and Neutron Star-Black Hole binaries (**NS-BH**)

	BNS	NS-BH	BBH
Initial LIGO (2002-06)	0.02	0.006	0.01
Adv. LIGO (2014+)	40	10	20
ET	Millions	100,000	Millions

GRB Progenitors

- Intense flashes of gamma-rays:
 - Most luminous EM source since the Big Bang
 - X-ray, UV and optical afterglows
- Bimodal distribution of durations
 - Short GRBs
 - Duration: $T_{90} < 2$ s
 - Mean redshift of 0.5
 - Long GRBs
 - Duration $T_{90} > 2$ s
 - Higher z , track Star Form. Rate.



• Long GRBs

- Core-collapse SNe, GW emission not well understood

- Could emit burst of GW

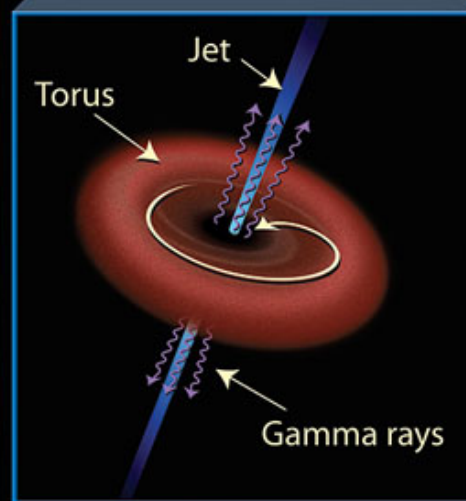
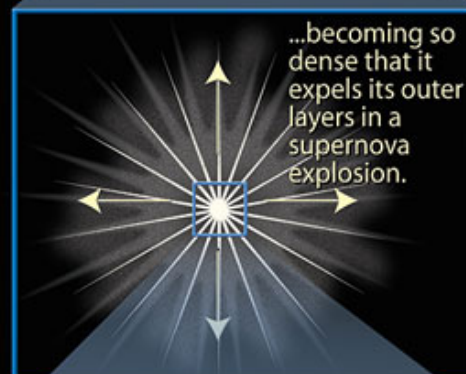
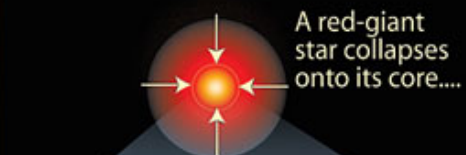
• Short GRBs

- Could be the end state of the evolution of compact binaries

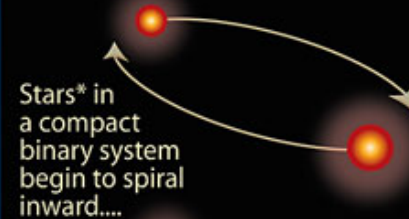
- BNS, NS-BH

Gamma-Ray Bursts (GRBs): The Long and Short of It

Long gamma-ray burst (>2 seconds' duration)



Short gamma-ray burst (<2 seconds' duration)



...eventually colliding.

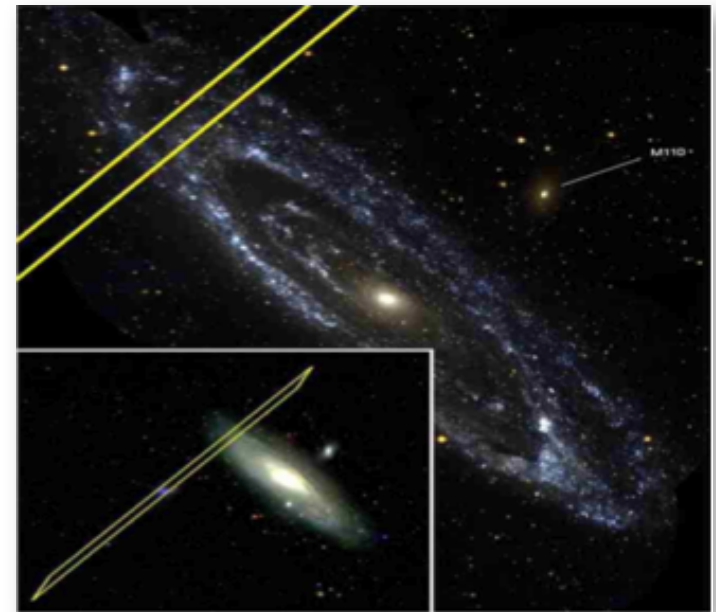
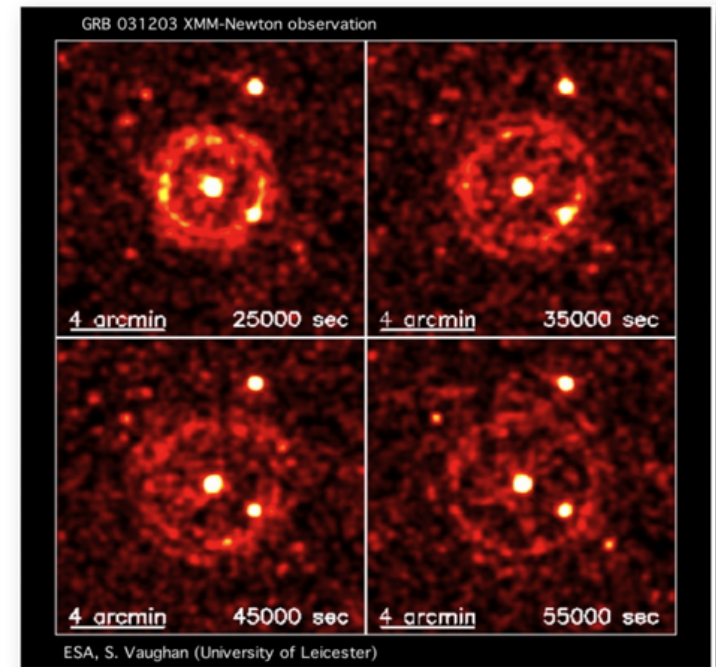
The resulting torus has at its center a powerful black hole.

*Possibly neutron stars.

Origin of GRB 070201 from LIGO Observations

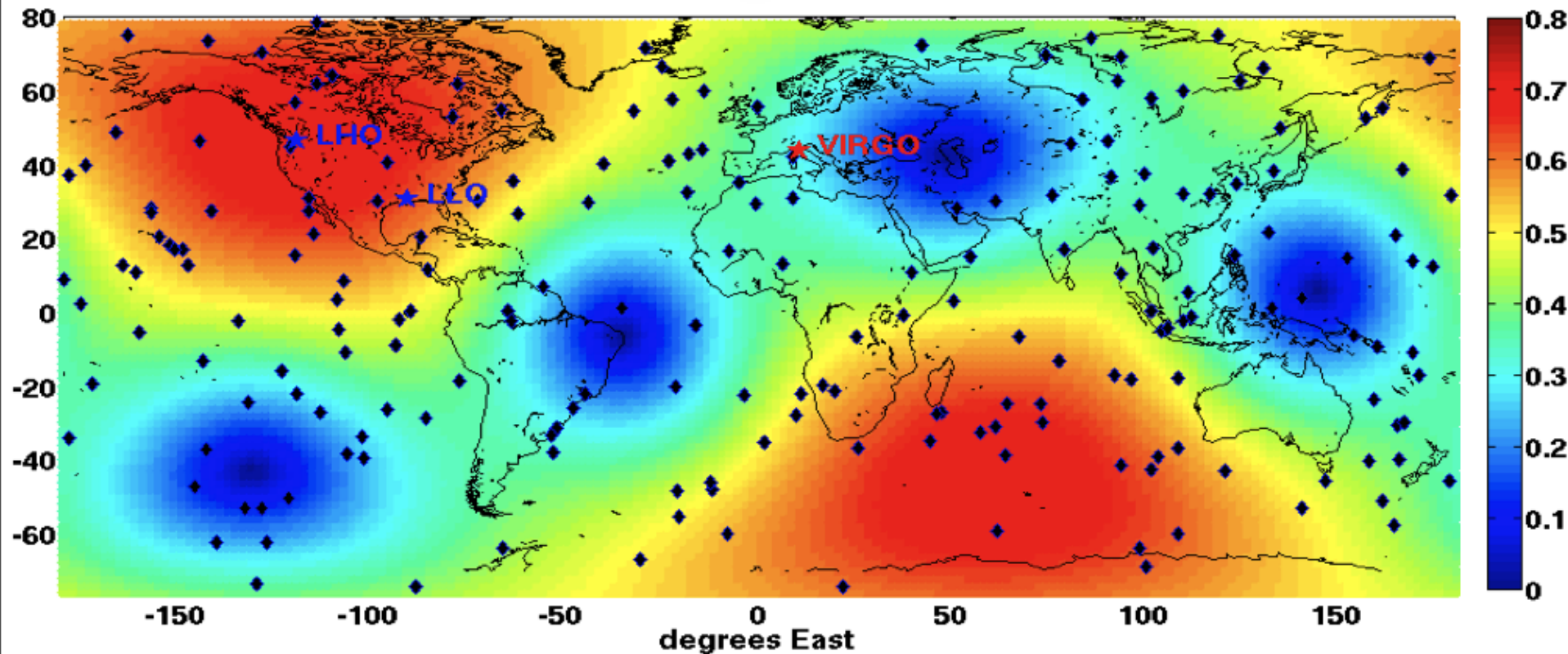
- LSC searched for binary inspirals and did not find any events: results in ApJ 681 1419 2008
- Null inspiral search result excludes binary progenitor in M31
- Soft Gamma-ray Repeater (SGR) models predict energy release $\leq 10^{46}$ ergs.
- SGR not excluded by GW limits

LSC, *Astrophys. J.* **681**, (2008) 1419



Search for GRBs during all of S5

- Nov 2005 - Oct 2007: **212** GRBs
- LSC-Virgo searched for **137** GRBs with 2 or more LIGO-Virgo detectors: ~25% with redshift, ~10% short duration: **Null result**
- Polarization-averaged antenna response of LIGO-Hanford, dots show location of GRBs during S5-VSR I



Spin-down limit on the Crab pulsar

LSC, ApJ Lett., **683**, (2008) 45

- 2 kpc away, formed in a spectacular supernova in 1054 AD
- Losing energy in the form of particles and radiation, leading to its spin-down

spin frequency of $\nu = 29.78 \text{ Hz}$

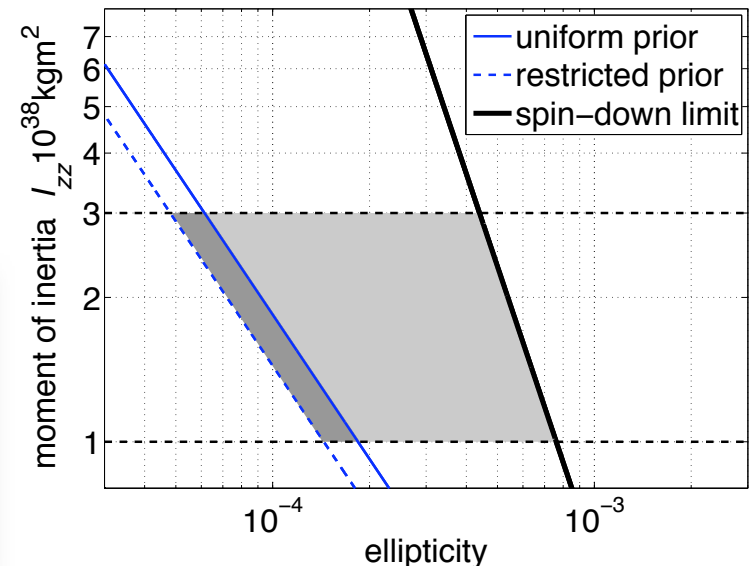
spin-down rate, $\dot{\nu} \approx -3.7 \times 10^{-10} \text{ Hz s}^{-1}$

