Testing General Relativity: The Advantage of 1 Hz Vs 10 Hz Cutoff

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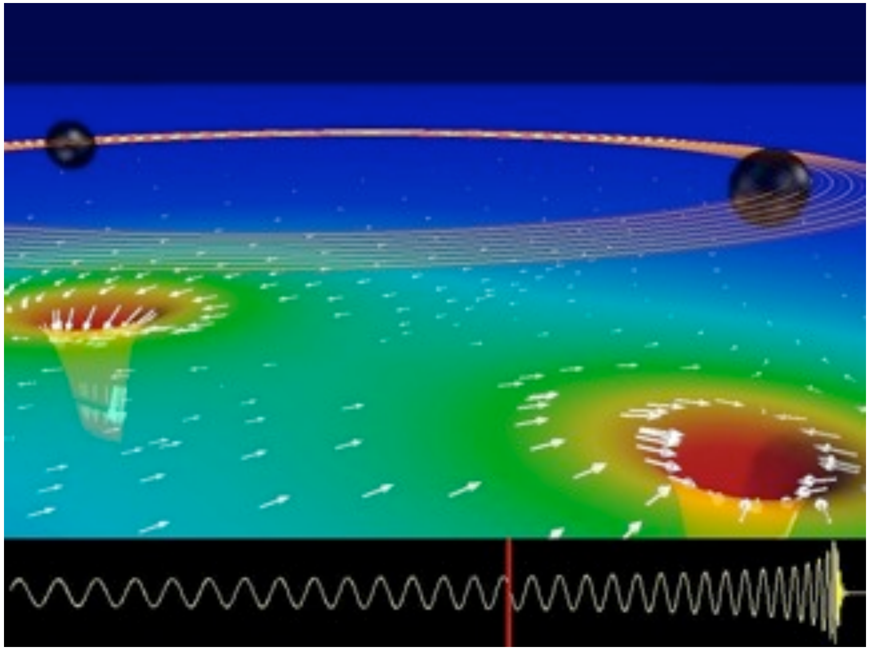
Goal of the talk

To show that gravitational-wave observations of compact binaries offer the best possible tests of general relativity, indeed any metric theory of gravity, beyond the solar system tests and binary pulsar tests.



A metric theory of gravity

- Tests of the equivalence principle have confirmed that the only possible theories of gravity are the so-called metric theories
- A metric theory of gravity is one in which
  - there exists a symmetric metric tensor
  - test bodies follow geodesics of this metric
  - in local Lorentz frames, non-gravitational laws of physics are those of special relativity
  - All non-gravitational fields couple in the same manner to a single gravitational field - that is "universal coupling"
    - Metric is a property of the spacetime
- The only gravitational field that enters the equations of motion is the metric
  - Other fields (scalar, vector, etc.) may generate the spacetime curvature associated with the metric but they cannot directly influence the equations of motion

Will, LRR

Parametrized post-Newtonian formalism

- In slow-motion, weak-field limit all metric theories of gravity have the same structure
  - Can be written as an expansion about the Minkowski metric in terms of dimensionless gravitational potentials of varying degrees of smallness
- Potentials are constructed from the matter variables
- The only way that one metric theory differs from another is in the numerical values of the coefficients that appear in front of the metric potentials
  - Current PPN formalism has 10 parameters
- Testing a metric theory of gravity amounts to constraining the PPN parameters

Will, LRR

Why compact binaries?

- Black holes and neutron stars are the most compact objects
  - Surface potential energy of a test particle is equal to its rest mass energy

$$\frac{GmM}{R} \sim mc^2$$

- Being the most compact objects, they are also the most luminous sources of gravitational radiation
  - The luminosity of a binary could increase a million times in the course of its evolution through a detector's sensitivity band
  - The luminosity of a merging binary black hole (no matter how small or large) outshines the luminosity in all visible matter in the Universe

BBH Signals as Testbeds for GR

- Gravity gets ultra-strong during a BBH merger compared to any observations in the solar system or in binary pulsars
  - In the solar system: $\varphi/c^2 \sim 10^{-6}$
  - In a binary pulsar it is still very small: $\varphi/c^2 \sim 10^{-4}$
  - Near a black hole $\varphi/c^2 \sim 1$
  - Merging binary black holes are the best systems for strong-field tests of GR
- Dissipative predictions of gravity are not even tested at the 1PN level
  - In binary black holes even $(v/c)^7$ PN terms might not be adequate for high-SNR (~100) events

Future tests of GR with GW observations

Testing GR with a compact binary: How does a binary pulsar test GR?

- Non-orbital parameters
  - position of the pulsar on the sky; period of the pulsar and its rate of change
- Five Keplerian parameters, e.g.
  - Eccentricity e
  - Orbital period P_b
  - Semi-major axis projected along the line of sight $a_p \sin i$
- Five post-Keplerian parameters
  - Average rate of periastron advance $\langle d\omega/dt \rangle$
  - Amplitude of delays in arrival of pulses γ
  - Rate of change of orbital period dP_b/dt
  - "range" and "shape" of the Shapiro time delay

