Synergy between observations and Numerical Relativity in the ET era

Luciano Rezzolla

Albert Einstein Institute, Potsdam, Germany

Dept. of Physics and Astronomy, Louisiana State Univ. Louisiana, USA

Observatoire de la CÔTE d'AZUR

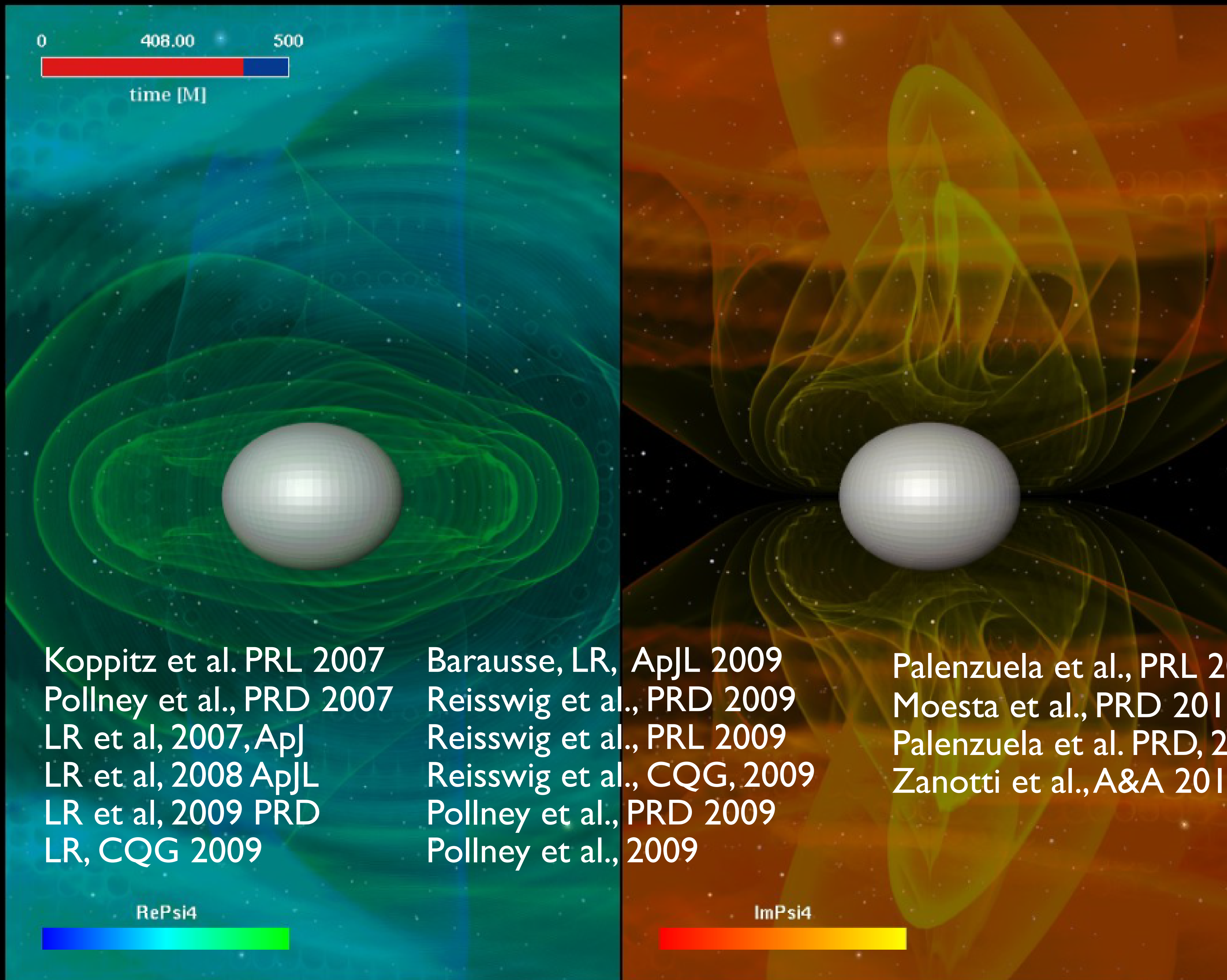
ET WG4

Nice, 1-2/09/10

Plan of the talk

- Synergies with binary black holes (BHs)
    - ★hybrid waveforms
    - ★NINJA projects
- Synergies with binary neutron stars (NSs)
    - ★equal/unequal mass, no magnetic field
    - ★equal mass, magnetic field
    - ★interfacing with EOB