$\dot{E} = 4\pi^2 I_{zz} \nu |\dot{\nu}| \approx 4.4 \times 10^{31} \text{ W}$

$h_0^{\text{sd}} = 8.06 \times 10^{-19} I_{38} r_{\text{kpc}}^{-1} (|\dot{\nu}|/\nu)^{1/2}$

- LSC have searched for gravitational waves in data from the fifth science run of LIGO detectors
- Lack of GW at S5 sensitivity means a limit on ellipticity a factor 4 better than spin-down upper limit - **less than 4% of energy in GW**

$$h_0^{95\%} = 3.4 \times 10^{-25}. \quad \varepsilon = 1.8 \times 10^{-4}$$



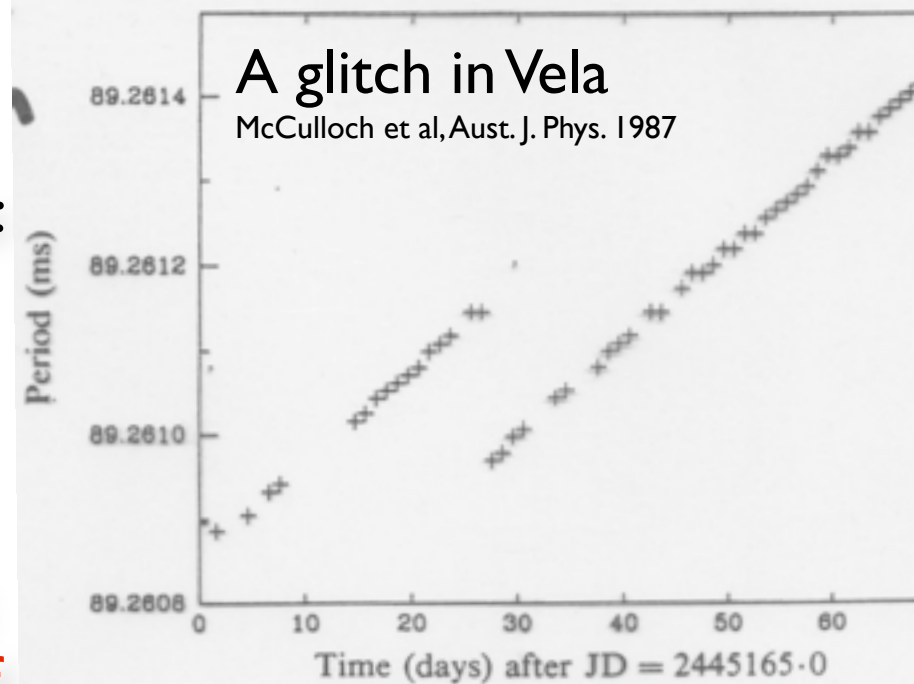
Pulsar Glitches

- Pulsars have stable rotation rates:
 - However, observe secular increase in pulse period
- Glitches are sudden dips in period
- Vela glitches once every few yrs
- Could be the result of transfer of angular momentum from core to crust
- At some critical lag rotation rate superfluid core couples to the crust imparting energy to the crust

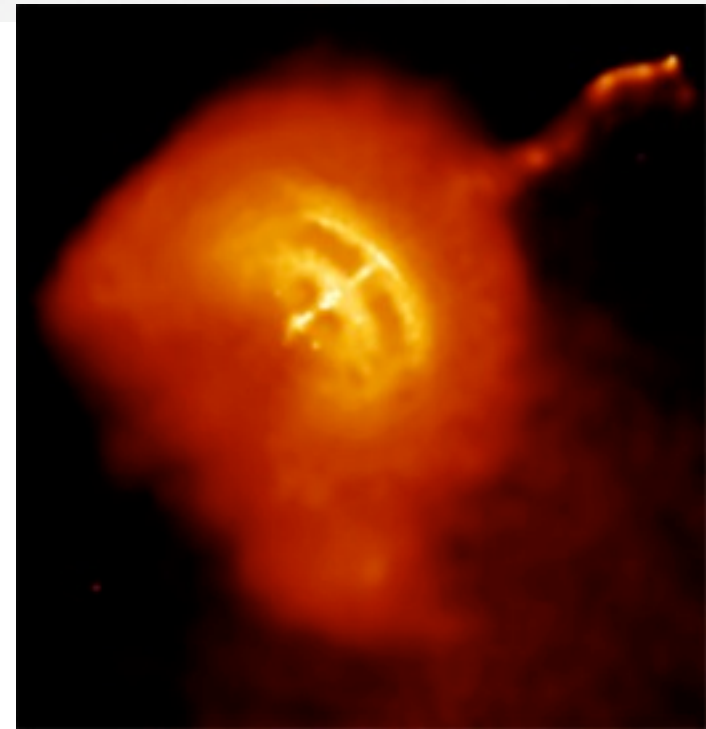
$$\Delta J \sim I_* \Delta \Omega \quad \Delta E = \Delta J \Omega_{\text{lag}}$$

$$\Delta \Omega / \Omega \sim 10^{-6}$$

$$\Delta E \sim 10^{-13} - 10^{-11} M_{\odot} c^2$$

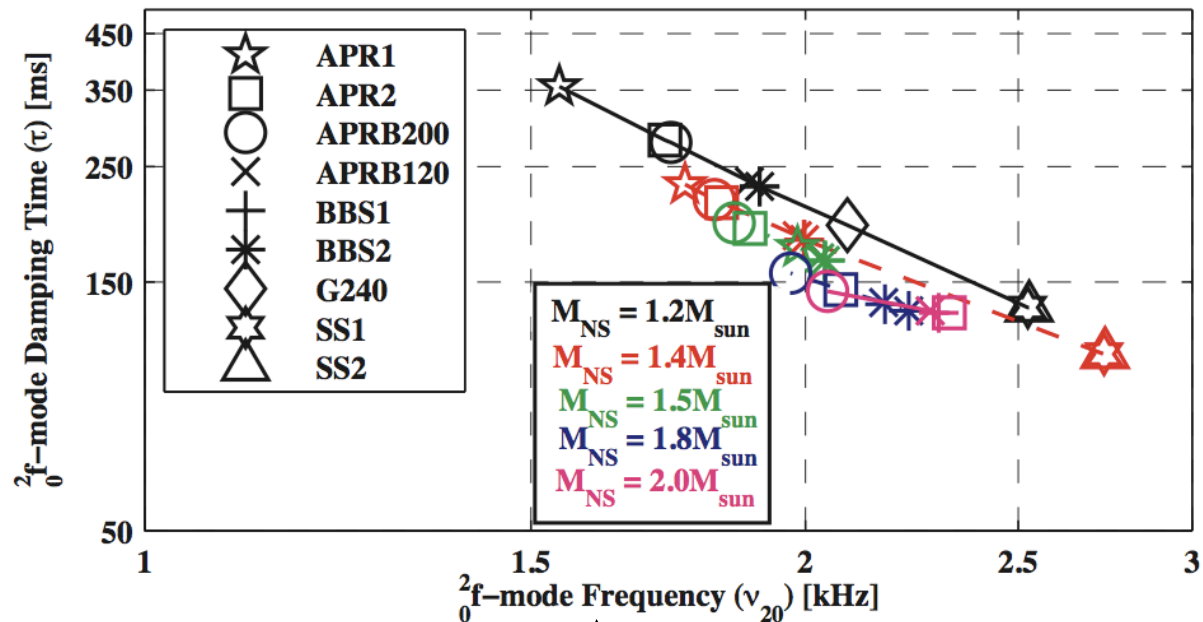
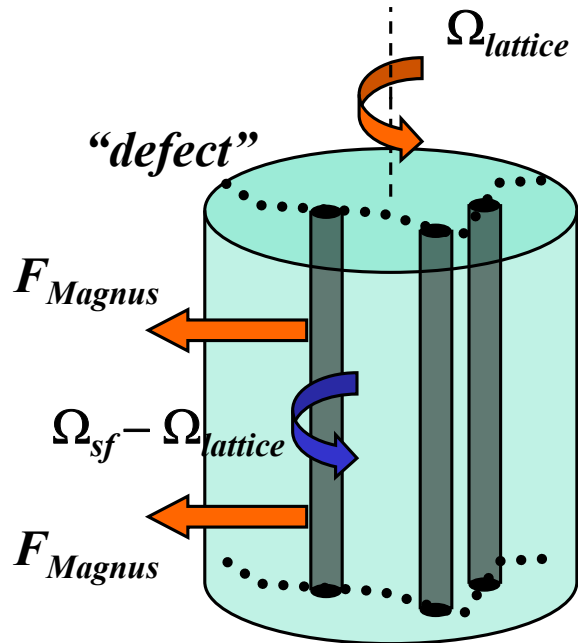


A composite Vela image



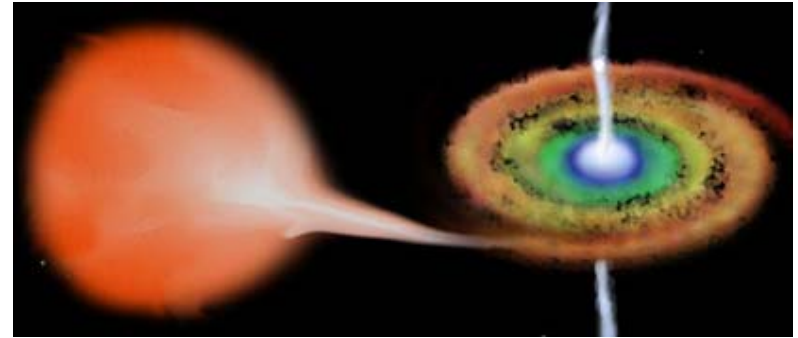
NS Normal Mode Oscillations

- Sudden jolt due to a glitch, and superfluid vortex unpinning, could cause oscillations of the core, emitting gravitational waves
- These normal mode oscillations have characteristic frequencies and damping times that depend on the equation-of-state
- Detecting and measuring normal modes could reveal the equation-of-state of neutron stars and their internal structure



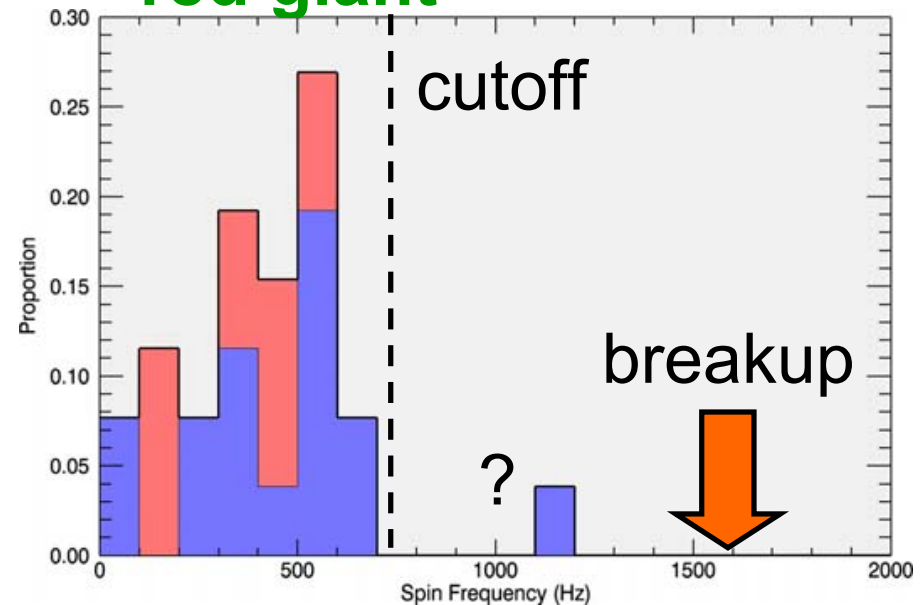
Accreting Neutron Stars

- Spin frequencies of accreting NS seems to be stalled below 700 Hz
 - Well below the break-up speed
- What could be the reason for this stall?
 - Balance of accretion torque with GW back reaction torque
- Could be explained if ellipticity is $\sim 10^{-8}$
 - Could be induced by mountains or relativistic instabilities, e.g. r-modes



