Parametrized tests of post-Newtonian theory with Einstein Telescope

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Tests of General Relativity

Most important Tests till date: [Will, 2001 for a review]

- Weak-field regime using Solar system observations.
  - Use of parametrized post-Newtonian (PPN) formalism.
  - Deviation of a general metric theory of gravity in the weak field limit from Newtonian theory was parametrized in terms of 10 free parameters
  - These parameters are constrained to very good accuracy with various solar system observations.
- Strong field & Radiative regime using binary pulsar observations:
  - Strong fields involving compact objects of $v \sim 10^{-3}c$.
  - Use of parametrized post-Keplerian (PPK) formalism as applied to timing equation.
  - Various Keplerian & post-Keplerian parameters are functions of the individual masses of the binary and determination of more than 2 of these $\Rightarrow$ consistency tests in the $m_1 - m_2$ plane.
- General relativity passes these tests in flying colours!

These tests were so successful because of solid theoretical platforms of PPN and PPK formalisms.

Stretching general relativity further: Tests with GWs

What if

* General relativity breaks down when the gravitational fields are stronger than those of binary pulsars.
* There is a scalar field coupled with the metric? [Scalar-tensor field theories]
* Graviton has a mass which is so small that it starts to show up in the very strong field regime. [Massive Graviton Theories]
- Gravity is described by some other theory.

Gravitational Waves

- Gravitational Waves have direct imprints of all the strong field effects
- How well can GW observations constrain deviations from GR?

Inspiralling compact binaries and testing general relativity

Adiabatic inspiral phase of a compact binary coalescence is well modelled using post-Newtonian (PN) formalism.

- Determination of coefficients in phasing formula can lead to meaningful tests
  - Detectability of tails [Blanchet & Sathyaprakash, 1994].
  - Measuring the dipolar content of the gravitational wave and test scalar-tensor theories [Will, 1994; Krolak et al, 1995, Damour & Esposito-Farése, 1998].
  - Parametrizing the 1PN coefficient of the phasing formula capturing the compton wavelength of the massive graviton and bounding its value from GW observations [Will, 1998].

The question

Can these tests be generalized, without having to know a priori the parameters of the underlying theory of gravity?

Parametrized test of PN theory

Phasing formula in the restricted waveform approximation

$$\tilde{h}(f) = \frac{1}{\sqrt{30}\,\pi^{2/3}} \frac{\mathcal{M}^{5/6}}{D_L} f^{-7/6} e^{i\psi(f)},$$

and to 3.5PN order the phase of the Fourier domain waveform is given by

$$\psi(f) \;=\; 2\pi f t_c - \phi_c - \frac{\pi}{4} + \sum_{k=0}^{7} (\psi_k + \psi_{kl} \ln f)\, f^{\frac{k-5}{3}},$$

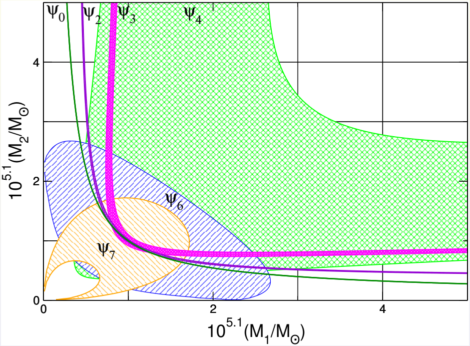
Log terms in the PN expansion

- Phasing coefficients are functions of component masses of the binary: $\psi_k(m_1, m_2)$ & $\psi_{kl}(m_1, m_2)$ [Spins negligible]
- Independent determination of 3 or more of the phasing coefficients ⇒ Tests of PN theory[KGA, Iyer, Qusailah & Sathyaprakash, 2006].

Basic Idea

- Parametrize the phasing formula in terms of various phasing coefficients where all of them are treated as independent.
- See how well can different parameters be extracted.
- Those which are well estimated, plot them (ψ_k & ψ_{kl}) in the $m_1 - m_2$ plane (similar to binary pulsar tests) with the widths of various curves proportional to $1 - \sigma$ error bars.