Measured effects depend only on the two masses of the binary

- Average rate of periastron advance

$$\langle \dot{\omega} \rangle = \frac{6\pi f_b (2\pi M f_b)^{2/3}}{(1-e^2)}$$

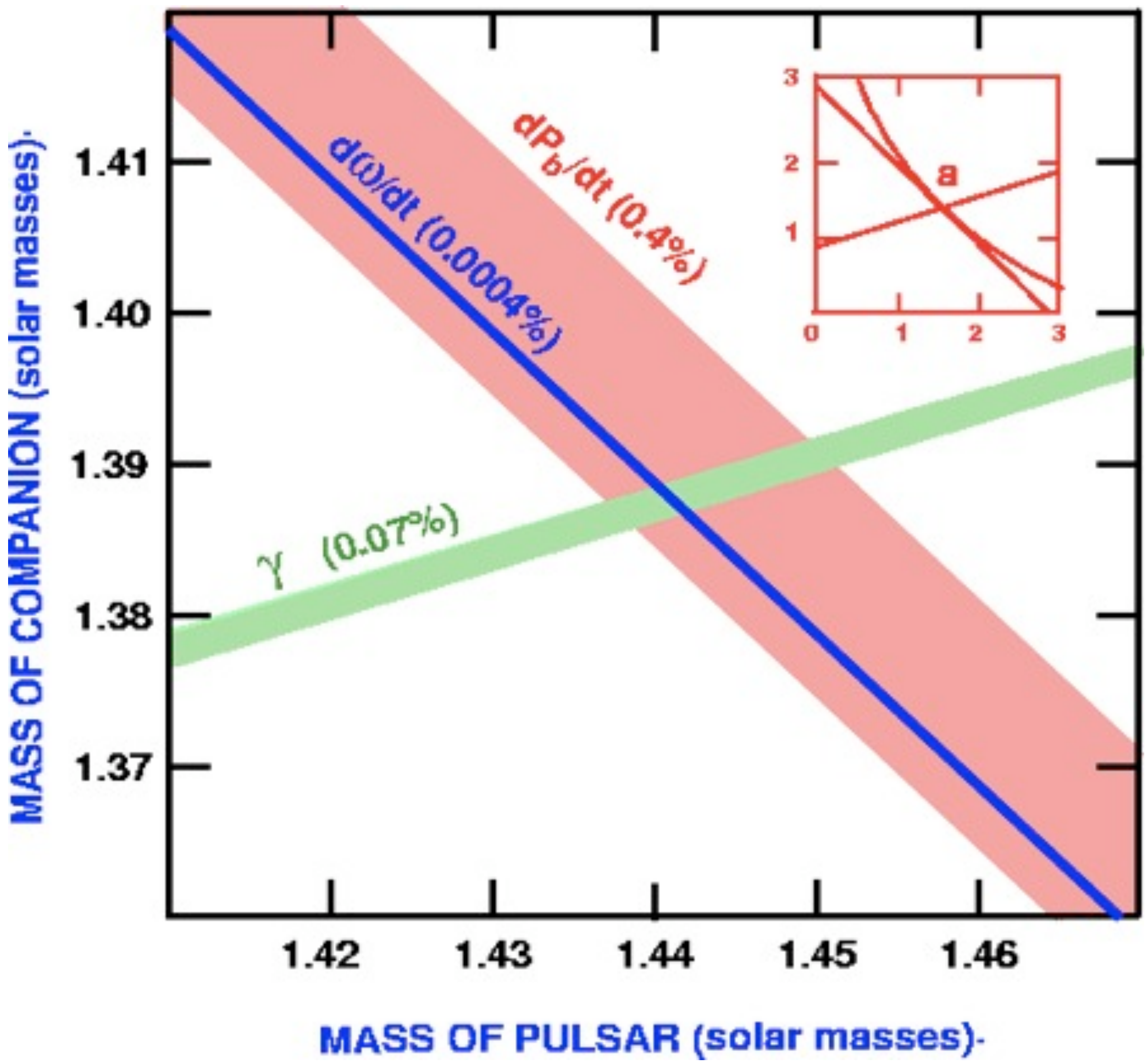
- Amplitude of delays in arrival times

$$\gamma = \frac{(2\pi M f_b)^{2/3}}{2\pi f_b} \frac{e m_2}{M} \left(1 + \frac{m_2}{M}\right)$$

- Rate of change of the orbital period

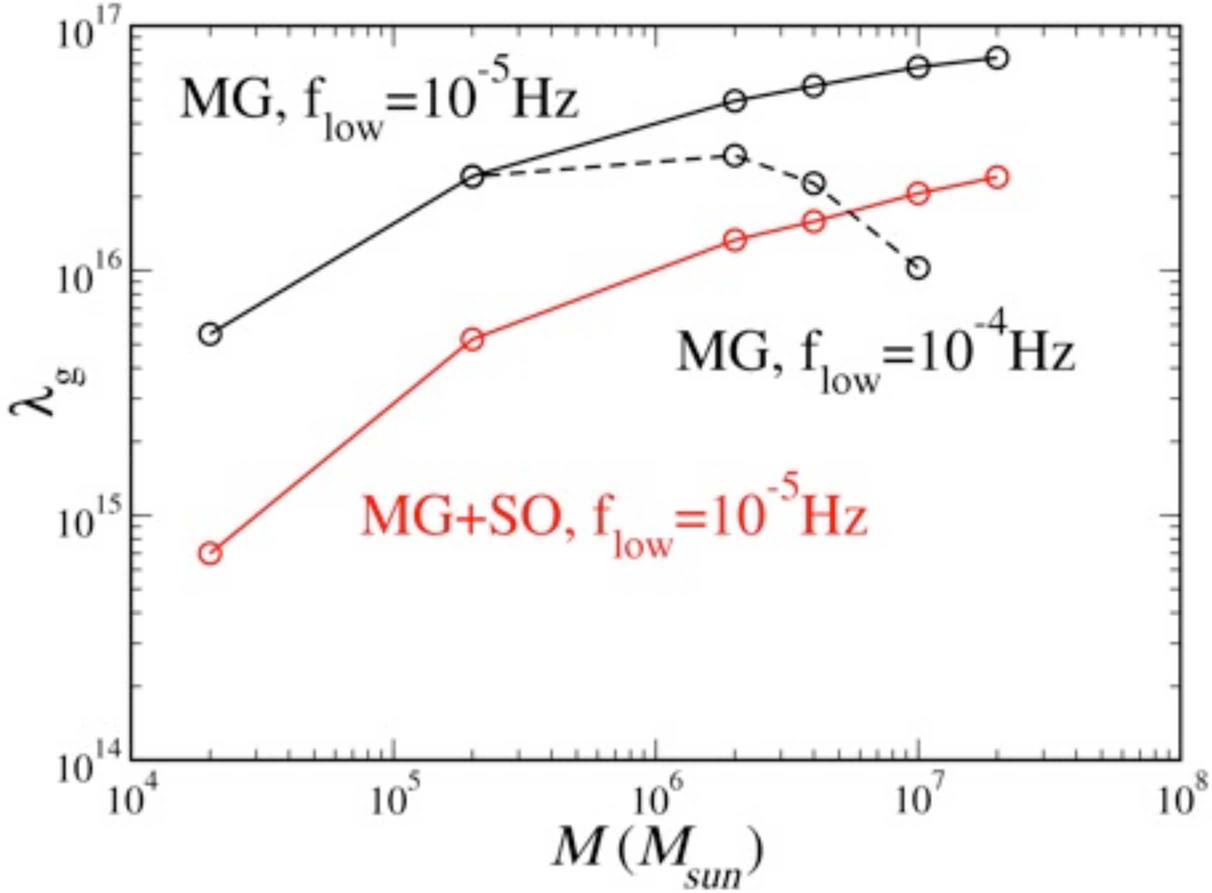
$$\dot{P}_b = -\frac{192}{5} (2\pi \mathcal{M} f_b)^{5/3} F(e)$$