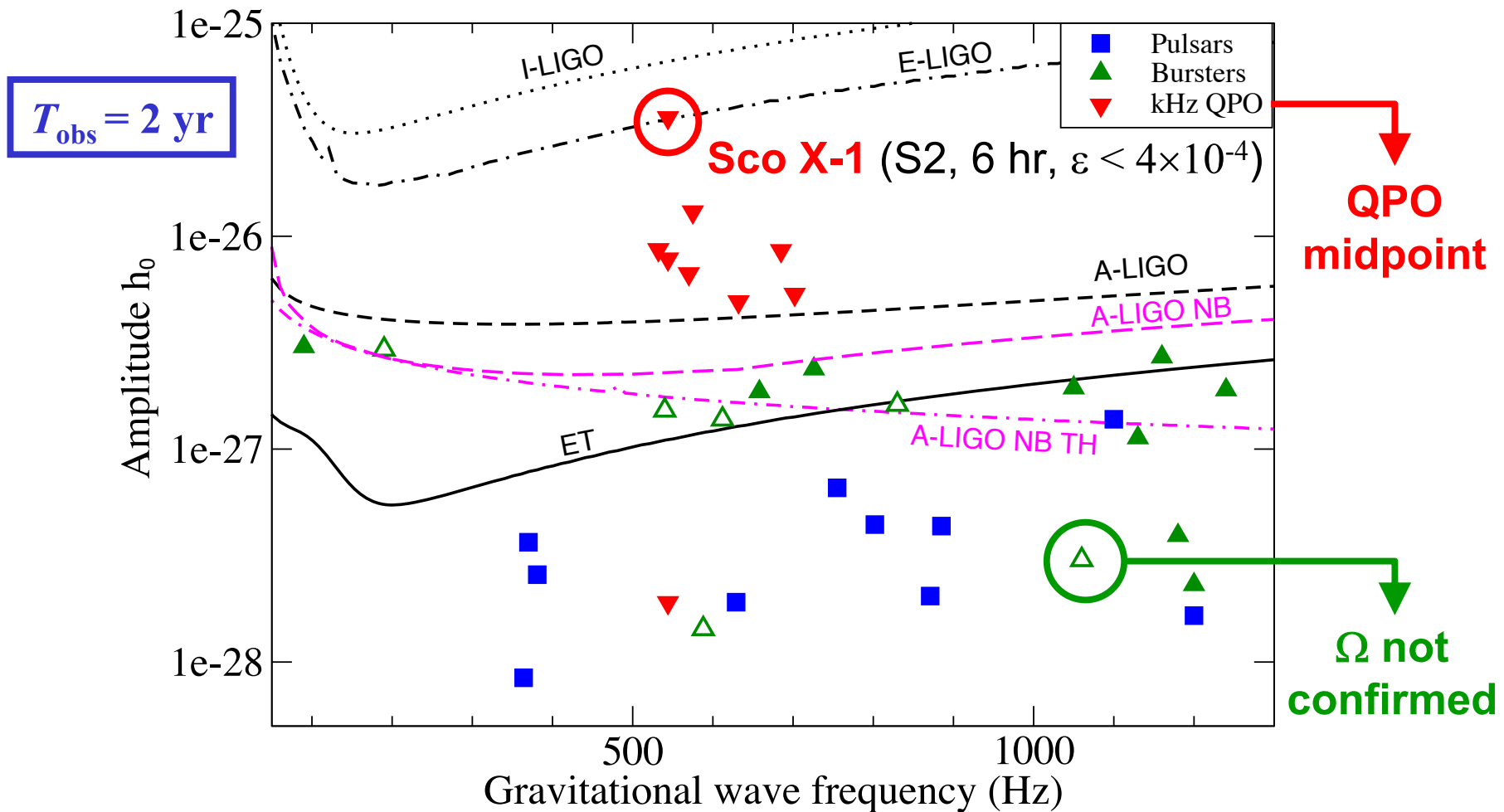
$< 1M_{\text{Sun}}$
red giant

NS



pulses & burst oscillations

Sensitivity to Accreting NS



Cosmology

Cosmology

- Cosmography

- H_0 , dark matter and dark energy densities, dark energy EoS w

- Black hole seeds

- Black hole seeds and their hierarchical growth

- Anisotropic cosmologies

- In an anisotropic Universe the distribution of H on the sky could show residual quadrupole and higher-order anisotropies

- Primordial gravitational waves

- Quantum fluctuations in the early Universe, stochastic BG

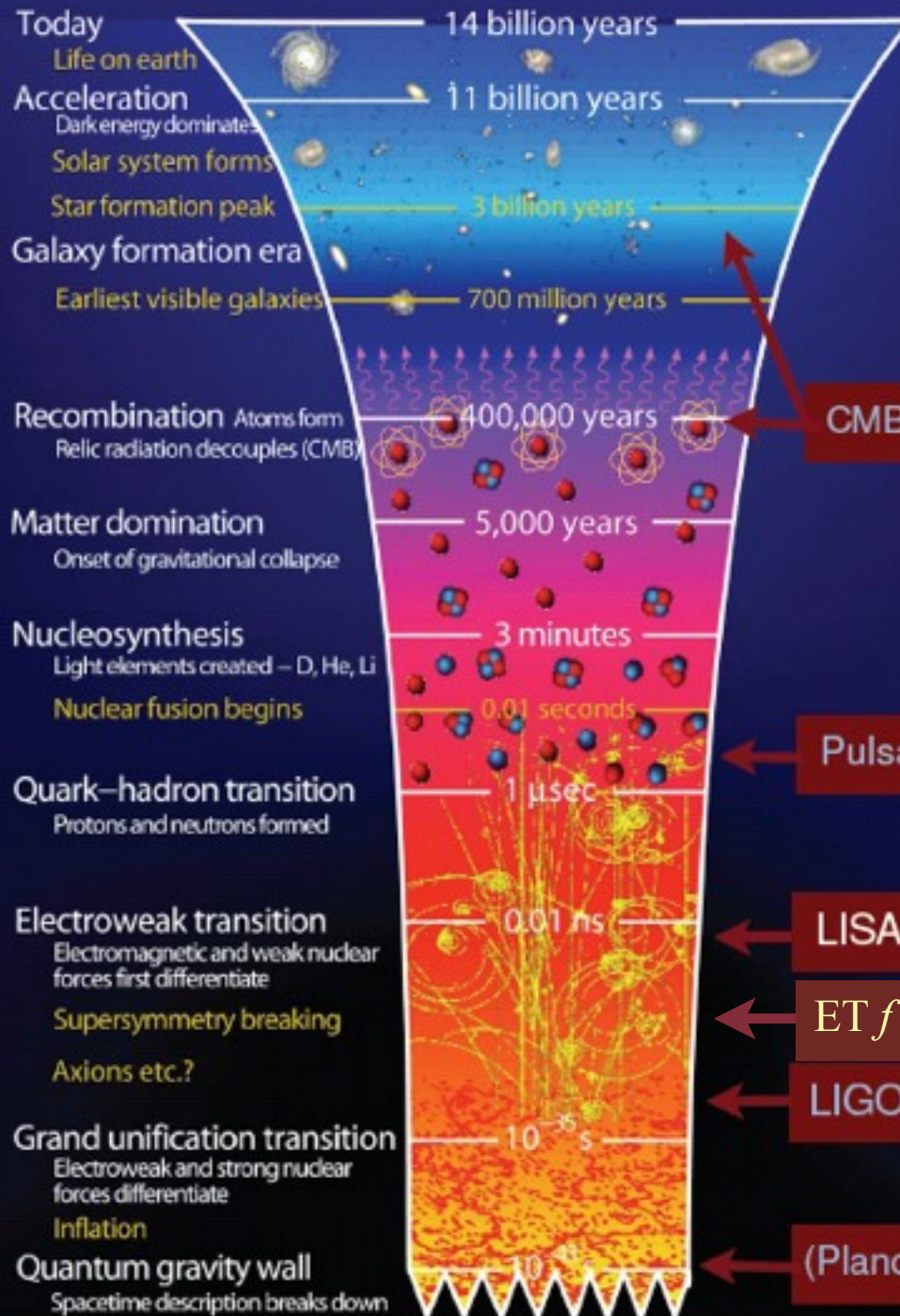
- Production of GW during early Universe phase transitions

- Phase transitions, pre-heating, re-heating, etc.

Stochastic Backgrounds

- ✧ Primordial background
 - ✧ Quantum fluctuations produce a background GW that is amplified by the background gravitational field
- ✧ Phase transitions in the Early Universe
 - ✧ Cosmic strings - kinks can form and “break” producing a burst of gravitational waves
- ✧ Astrophysical background
 - ✧ A population of Galactic white-dwarf binaries produces a background above instrumental noise in LISA

A brief history of the Universe



CMB $f < 3 \times 10^{-17} \text{ hHz}$ probes $300,000 \text{ yrs} < t_e < 14 \text{ Gys}$

Pulsars $f \sim 10^{-8} \text{ Hz}$ probe $t_e \sim 10^{-4} \text{ s}$ ($T \sim 50 \text{ MeV}$)

LISA $f \sim 10^{-3} \text{ Hz}$ probes $t_e \sim 10^{-14} \text{ s}$ ($T \sim 10 \text{ TeV}$)

ET $f \sim 10 \text{ Hz}$ probes $t_e \sim 10^{-20} \text{ s}$ ($T \sim 10^6 \text{ GeV}$)

LIGO $f \sim 100 \text{ Hz}$ probes $t_e \sim 10^{-24} \text{ s}$ ($T \sim 10^8 \text{ GeV}$)

(Planck scale $f \sim 10^{11} \text{ Hz}$ has $t_e \sim 10^{-43} \text{ s}$ ($T \sim 10^{19} \text{ GeV}$))

Searching for a Stochastic Background

$$\Omega_{\text{gw}}(f) = \frac{1}{\rho_{\text{crit}}} \frac{d\rho_{\text{gw}}}{d \ln f}$$