Binary Black Holes



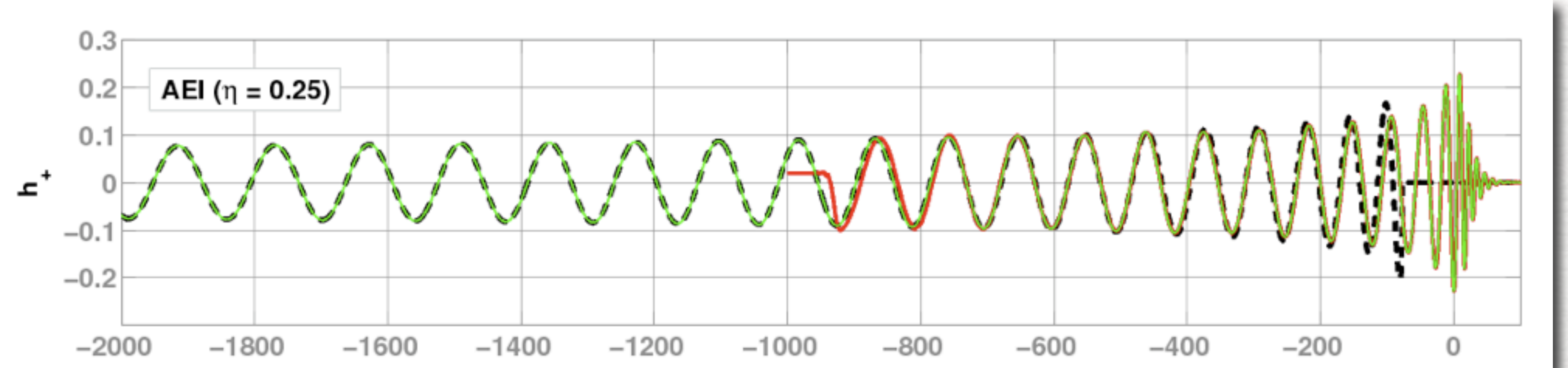
Koppitz et al. PRL 2007
Pollney et al., PRD 2007
LR et al, 2007, ApJ
LR et al, 2008 ApJL
LR et al, 2009 PRD
LR, CQG 2009

Barausse, LR, ApJL 2009
Reisswig et al., PRD 2009
Reisswig et al., PRL 2009
Reisswig et al., CQG, 2009
Pollney et al., PRD 2009
Pollney et al., 2009

Palenzuela et al., PRL 2009
Moesta et al., PRD 2010
Palenzuela et al. PRD, 2010
Zanotti et al., A&A 2010