Test of GR in PSR 1913+16



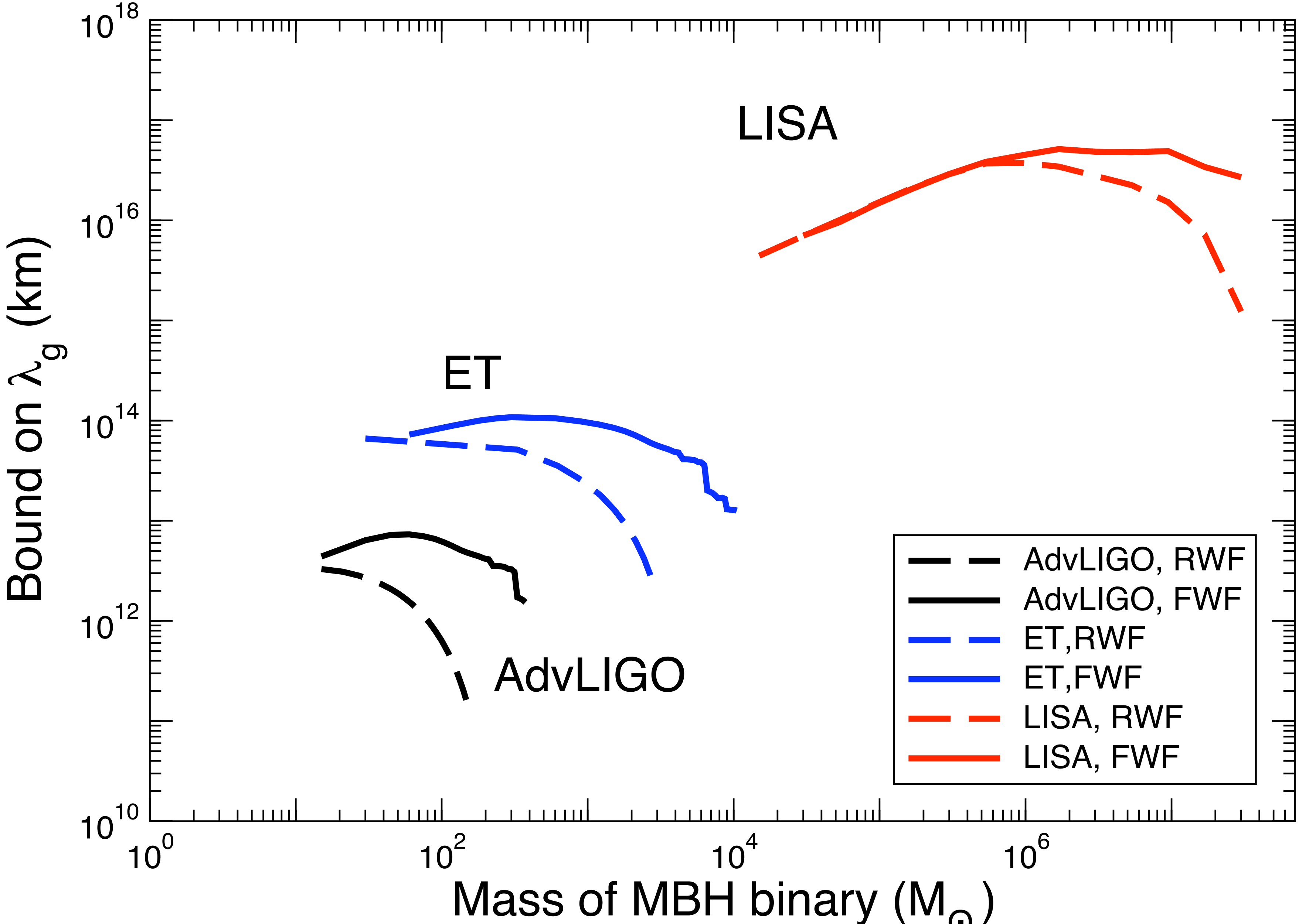
Bound on λ_g as a function of total mass

- Limits based on GW observations will be five orders-of-magnitude better than solar system limits
- Still not as good as (model-dependent) limits based on dynamics of galaxy clusters



Berti, Buonanno and Will (2006)

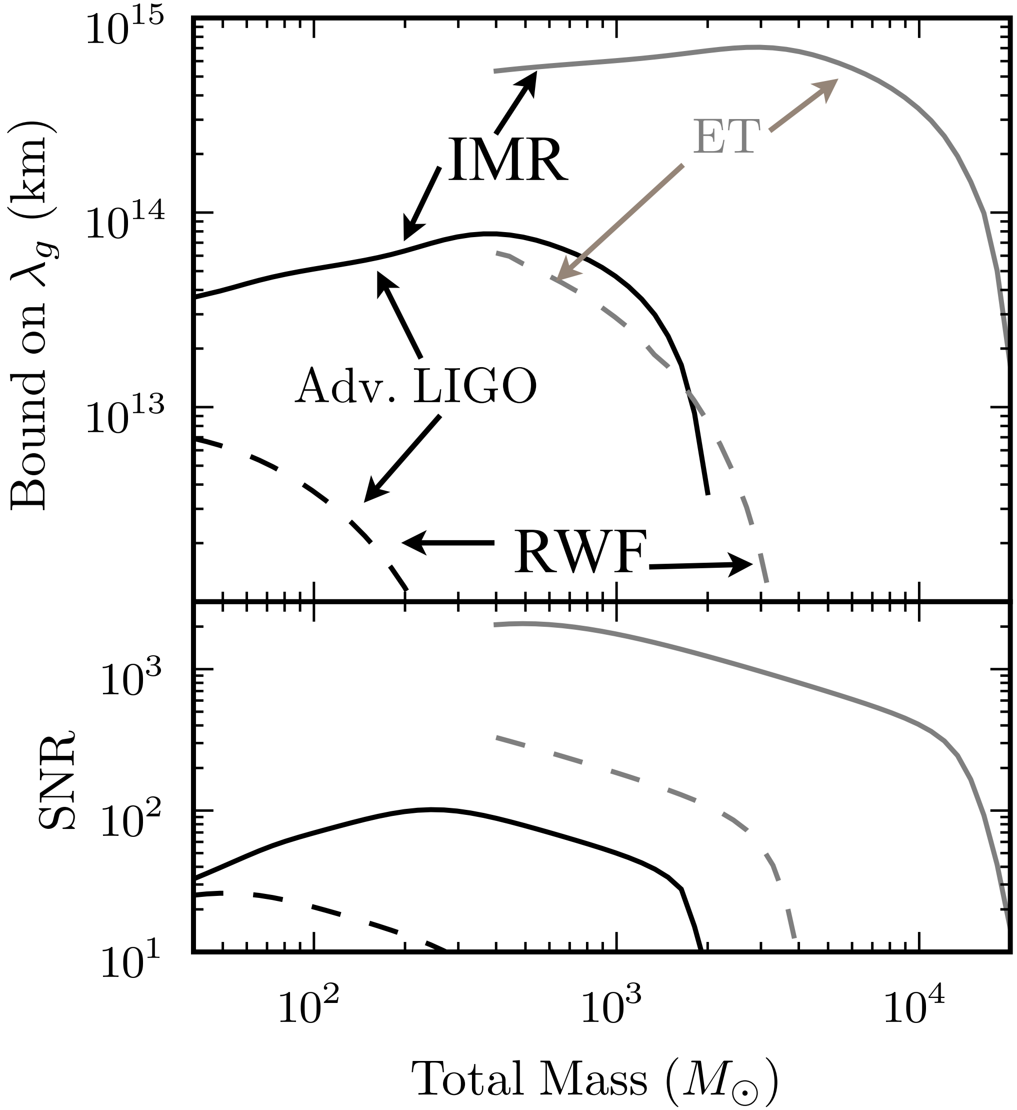
Arun and Will (2009)