● Nucleosynthesis upper-limit

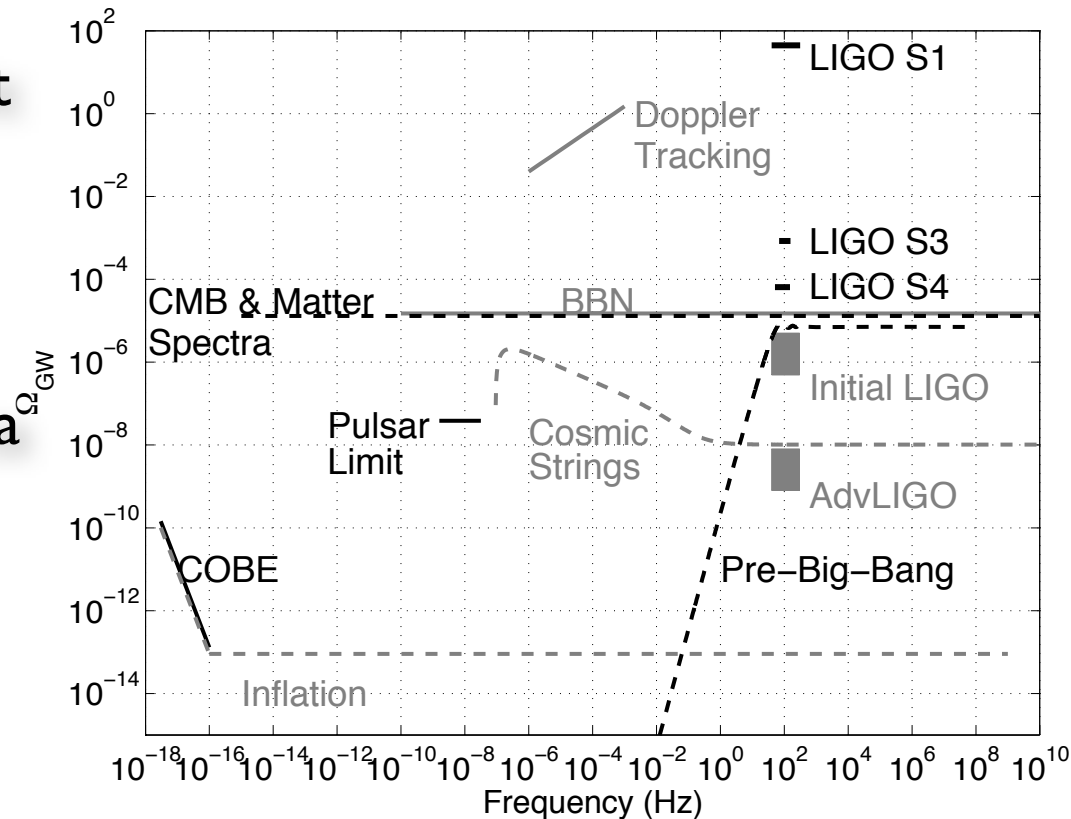
$$\int \frac{df}{f} \Omega_{\text{gw}}(f) \lesssim 1.5 \times 10^{-5}.$$

● Upper limit from LIGO data from the 4th Science run

$$\Omega_{\text{gw}}(f) < 6.5 \times 10^{-5}$$

● S5 data will improve this better than the nucleosynthesis limit

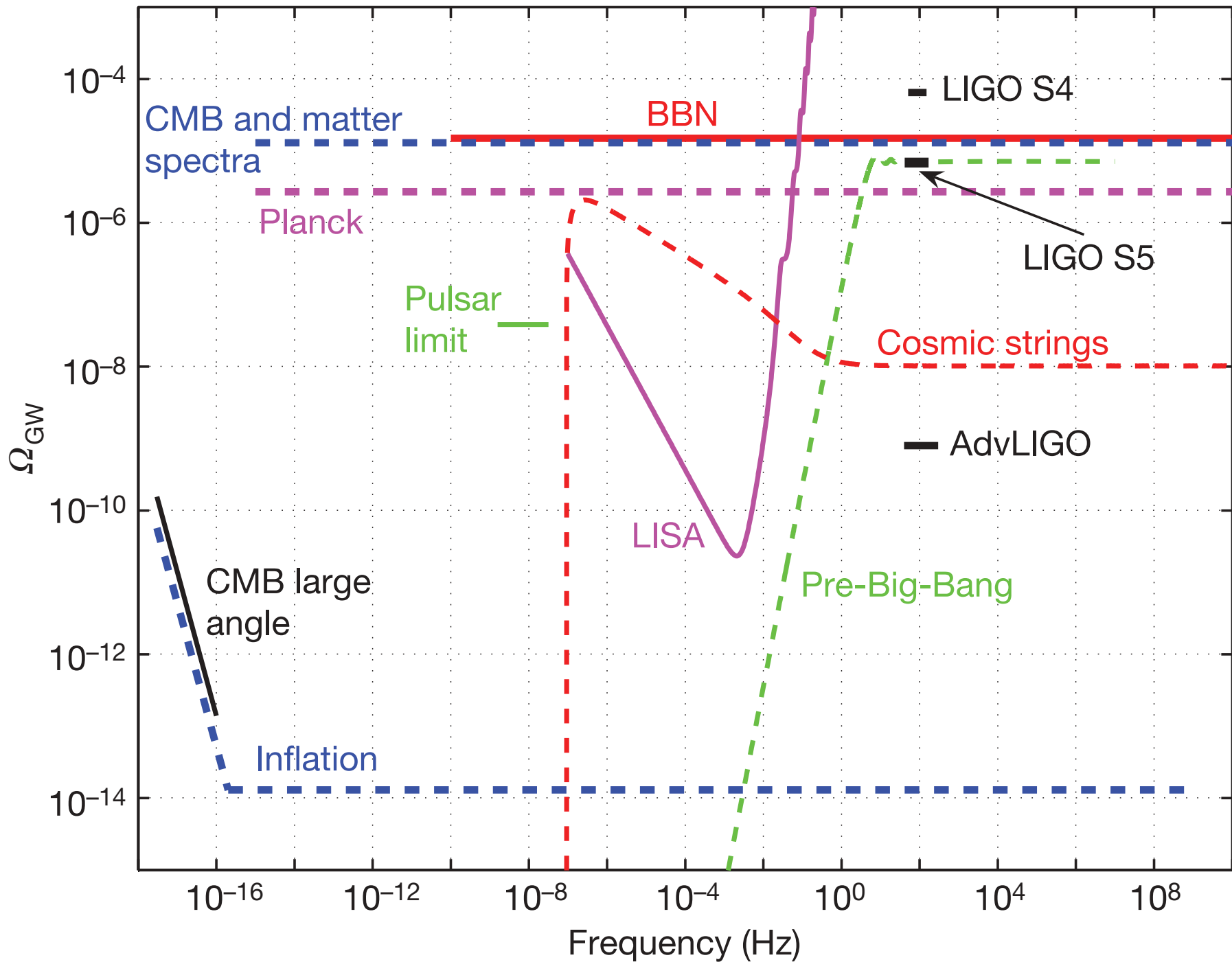
LSC, *Astrophys.J.* 659 (2007) 918



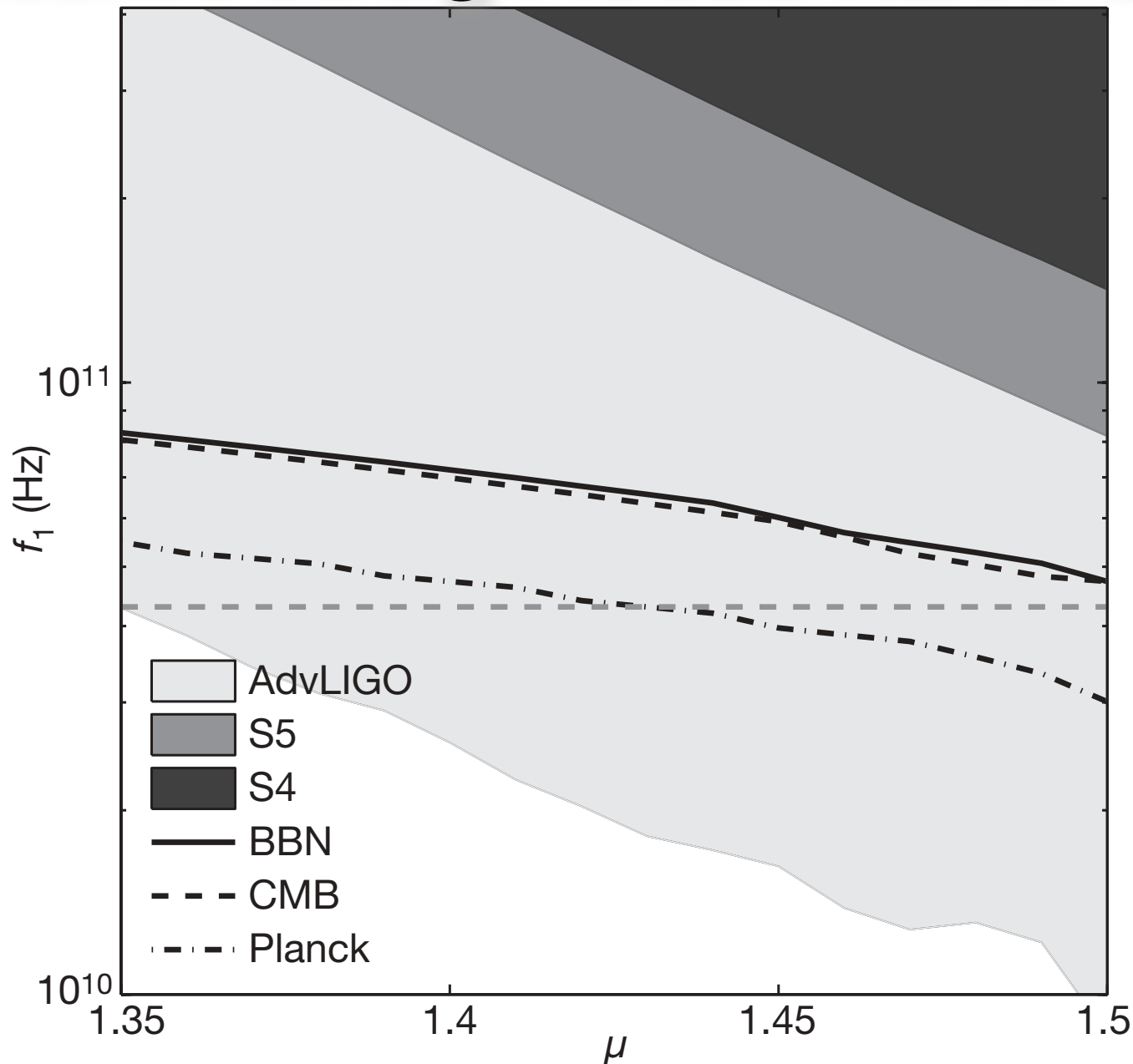
LETTERS

An upper limit on the stochastic gravitational-wave background of cosmological origin

The LIGO Scientific Collaboration* & The Virgo Collaboration*



Cosmic String Models Ruled Out



Cosmological parameters

- Luminosity distance Vs. red shift depends on a number of cosmological parameters H_0 , Ω_M , Ω_b , Ω_Λ , w , etc.