A hybrid waveform

Ajith et al., CQG 2007
Ajith et al., PRD 2008



Red line is the numrel waveform

Black dashed line is the 3.5PN waveform

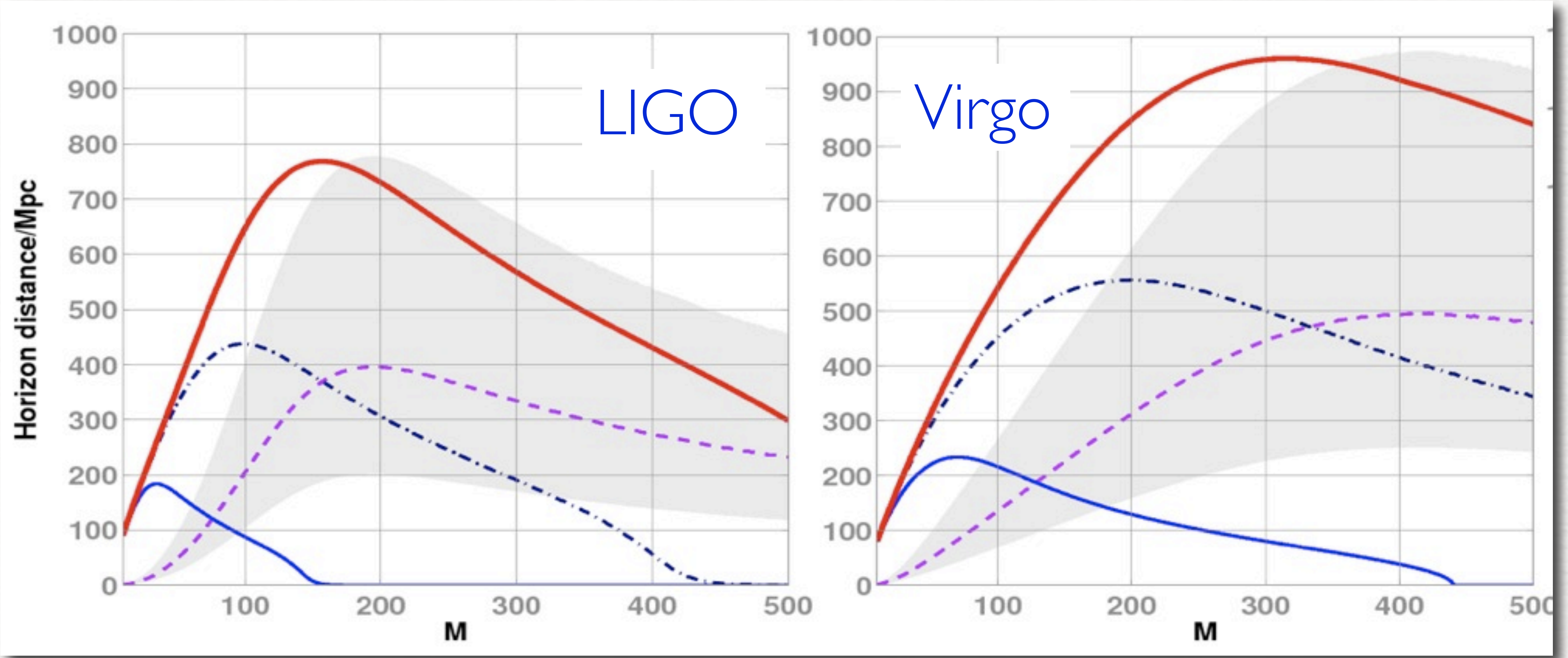
Green line is the hybrid waveform

Once the hybrid waveform is computed, it can be parametrized in the Fourier domain via 10 phenomenological parameters (4 for the amplitude, 6 for the phase).

The goal is to reduce them to the 2 physical ones: M mass of the binary and $\eta = M_1 M_2/(M_1+M_2)^2$ symmetric mass-ratio

What is this good for?

Red line: complete (inspiral, merger, ringdown) template
Blue line: PN template truncated at ISCO
Black line: EOB template truncated at light-ring
Purple line: uses ringdown templates