Improving bounds with IMR Signals

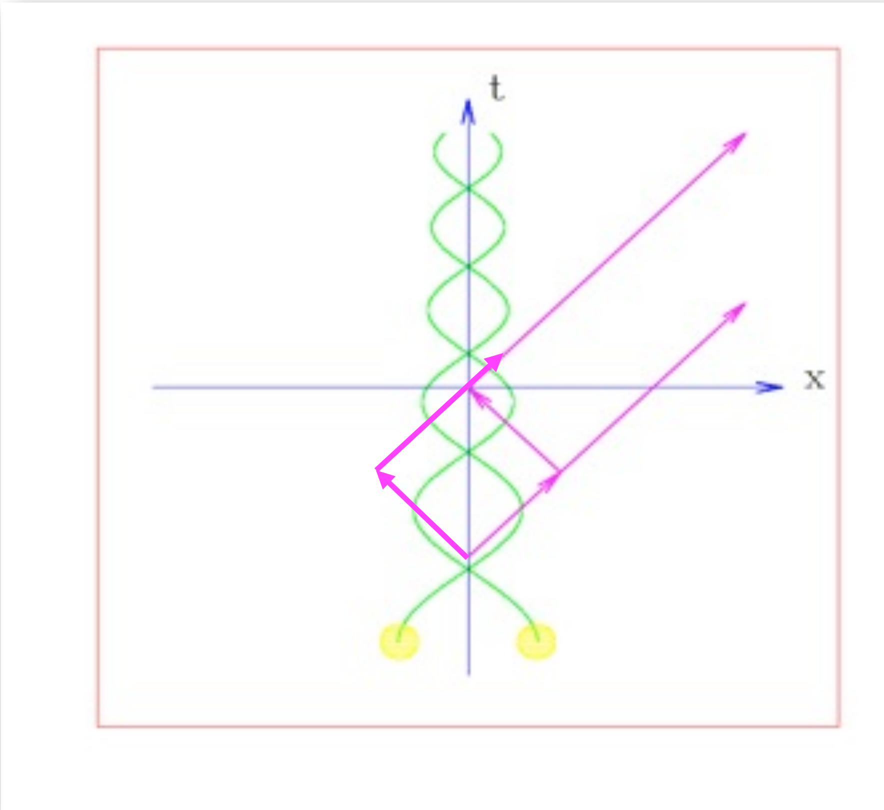
Keppel and Ajith (2010)

- By including the merger and ringdown part of the coalescence it is possible to improve the bound on graviton wavelength
- Equal mass compact binaries assumed to be at 1 Gpc
- ET can achieve 2 to 3 orders of magnitude better bound than the best possible model-independent bounds



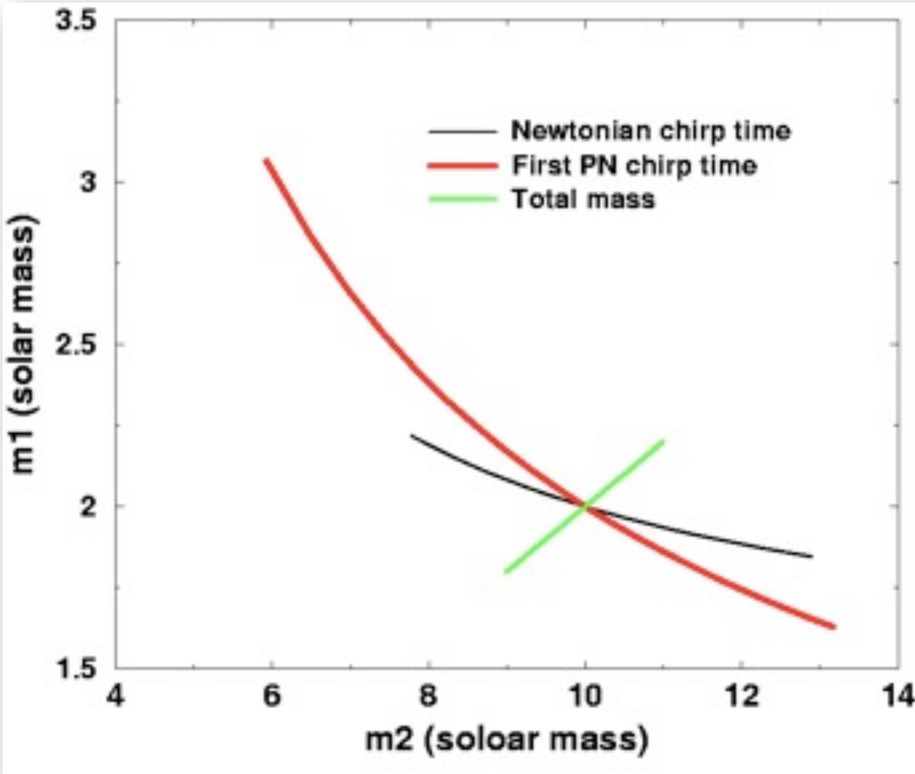
Testing the tail effect

Gravitational wave tails



Blanchet and Schaefer (1994)

Testing the presence of tails



Blanchet and Sathyaprakash (1995)

Testing general relativity with post-Newtonian theory

Post-Newtonian expansion of orbital phase of a binary contains terms which all depend on the two masses of the binary

$$H(f) = \frac{\mathcal{A}(M, \nu, \mathrm{angles})}{D_L} f^{-7/6} \exp\left[-i\psi(f)\right]$$

$$\psi(f) = 2\pi f t_C + \varphi_C + \sum_k \psi_k f^{(k-5)/3}$$

$$\psi_k = \frac{3}{128} (\pi M)^{(k-5)/3} \alpha_k(\nu)$$

$$\alpha_0 = 1, \quad \alpha_1 = 0, \quad \alpha_2 = \frac{3715}{756} + \frac{55}{9}\nu, \ldots$$

Testing general relativity with post-Newtonian theory

- Post-Newtonian expansion of orbital phase of a binary contains terms which all depend on the two masses of the binary