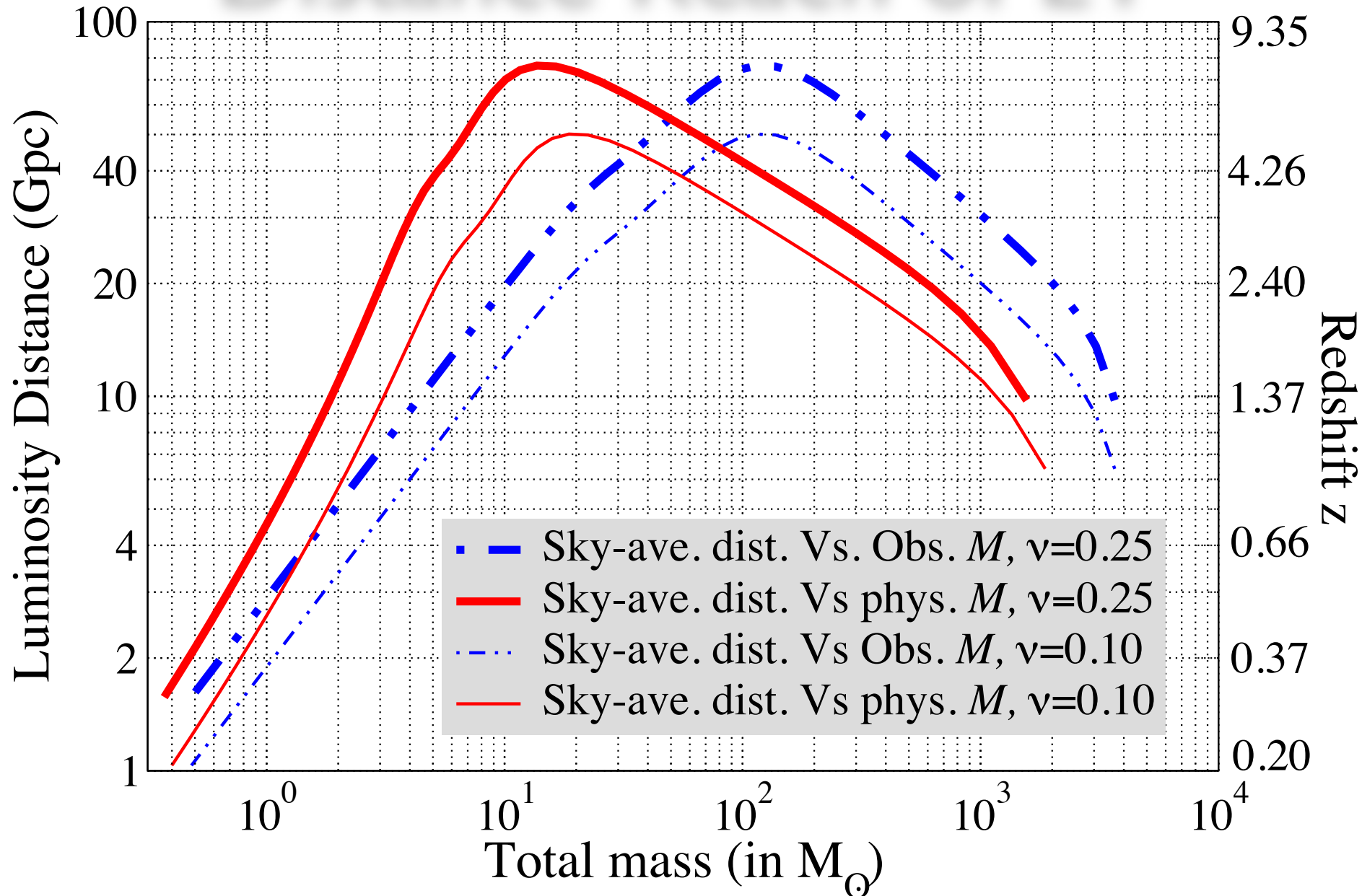
$$D_L(z) = \frac{c(1+z)}{H_0} \int_0^z \frac{dz}{[\Omega_M(1+z)^3 + \Omega_\Lambda(1+z)^{3(1+w)}]^{1/2}}$$

- Einstein Telescope will detect 1000's of compact binary mergers for which the source can be identified (e.g. GRB) and red-shift measured.
- A fit to such observations can determine the cosmological parameters to better than a few percent.

Compact Binaries are Standard Sirens

- Amplitude of gravitational waves depends on $h \propto \frac{\mathcal{M}^{5/6}}{D_L}$
- Chirp-mass = $\mu^{3/5} M^{2/5}$
- Gravitational wave observations can measure both
 - Amplitude (this is the strain caused in our detector)
 - Chirp-mass (because the chirp rate depends on the chirp mass)
- Therefore, binary black hole inspirals are **standard sirens**
 - From the apparent luminosity (the strain) we can conclude the luminosity distance
- However, GW observations alone **cannot determine the red-shift** to a source
- Joint gravitational-wave and optical observations can facilitate a new cosmological tool

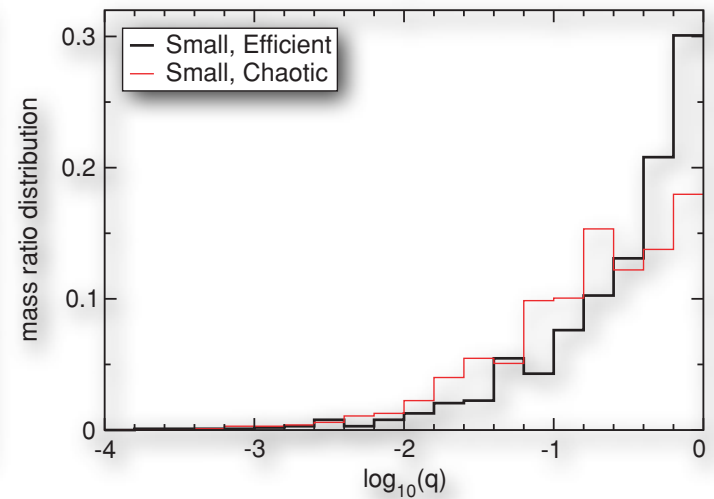
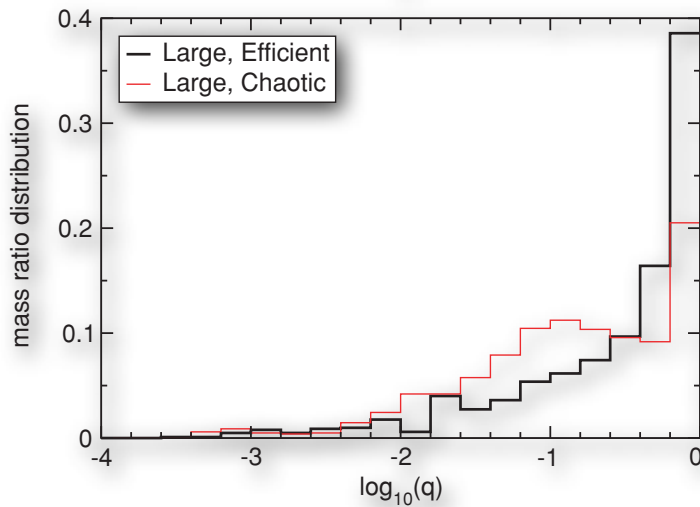
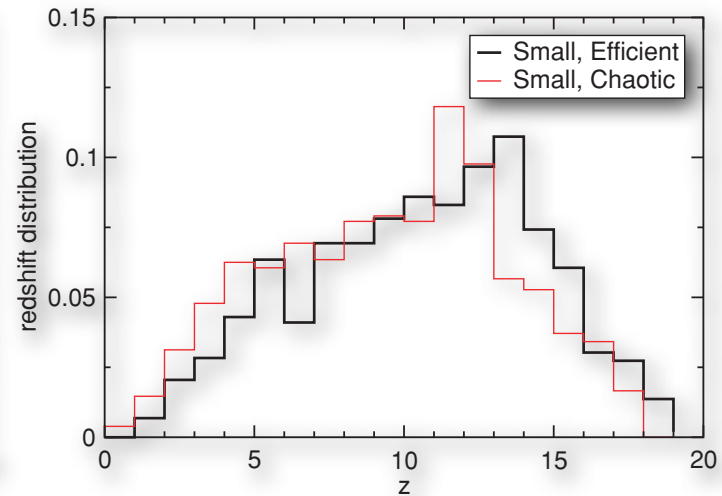
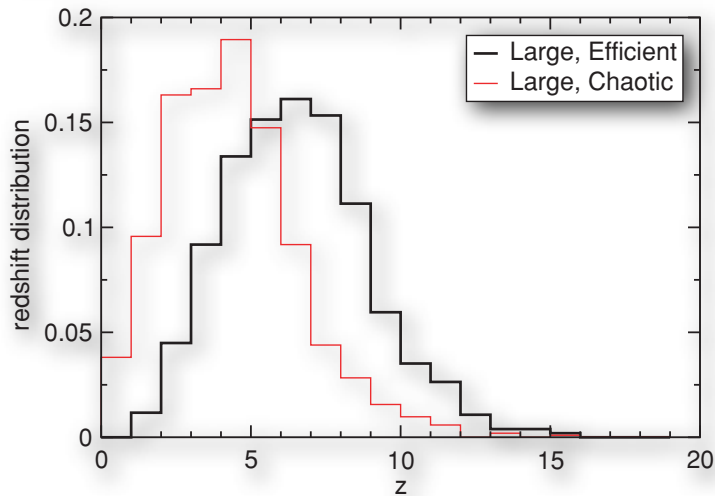
Distance Reach of ET



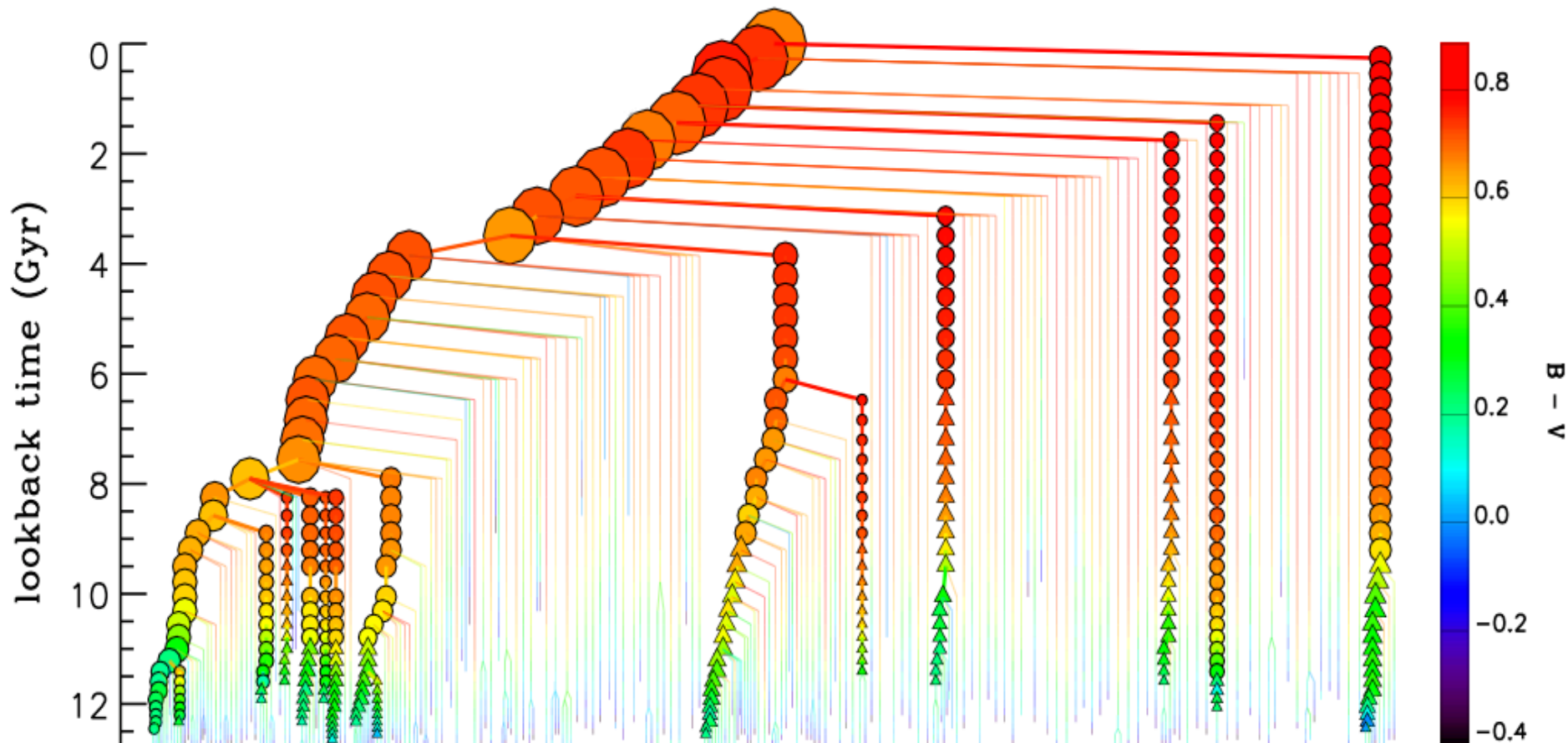
Models of Black Hole Seeds and Their Evolution