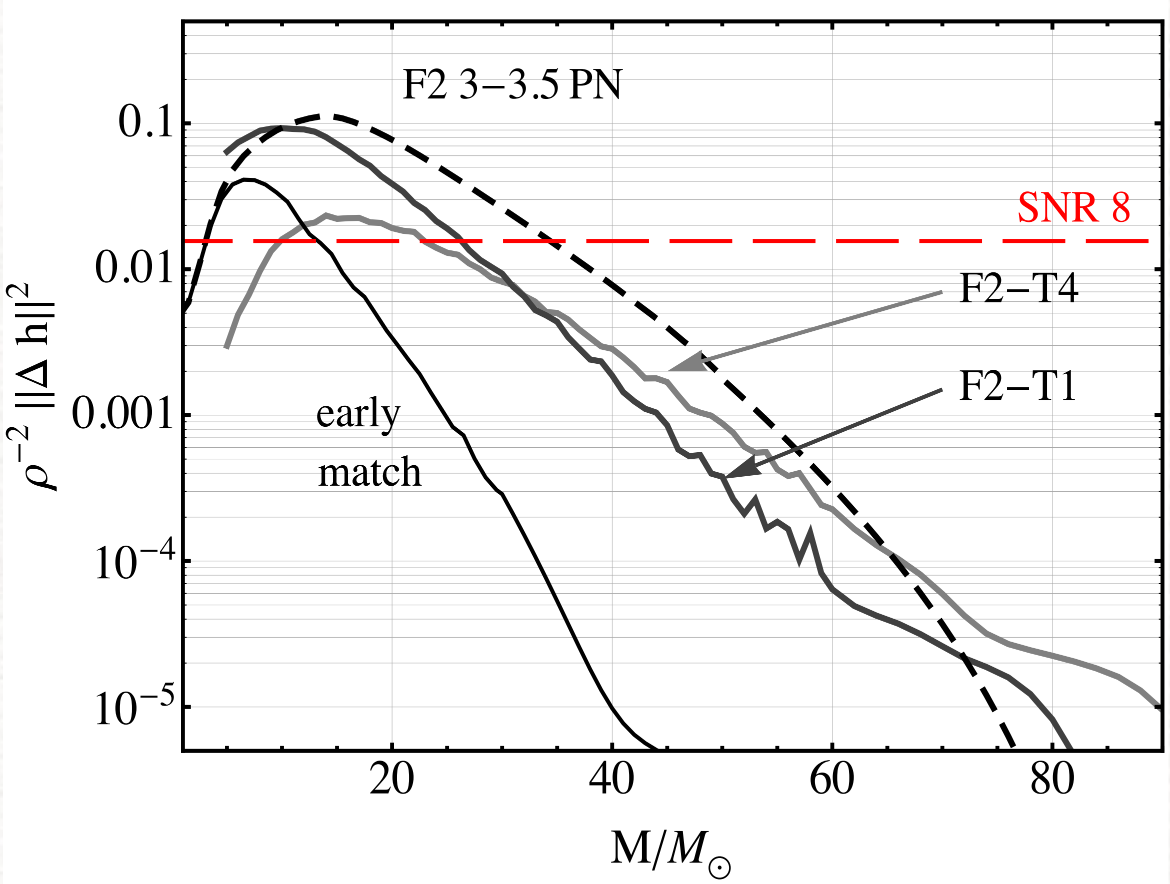


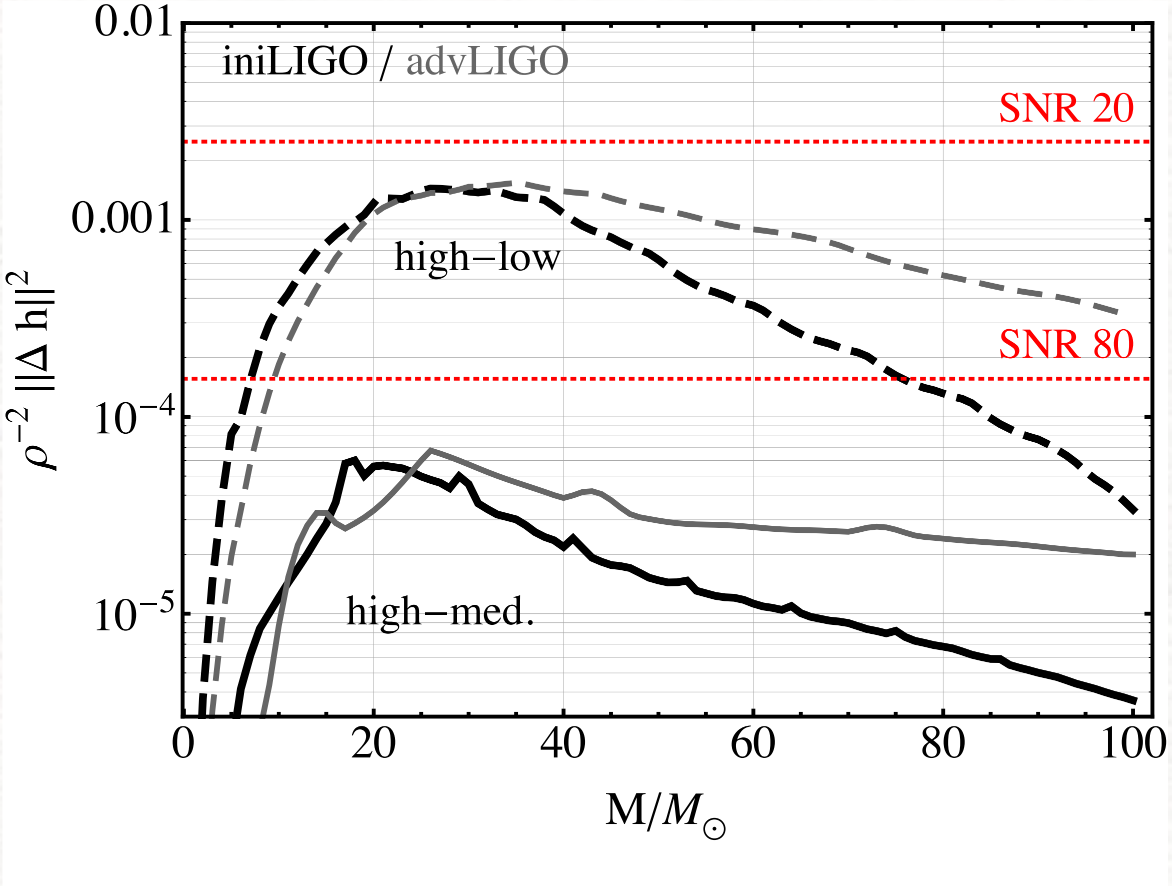
Expanding to aligned-spins binaries

Santamaria et al., PRD 2010

Accuracy of hybrid waveforms

- PN is biggest source of error: we need either longer NR waveforms or better PN approximation