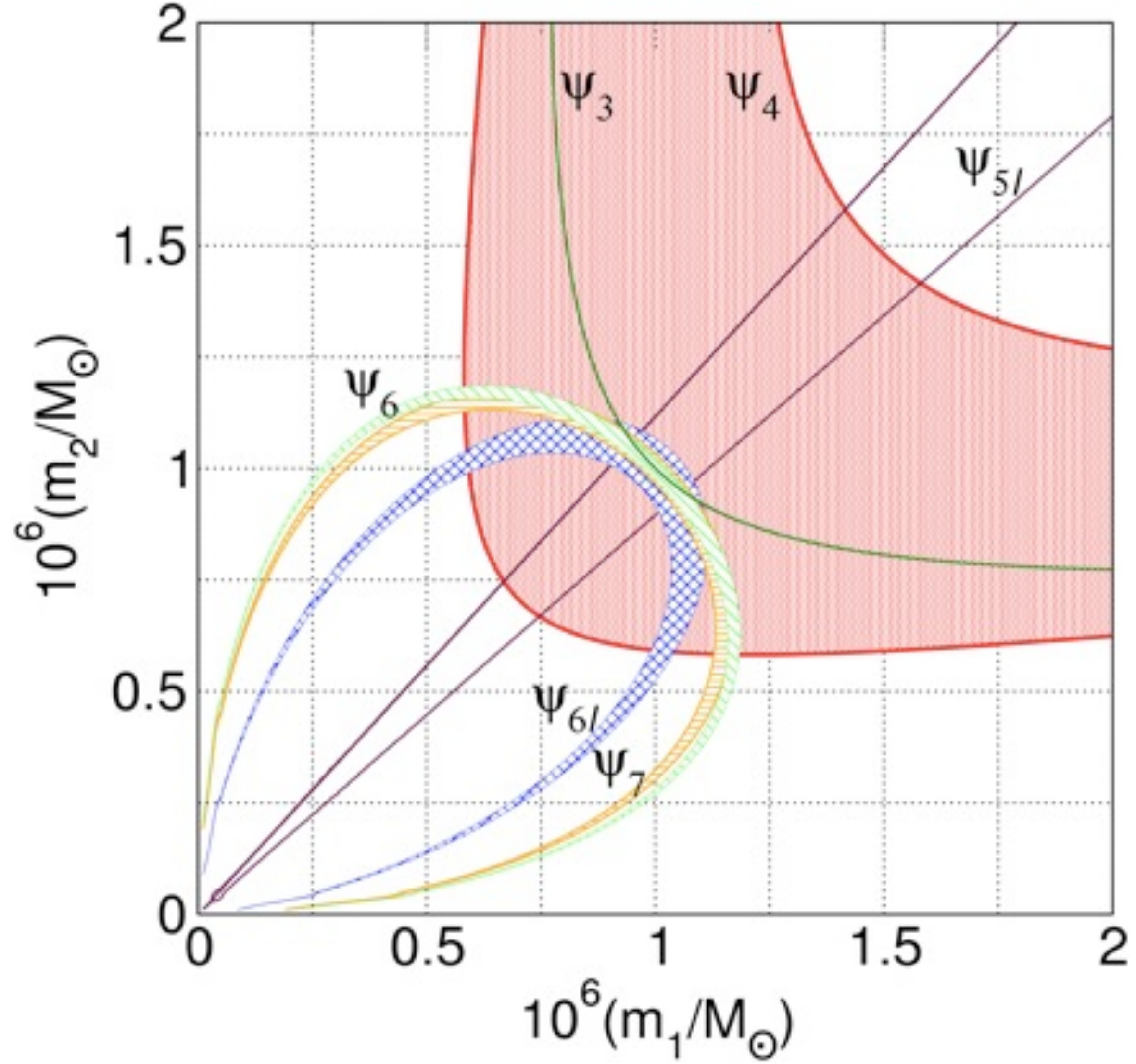
$$\psi_k = \frac{3}{128}(\pi M)^{(k-5)/3}\alpha_k(\nu)$$

- Different terms arise because of different physical effects
- Measuring any two of these will fix the masses
- Other parameters will have to consistent with the first two

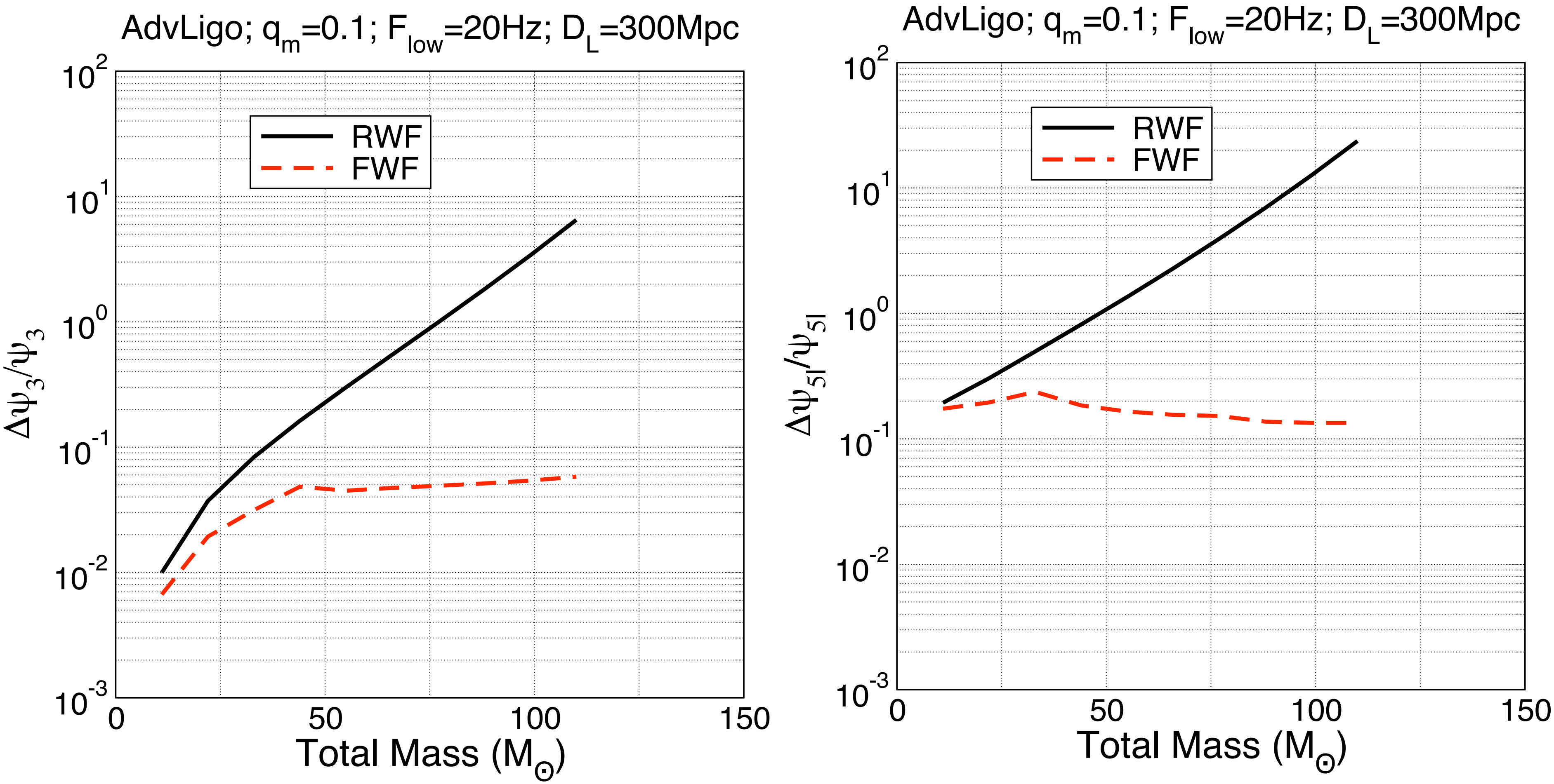
Arun, Iyer, Qusailah, Sathyaprakash (2006a, b)

Testing post-Newtonian theory

Arun, Iyer, Qusailah, Sathyaprakash (2006a, b)