[KGA, Iyer, Qusailah, Sathyaprakash, 2006a]



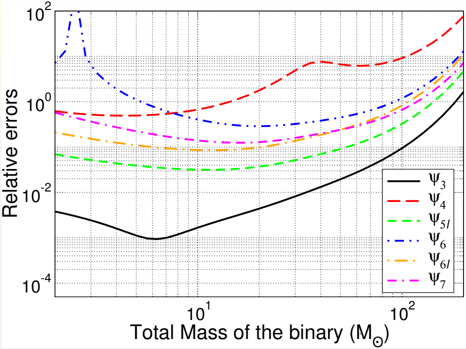
Issues

Highly correlated parameters & Ill-conditioned Fisher matrix for a large parameter space.

Alternative Proposal

[KGA, Iyer, Qusailah & Sathyaprakash, 2006b]

- Treat two parameters as basic variables in terms of which one can parametrize all other parameters EXCEPT one which is the *test* parameter.
- This way, dimensionality of the parameter space is considerably reduced.
- Thus, one will have ${}^{8}C_3$ tests, not all of them independent.
- The best choice to be used as basic variables are the leading two coefficients at 0PN & 1PN, which are the best determined ones.
- Then one will have 6 tests.



- Used an earlier EGO noise PSD (similar to one of the ET noise PSDs).
- All parameters except ψ_4 determined quite well over a large range of masses.

Present work

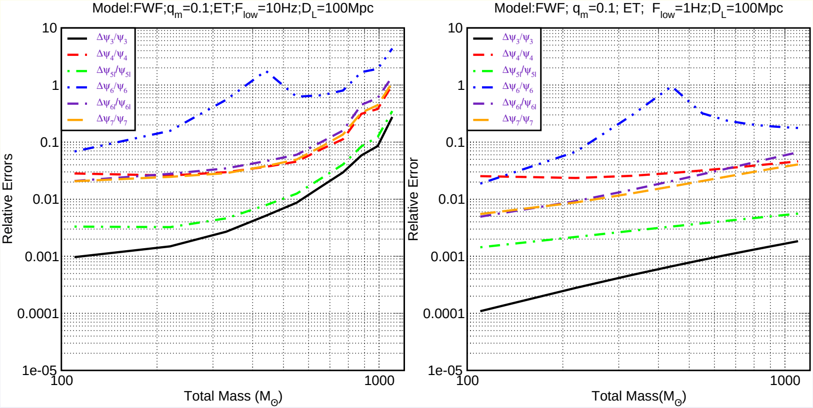
Use of Full Waveforms

- Revisit the earlier estimates more carefully using the ET noise PSD.
- Use of 3PN accurate amplitude corrected waveforms (as opposed to restricted waveforms).
- Effect of low frequency sensitivity on the Test of GR.
- Consideration of unequal mass systems.

Details

- We parametrize the mass dependences (through $\delta = \frac{|m1-m2|}{(m1+m2)}$ and ν) in the *amplitude* terms by ψ_0 & ψ_2 which are used as the basic variables to parametrize all phasing coefficients except the one to be used as test parameter.
- Final parameter space is spanned by: $\{\psi_1, \psi_2, \psi_T, t_c, \phi_c\}$

Results, FWF: 10Hz Cut-off Vs 1Hz Cut-off

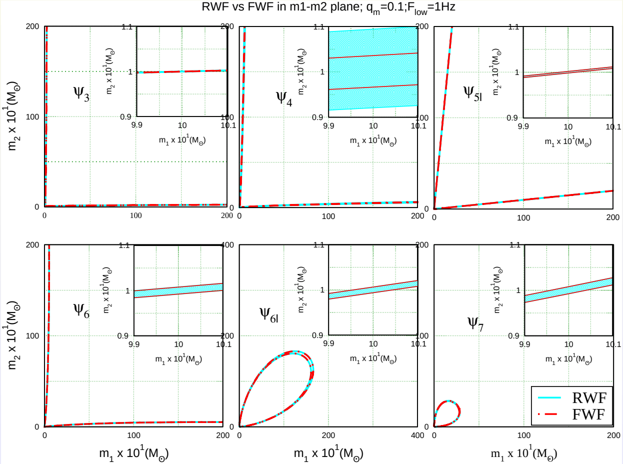


Features

- ψ_3 makes the best use of lowered seismic cut-off.
- Improvement in other parameters is less dramatic due to lower seismic cut-off.
- Improvements are significant for masses $> 500 M_{\odot}$.

$m_1 - m_2$ plots: 1Hz Cut-off (RWF Vs FWF)

System: $10 - 100 M_\odot$ at 100 Mpc.