- Plots show "distance" between various hybrids





difference between different PN approximants: distinguishable even at SNR ~ 8 for AdvLIGO

comparison of different NR resolutions: *indistinguishable even at SNR ~ 20*

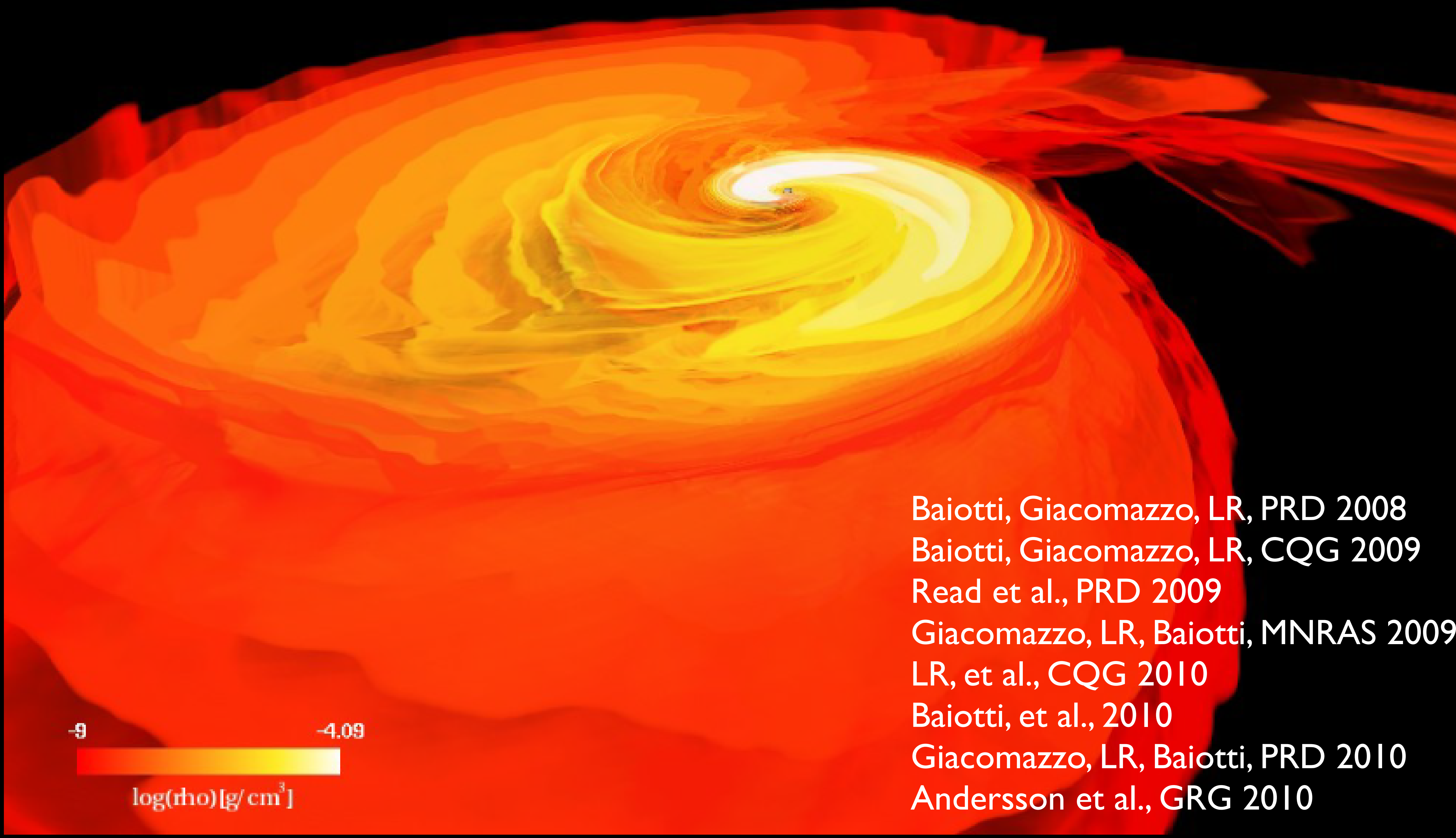
Numerical Injection Analysis (NINJA)