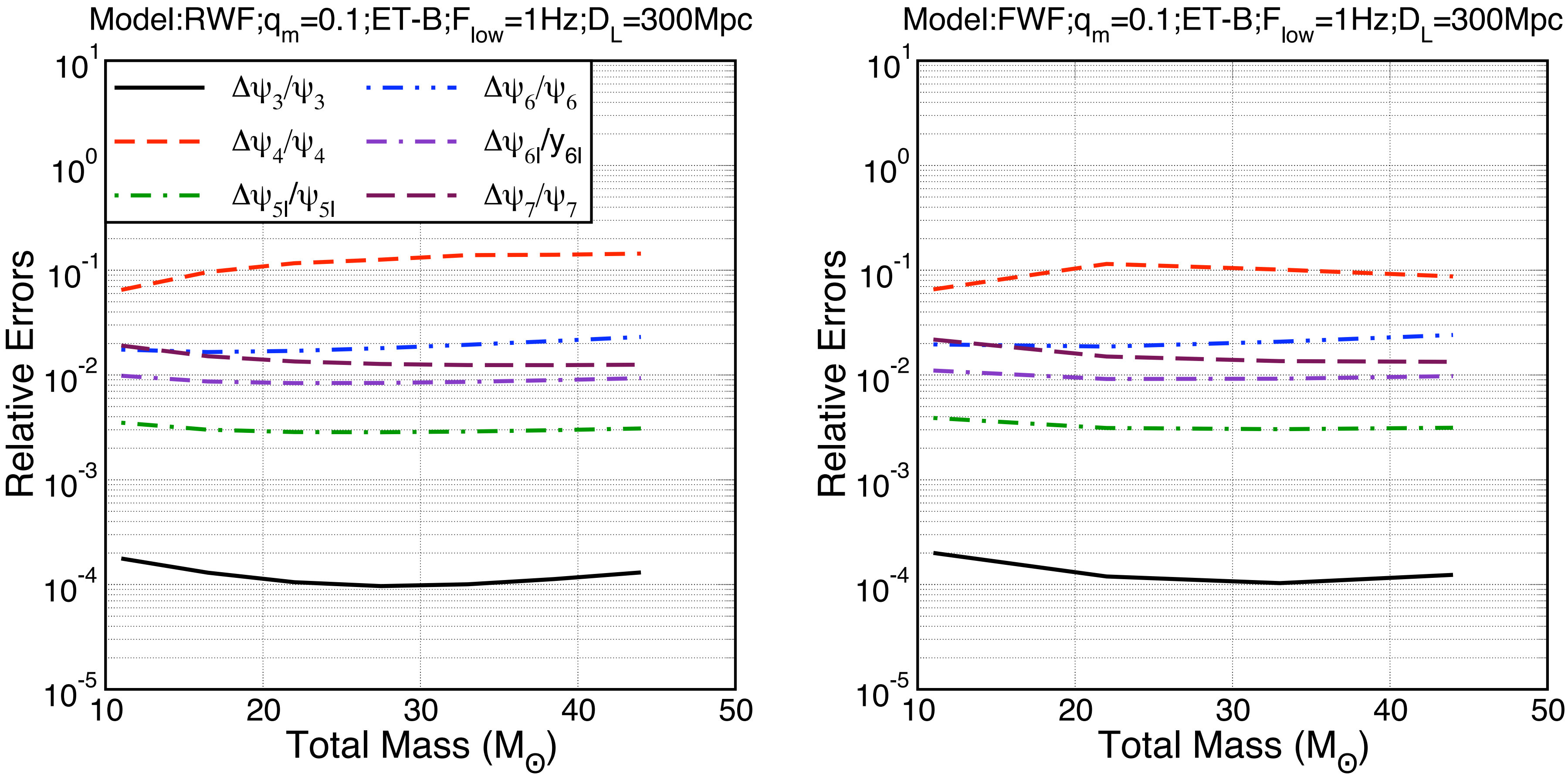


Confirming the presence of tail- and log-terms with Advanced LIGO



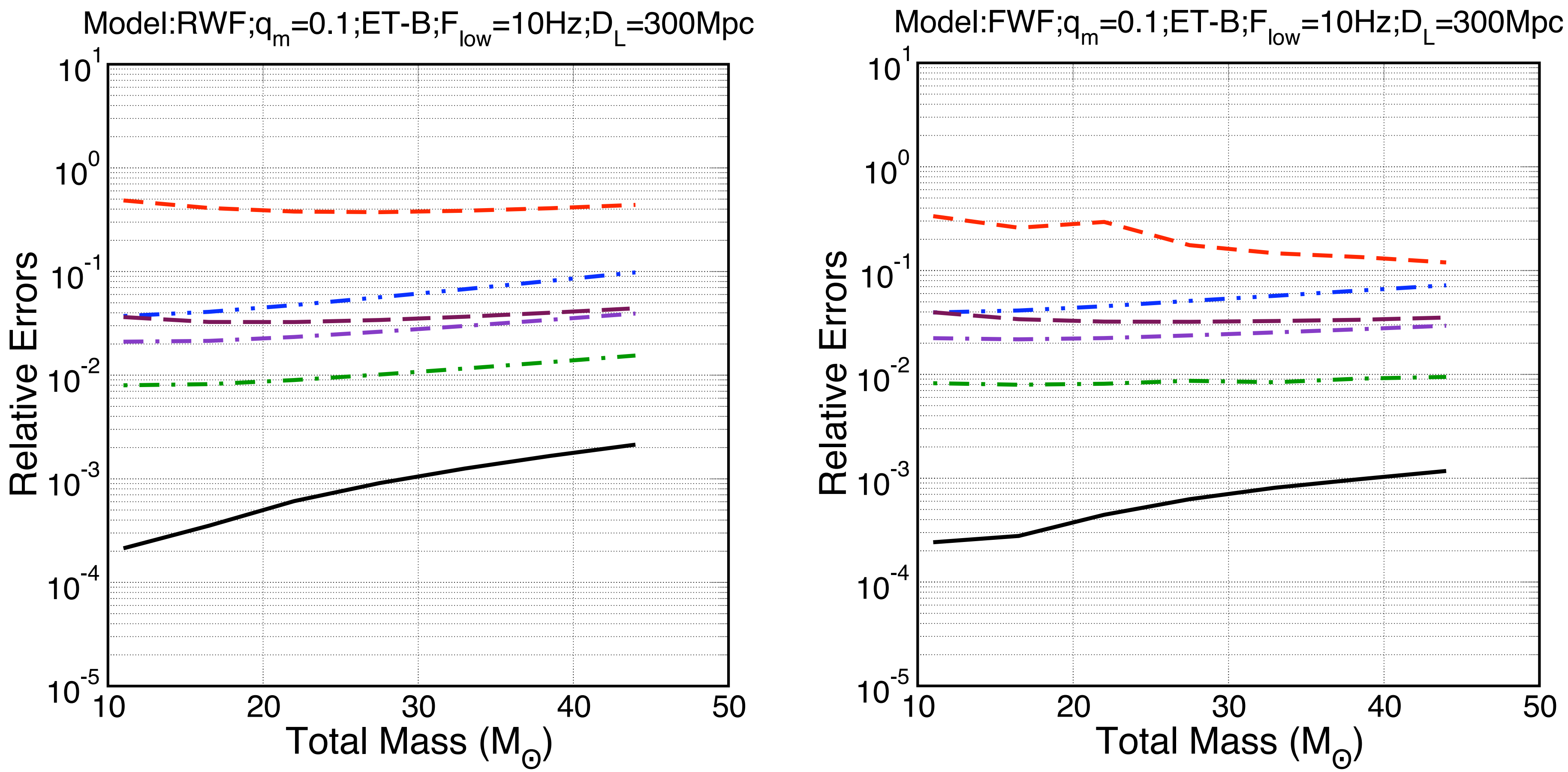
Arun, Mishra, Iyer, Sathyaprakash (2010)