Class. Quantum Grav. **26** (2009) 094027

K G Arun *et al*



Hierarchical Growth of Black Holes in Galactic Nuclei



- Initially small black holes may grow by hierarchical merger
- ET could observe seed black holes if they are of order 1000 solar mass

Fundamental Physics

Fundamental Physics

- Properties of gravitational waves
 - Test wave generation formula beyond quadrupole approx.
 - Number of GW polarizations?
 - Do gravitational waves travel at the speed of light?
- Equation-of-State of dark energy
 - GW from inspiralling binaries are standard sirens
- Equation-of-State of supra-nuclear matter
 - Signature NS of EoS in GW from binary neutron star mergers
- Black hole no-hair theorem and cosmic censorship
 - Are black hole candidates black holes of general relativity?
- Merger dynamics of spinning black hole binaries

Are Gravitons Massive?

- Coincident observation of a supernova and the associated gravitational radiation can be used to constrain the **speed of gravitational waves** to a fantastic degree:
- If Δt is the time difference in the arrival times of GW and optical radiation and D is the distance to the source then the fractional difference in the speeds is

$$\frac{\Delta v}{c} = \frac{\Delta t}{D/c} \simeq 10^{-14} \left(\frac{\Delta t}{1\text{sec}} \right) \left(\frac{D}{1\text{Mpc}} \right)$$

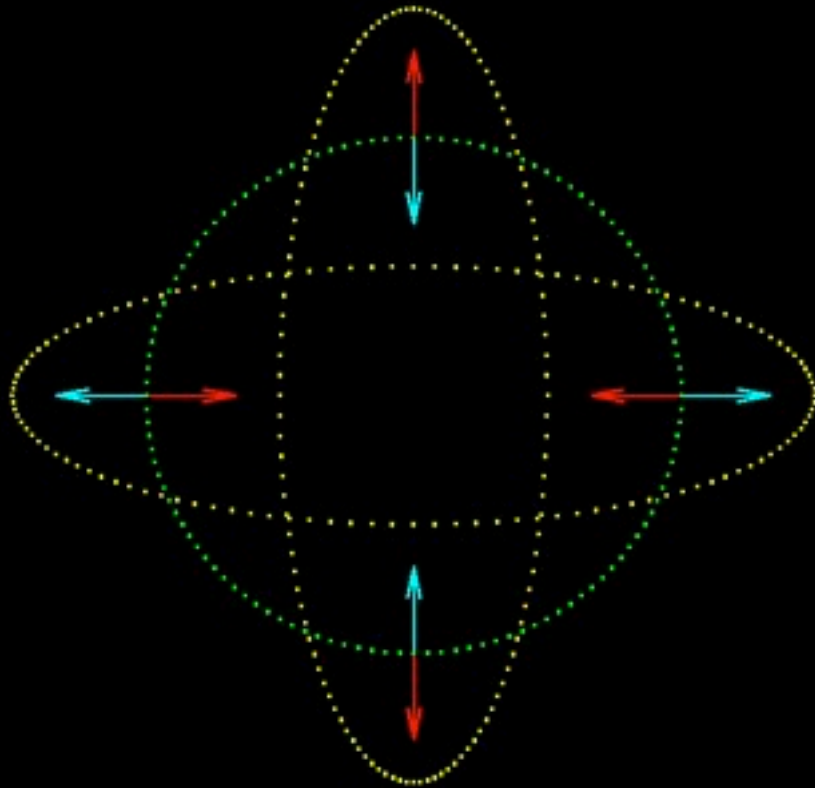
- Should also be possible to **constrain the mass of the graviton** as they alter GW phasing of inspiral waveform due to dispersion of gravitational waves; no EM counterpart needed

Counting the Polarization States

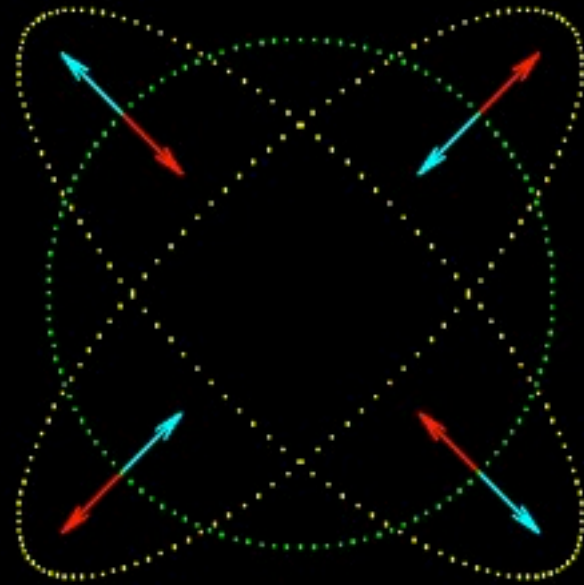
Only two states in GR: h_+ and h_x

Counting the Polarization States

Only two states in GR: h_+ and h_\times



Plus polarization



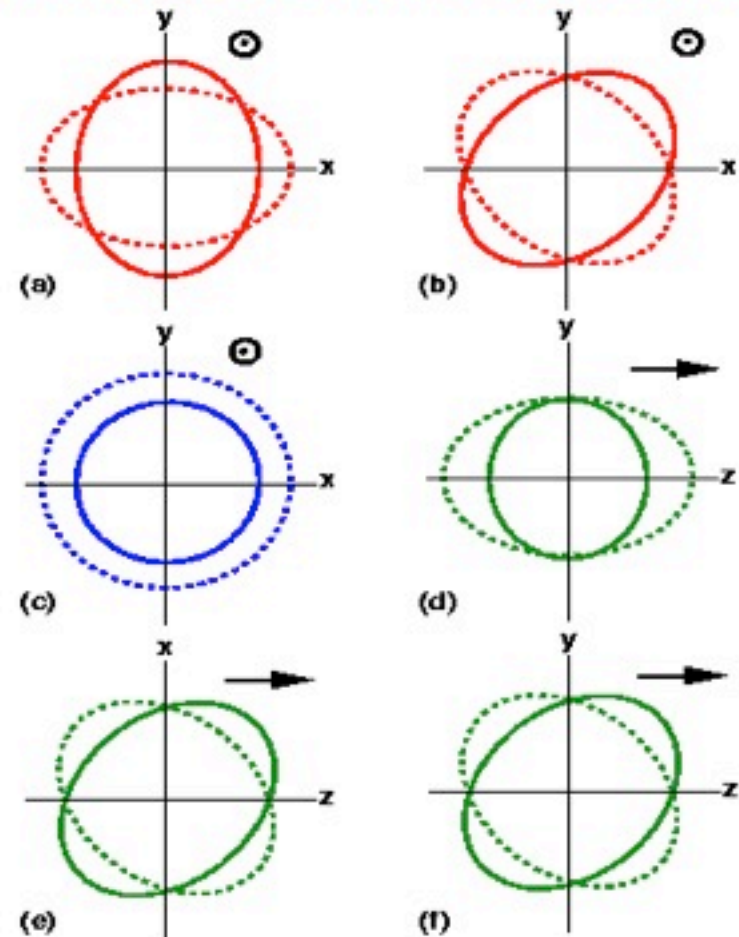
Cross polarization

Polarization States in a Scalar-Tensor Theory

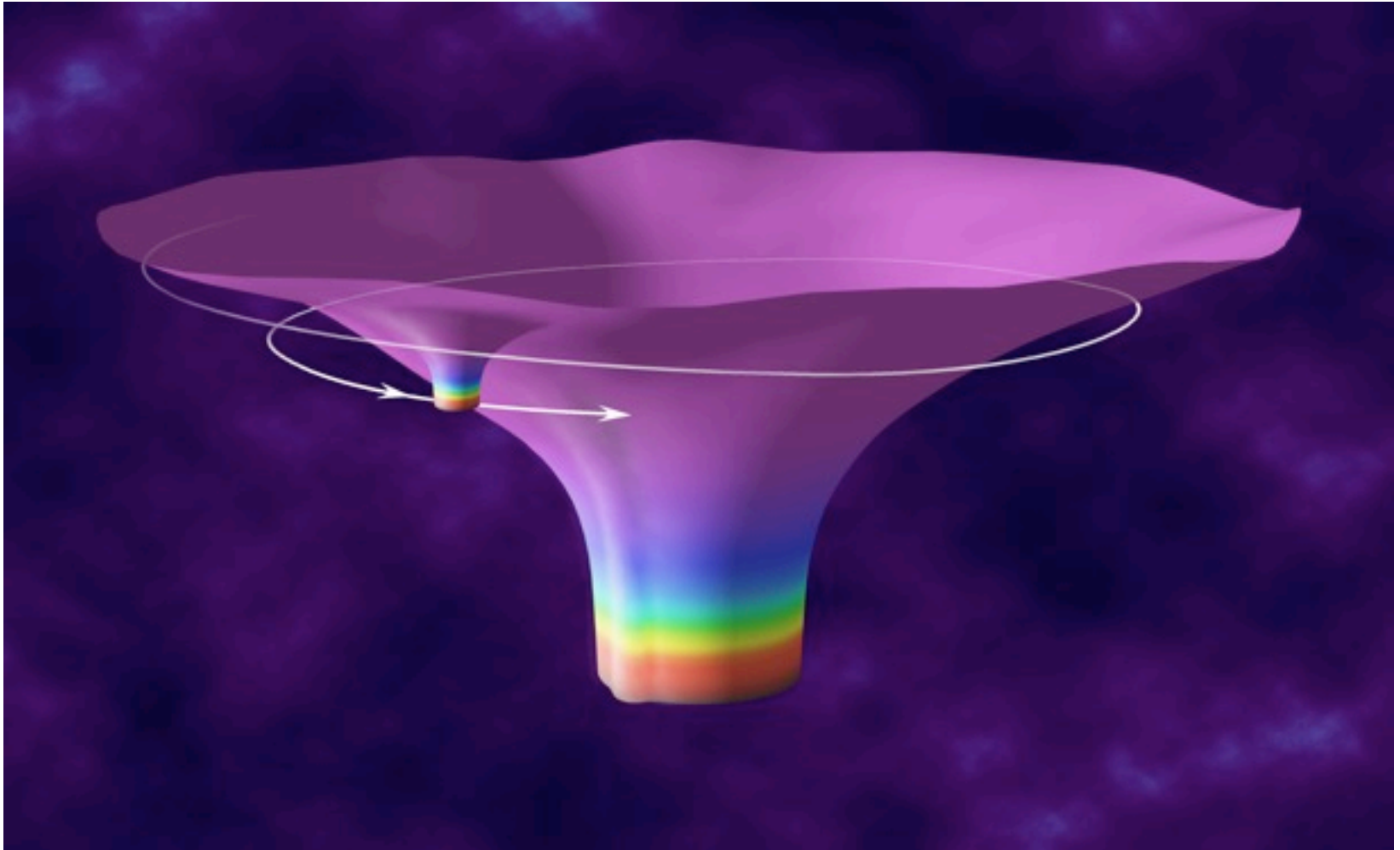
- Polarization tests are qualitative tests
- A single measurement is good enough to rule the theory out
- In Einstein's theory there are only two polarization states - the plus and the cross polarizations
- In a scalar-tensor theory of gravity, there are six different polarization modes