In short

- For this system, FWF doesn't affect the $m_1 - m_2$ plots except for ψ_4.
- Best test parameter is ψ_3
- Worst test parameter is ψ_4.

Still..

- Always use the FWF to avoid systematic errors.

Limitations of the proposed Test

- This proposal can test the **overall** consistency between various PN coefficients, but cannot pin point the inconsistent parameter as opposed to the earlier proposal which determines all the PN coefficients independently (but is not feasible due to large correlations).
- When different parameters are used as tests, though they are *independent* tests in principle, interpretation of the outcome in the $m_1 - m_2$ plane cannot yield more information other than the overall consistency.
- Cannot be used for probing very specific aspects such as logarithmic terms in the phasing etc.

Other closely related works

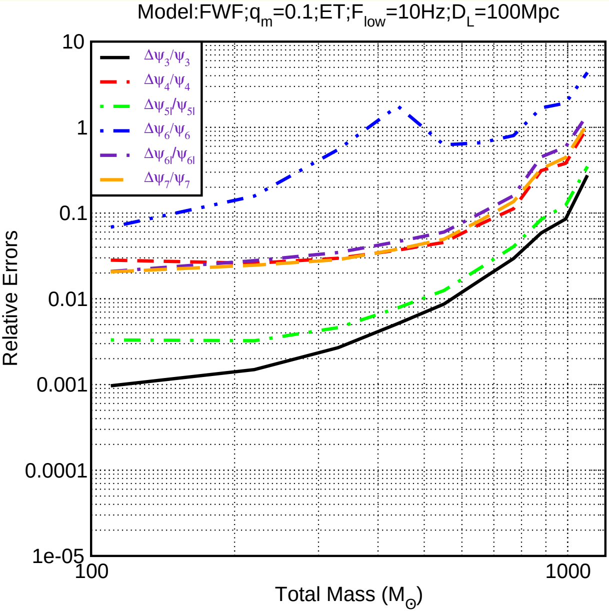
- Extracting the three- and four-graviton vertices from binary pulsars and coalescing binaries., Cannella et al, 2009:
  - Interpreting our tests as measurement of three and four graviton vertices.
- Fundamental Theoretical Bias in Gravitational Wave Astrophysics and the Parametrized Post-Einsteinian Framework., Yunes & Pretorius, 2009:
  - Biases in using GR templates for GW detection problem and implications.

Conclusions & Future directions

- A general way to test the consistency between various PN coefficients, without having to know the details of the underlying theory, is proposed.
- Einstein Telescope may have good sensitivity to test the consistency between various PN coefficients in the GW phasing.
- For masses for which the proposed test can be carried out effectively, use of FWF does not bring in dramatic improvement unless ψ_4 is used as test parameter (which is the worst determined parameter when used as test). (We plan to study this in more detail in the entire parameter space).
- Still it is strongly advised to use the FWF in these tests to avoid systematic biases due to incomplete waveforms.
- Lowering the seismic cut-off from 10Hz to 1Hz brings an order of magnitude improvement in the estimation of ψ_3, which is the best test parameter. (Currently lookin into how its effect in the $m_1 - m_2$ plane.)
- Effects of spin, residual orbital eccentricity and the merger + ringdown part of the waveforms, on this test have to be investigated (In progress).