PN parameter accuracies with ET
1 Hz lower cutoff



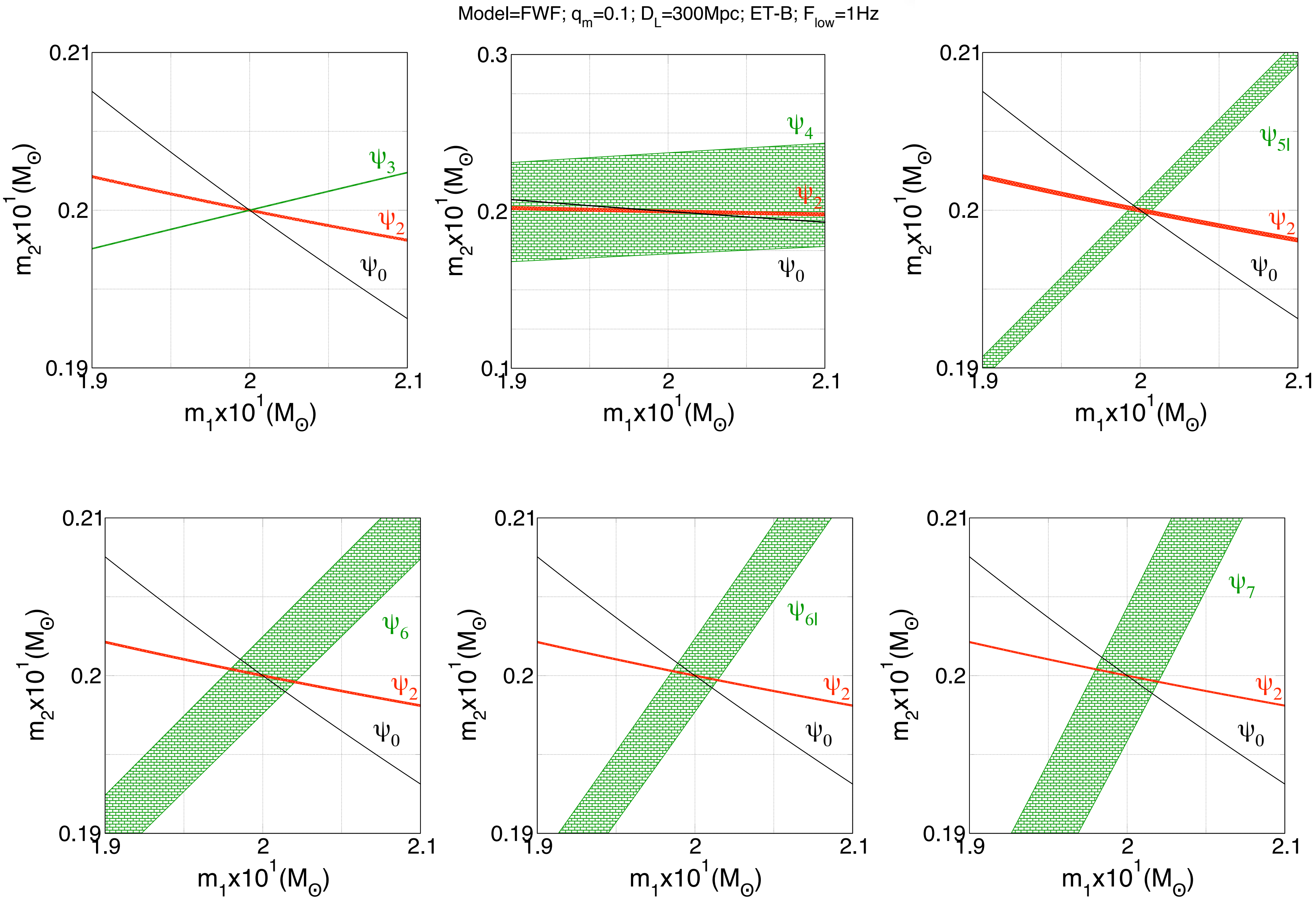
Arun, Mishra, Iyer, Sathyaprakash (2010)

PN parameter accuracies with ET
10 Hz lower cutoff



Arun, Mishra, Iyer, Sathyaprakash (2010)