Cliff Will, Living Rev. in Relativity

Gravitational-Wave Polarization



Capture of Small Black Holes by Intermediate-Mass Black Holes



Testing the No-Hair Theorem

Ryan



Image: AEI/Einstein Online

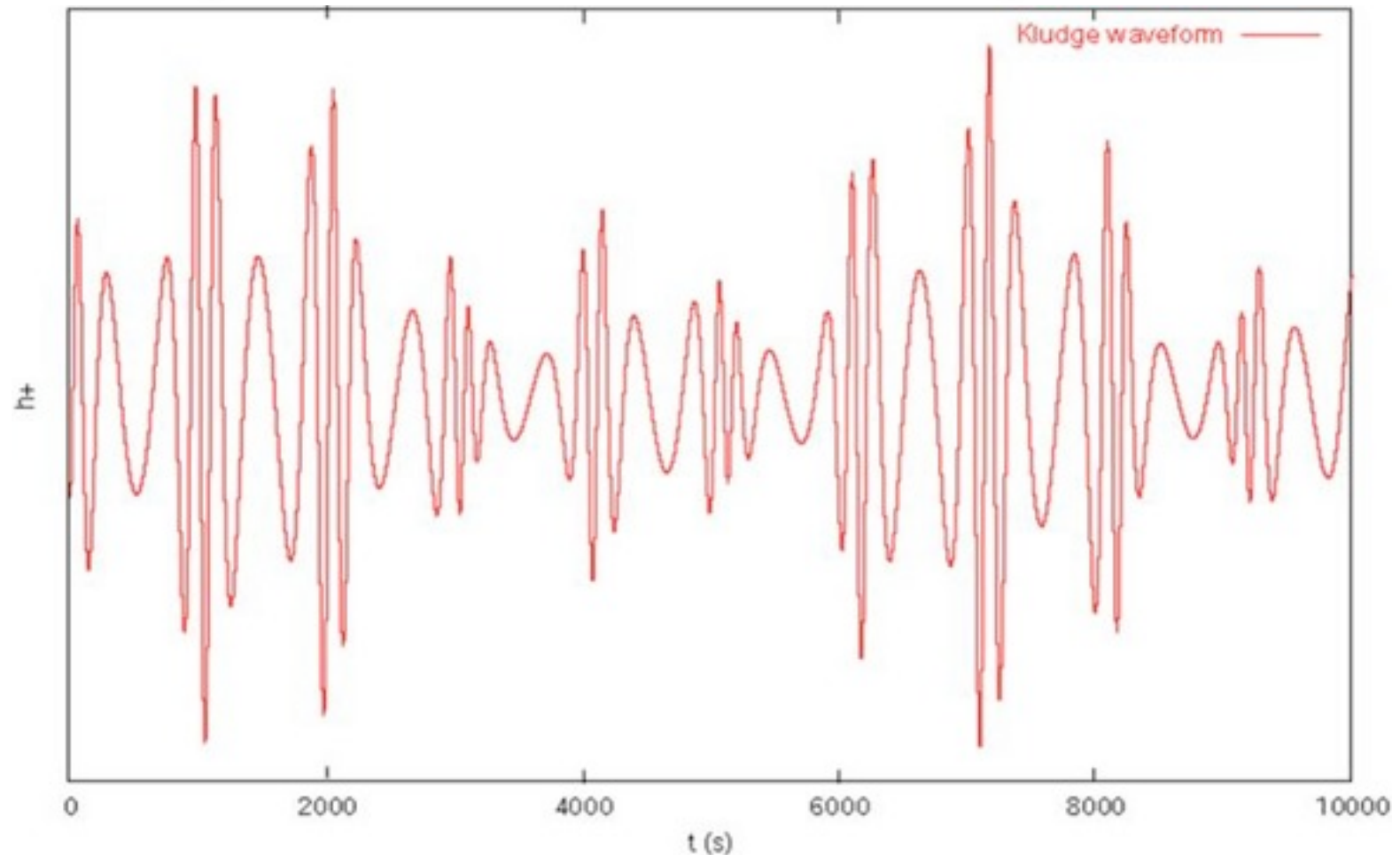
Testing the No-Hair Theorem

Ryan



Image: AEI/Einstein Online

Gravitational Capture and Testing Uniqueness of Black Hole Space-times



Glampedakis and Babak

Summary

- Was Einstein right?
 - Is the nature of gravitational radiation as predicted by Einstein?
 - Are black holes in nature black holes of GR?
 - Are there naked singularities?
- Unsolved problems in astrophysics
 - What is the origin of gamma ray bursts?
 - What is the structure of neutron stars and other compact objects?
- Cosmology
 - Measurement of Hubble parameter, dark matter density, etc.
 - Demography of massive black holes at galactic nuclei?
 - Phase transitions in the early Universe?
- Fundamental questions
 - What were the physical conditions at the big bang?
 - What is dark energy?

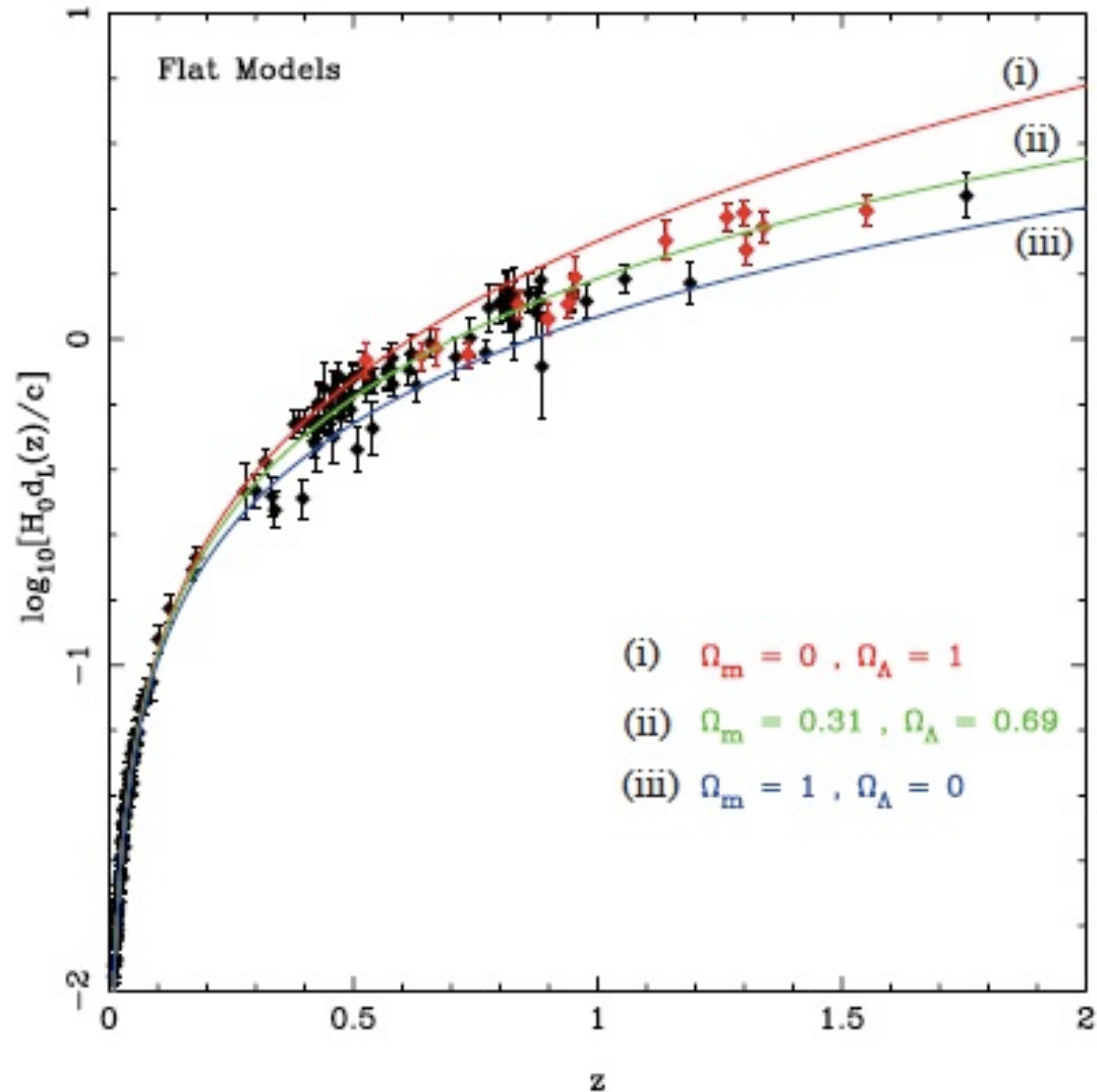
- **PREDICTIONS:**
- Within 5 to 10 years, gravitational wave astronomy will exist!
- Within 20 years, gravitational wave astronomy will be common-place with LIGO, LISA and PTAs being used just like radio/optical/xray... telescopes are today!