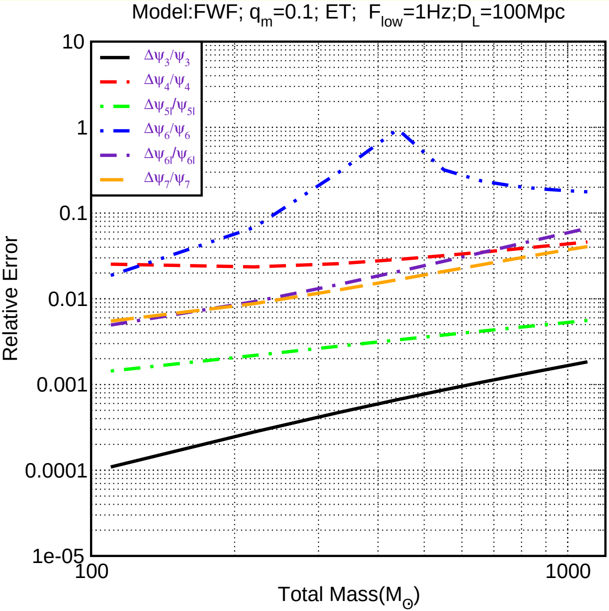
Back up slides

FWF: 10Hz



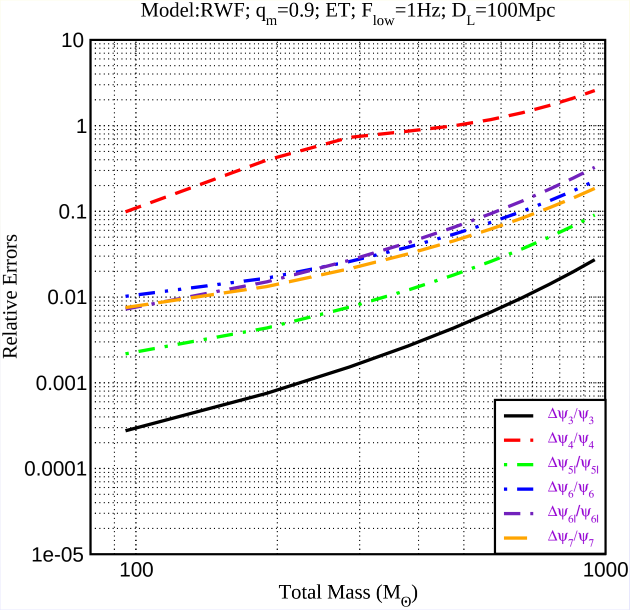
Model:FWF;q_m=0.1;ET;F_low=10Hz;D_L=100Mpc
Δψ3/ψ3
Δψ4/ψ4
Δψ5l/ψ5l
Δψ6/ψ6
Δψ6l/ψ6l
Δψ7/ψ7
10
1
0.1
0.01
0.001
0.0001
1e-05
100
1000
Relative Errors
Total Mass (M⊙)

FWF: 1Hz



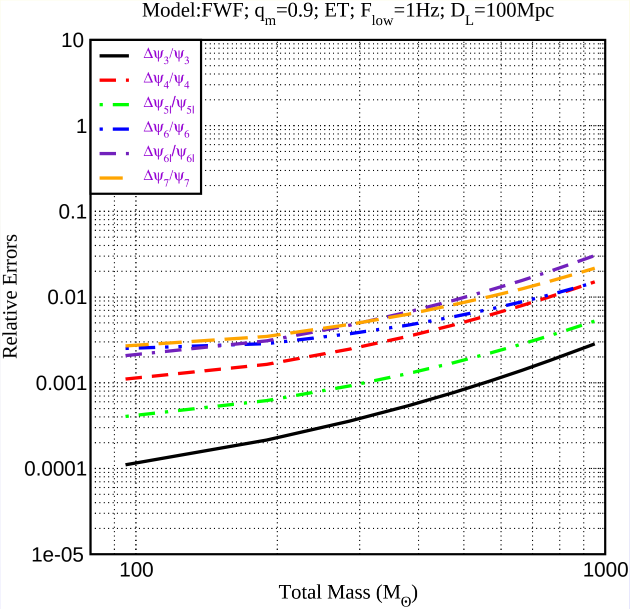
Model:FWF; q_m=0.1; ET; F_low=1Hz;D_L=100Mpc
Relative Error
10
1
0.1
0.01
0.001
0.0001
1e-05
100
1000
Total Mass(M_⊙)
Δψ3/ψ3
Δψ4/ψ4
Δψ5l/ψ5l
Δψ6/ψ6
Δψ6l/ψ6l
Δψ7/ψ7

RWF: 1Hz



Model:RWF; q_m=0.9; ET; F_low=1Hz; D_L=100Mpc
Relative Errors
10
1
0.1
0.01
0.001
0.0001
1e-05
100
1000
Total Mass (M_Θ)
Δψ3/ψ3
Δψ4/ψ4
Δψ5l/ψ5l
Δψ6/ψ6
Δψ6l/ψ6l
Δψ7/ψ7

FWF: 1Hz



Model:FWF; q_m=0.9; ET; F_low=1Hz; D_L=100Mpc
Δψ3/ψ3
Δψ4/ψ4
Δψ5l/ψ5l
Δψ6/ψ6
Δψ6l/ψ6l
Δψ7/ψ7
10
1
0.1
0.01
0.001
0.0001
1e-05
100
1000
Relative Errors
Total Mass (M_Θ)