Test as seen in the plane of component masses



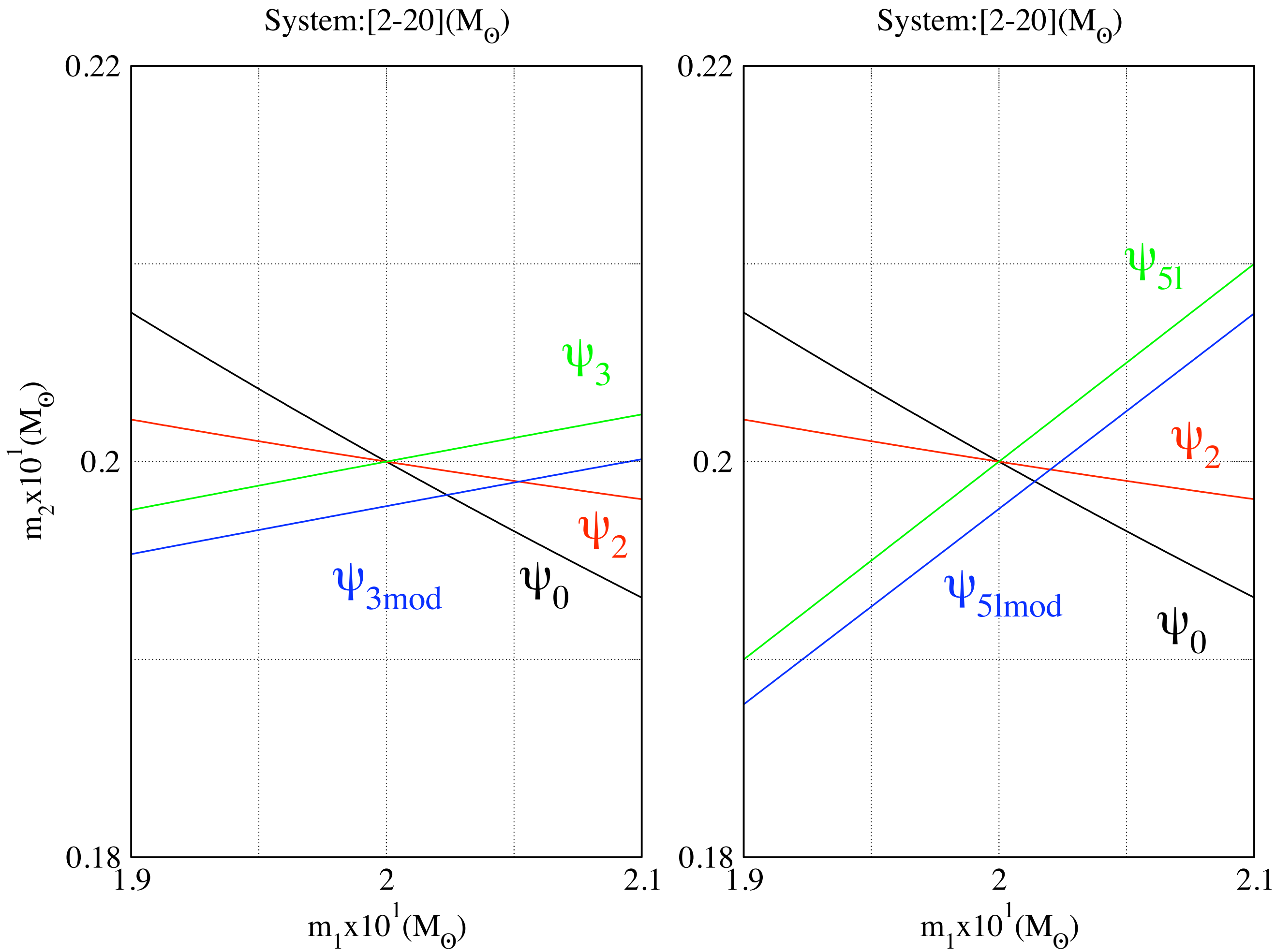
Power of a PN Test

- Suppose the GR k^{th} PN coefficient is $q_k(m_1,m_2)$ while the true k^{th} PN coefficient is $p_k(m_1,m_2)$
- The "measured value of the k^{th} PN coefficient is, say, p_0
- The curve $q_k(m_1,m_2)= p_0$ in the (m_1,m_2) plane will not pass through the masses determined from the other parameters

Arun, Mishra, Iyer, Sathyaprakash (2010)

Power of the PPN test

Effect of changing the coefficients ψ_3 and ψ_{51} by 1% on the test.

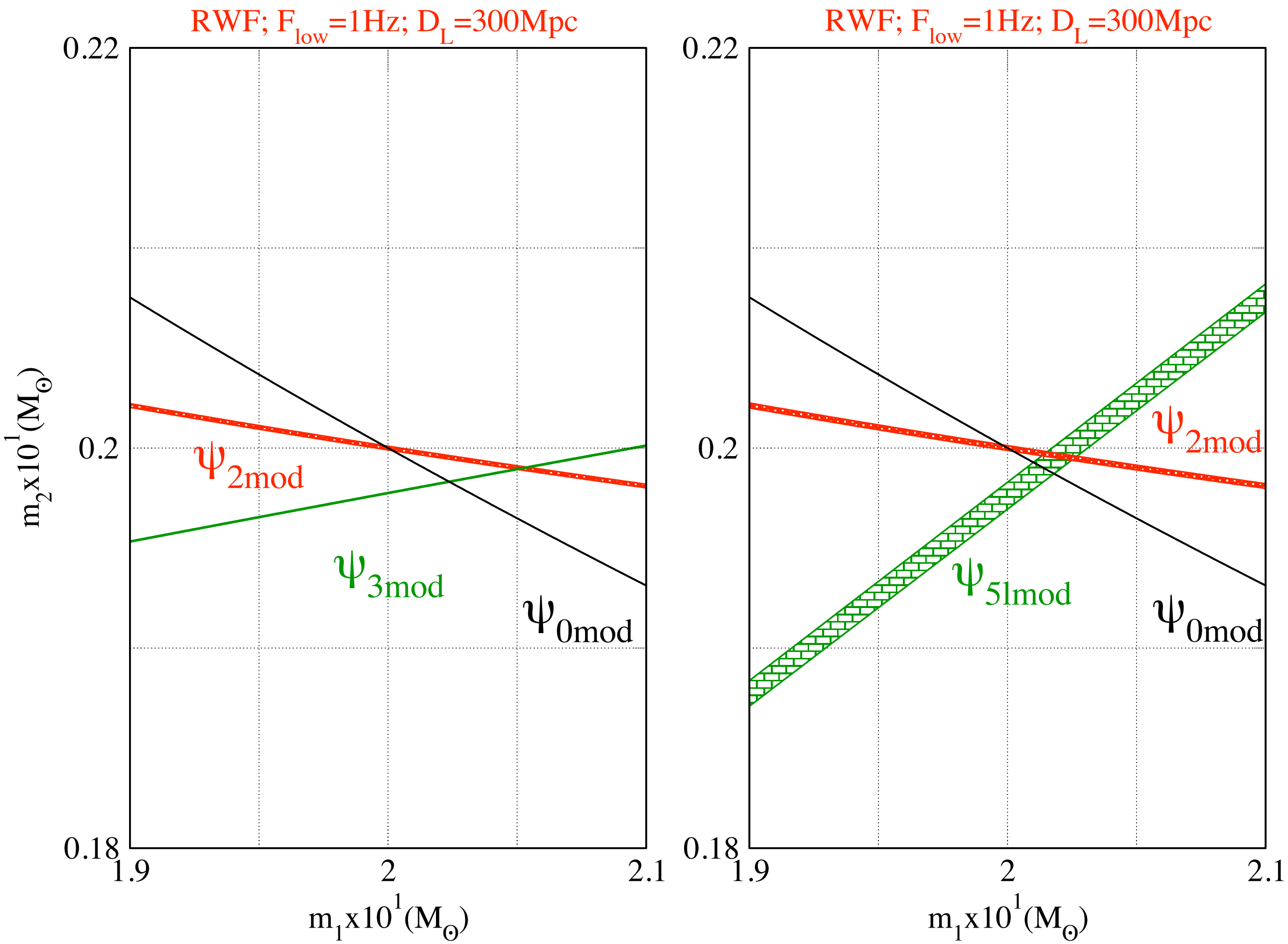


NOTE: Blue curve in the plot corresponds to the new ψ_k

Arun, Mishra, Iyer, Sathyaprakash (2010)

Efficacy of the PPN Test

Effect of changing the coefficients ψ_3 and ψ_{51} by 1% on the test.



NOTE: Reference System: (2-20) $(M_\odot)$

Arun, Mishra, Iyer, Sathyaprakash (2010)

Conclusions

- ET could put interesting bounds on the mass of gravitons
  - Could surpass the model-independent solar systems bounds by several orders of magnitude
- Gravitational-wave observations offer new tests of general relativity in the dissipative strongly non-linear regime
  - Advanced LIGO can already test tails of gravitational waves and the presence of the log-term in the PN expansion
  - Einstein Telescope will measure all known PN coefficients (except one at 2PN order) to a good accuracy