Max-Planck-Institut für Gravitationsphysik

Spare Slides

Luminosity Distance Vs Redshift

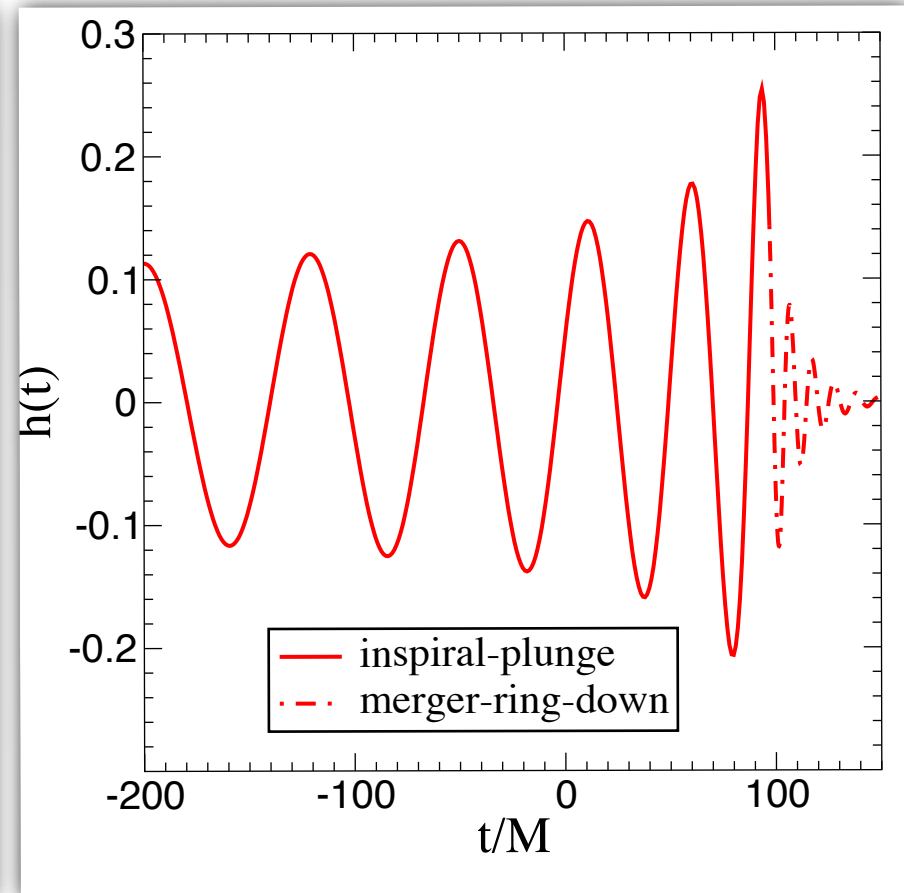
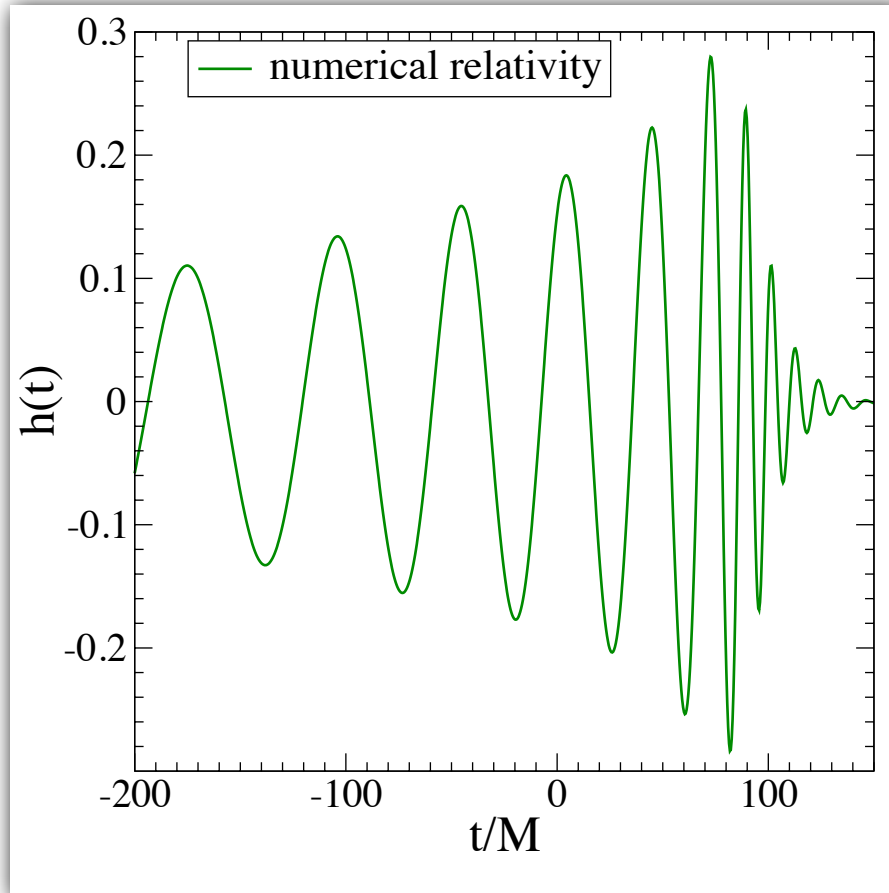


What do we know about the sources

Black Hole Mergers from Numerical Relativity

- After several decades NR is now able to compute accurate waveforms for use in extracting signals and science
 - New physics - e.g. super-kick velocities
 - Analytical understanding of merger dynamics
- We should be able to see further and more massive objects

Close Agreement b/w NR and EOB



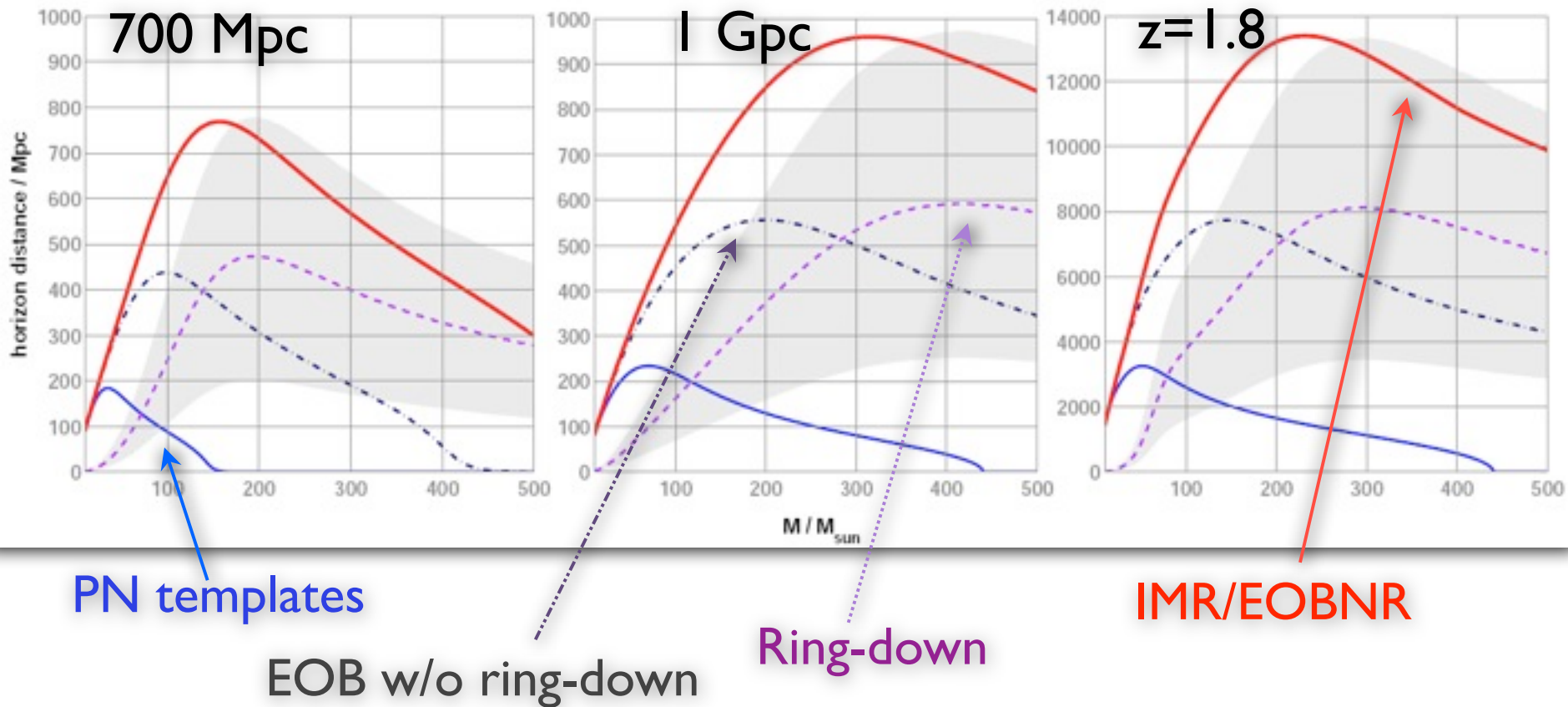
A new Effective One-Body (EOB) model by Damour, Iyer and Nagar (2009) in excellent agreement with Numerical Relativity simulations

How further can we see with Inspiral, Merger and Ringdown?

Initial LIGO

Virgo design

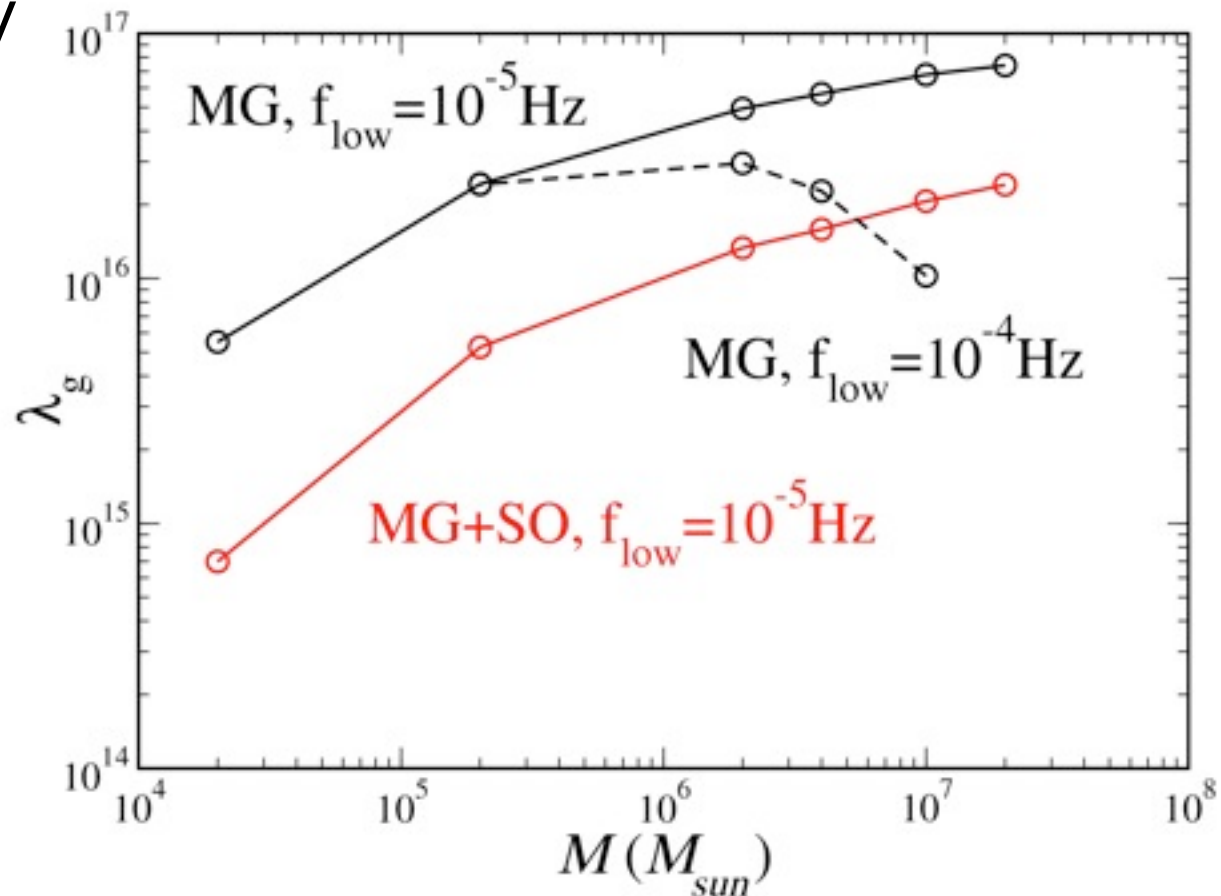
Advanced LIGO



Ajith et al

Bound on λ_g as a function of total mass

- Limits based on GW observations will be five orders-of-magnitude better than solar system limits
- Still not as good as (model-dependent) limits based on dynamics of galaxy clusters



Berti, Buonanno and Will (2006)