- **NINJA** projects aim at using NR waveforms in data-analysis pipelines
- **NINJA1**: involved10 DA groups and 9 NR groups using simulated data (`Aylott et al, CQG 2010 + 5 technical papers`). AEI has played key role thanks also to coordination of B. Krishnan.
- **NINJA1**: has shown that current LIGO-Virgo (LVC) search pipelines find signals at about the expected sensitivities.
- **NINJA1**: has shown that better statistics are needed and hence a more consistent quality control NR data.
- **NINJA1**: has also shown that simulated detector data is not sufficient: Gaussian noise prevented false alarm calculation
- Most importantly, **NINJA1** has demonstrated that a collaborative effort between DA and NR is possible and a lot of fun too!

Numerical Injection Analysis (NINJA)

- **`NINJA2`**: launched at the NRDA meeting at the AEI in 2008 and makes improvements on several aspects:
  - ✳ uses real data (Gaussian noise prevented false alarm calculation)
  - ✳ sets constraints on the length of NR waveforms (waveforms of NINJA1 were useful only for very large masses) but not on accuracy
  - ✳ uses waveforms for aligned-spin binaries
- Goals **`NINJA2`**:
  - ✳ systematic tests and comparisons of current searches injected in real LIGO-Virgo data
- Current status of **`NINJA2`**:
  - ✳ NR waveforms have been produced to specifications
  - ✳ the analyses are under way
  - ✳ Results expected next year
- **`NINJA-matter`**: still under development (limited man power) but technology is already in place

Binary Neutron Stars



Baiotti, Giacomazzo, LR, PRD 2008
Baiotti, Giacomazzo, LR, CQG 2009
Read et al., PRD 2009
Giacomazzo, LR, Baiotti, MNRAS 2009
LR, et al., CQG 2010
Baiotti, et al., 2010
Giacomazzo, LR, Baiotti, PRD 2010
Andersson et al., GRG 2010

The two-body problem: GR

Baiotti, Giacomazzo, LR, PRD (2008); Baiotti, Giacomazzo, LR, CQG (2009); Giacomazzo, LR, Baiotti, MNRAS (2009); LR, et al (CQG 2010); Giacomazzo, LR, Baiotti, PRD (2010)

Modelling binary black holes (BHs) and binary neutron stars (BNSs) is very different and not because the eqs are different

In the case of BHs we know what to **expect**:

BH + BH ⟶ BH + gravitational waves (GWs)

In the case of NSs the question is more **subtle** because in general the merger will lead to an hyper-massive neutron star (HMNS), namely a self-gravitating object in metastable equilibrium:

NS + NS ⟶ HMNS + GWs + **... ?** ⟶ BH + GWs

It's in the intermediate stage that all the physics and complications are; the rewards are however high (GRBs, nuclear physics, etc).

"merger → HMNS → BH + torus"

Quantitative differences are produced by:

- **differences induced by the gravitational MASS:**
  a binary with smaller mass will produce a HMNS further away from the stability threshold and will collapse at a later time
- **differences induced by the EOS ("cold" or "hot"):**
  a binary with an EOS with large thermal capacity (ie hotter after merger) will have more pressure support and collapse later
- **differences induced by MASS ASYMMETRIES:**
  tidal disruption before merger; may lead to prompt BH
- **differences induced by MAGNETIC FIELDS:**
  the angular momentum redistribution via magnetic braking or MRI can increase/decrease time to collapse
- **differences induced by RADIATIVE PROCESSES:**
  radiative losses will alter the equilibrium of the HMNS

Cold vs Hot EOSs

Simplest example of a **"cold"** EOS is the polytropic EOS. This isentropic: internal energy (temperature) increases/decreases only by mechanical work (compression/expansion)

$$p = K\rho^{\Gamma}\,, \qquad \epsilon = \frac{K\rho^{\Gamma-1}}{\Gamma-1}$$

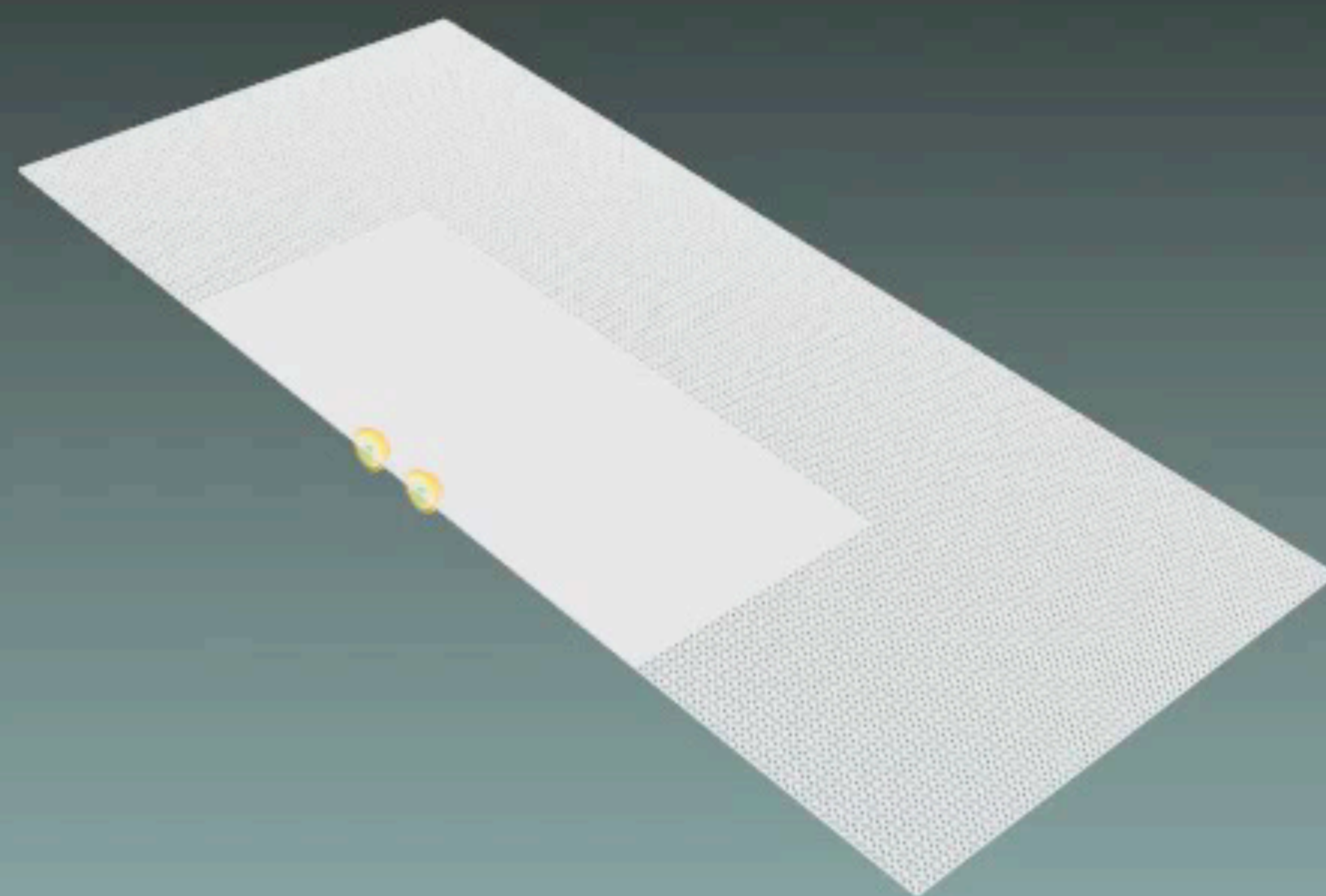
Simplest example of a **"hot"** EOS is the ideal-fluid EOS. This non-isentropic in presence of shocks: internal energy (i.e. temperature) can increase via shock heating.

$$p = \rho\epsilon(\Gamma-1)\,, \quad \partial_t\epsilon = \ldots$$

A **cold** EOS is optimal for the inspiral; a **hot** EOS is essential after the merger. Take them as extremes of possible behaviours

Animations: Kaehler, Giacomazzo, LR

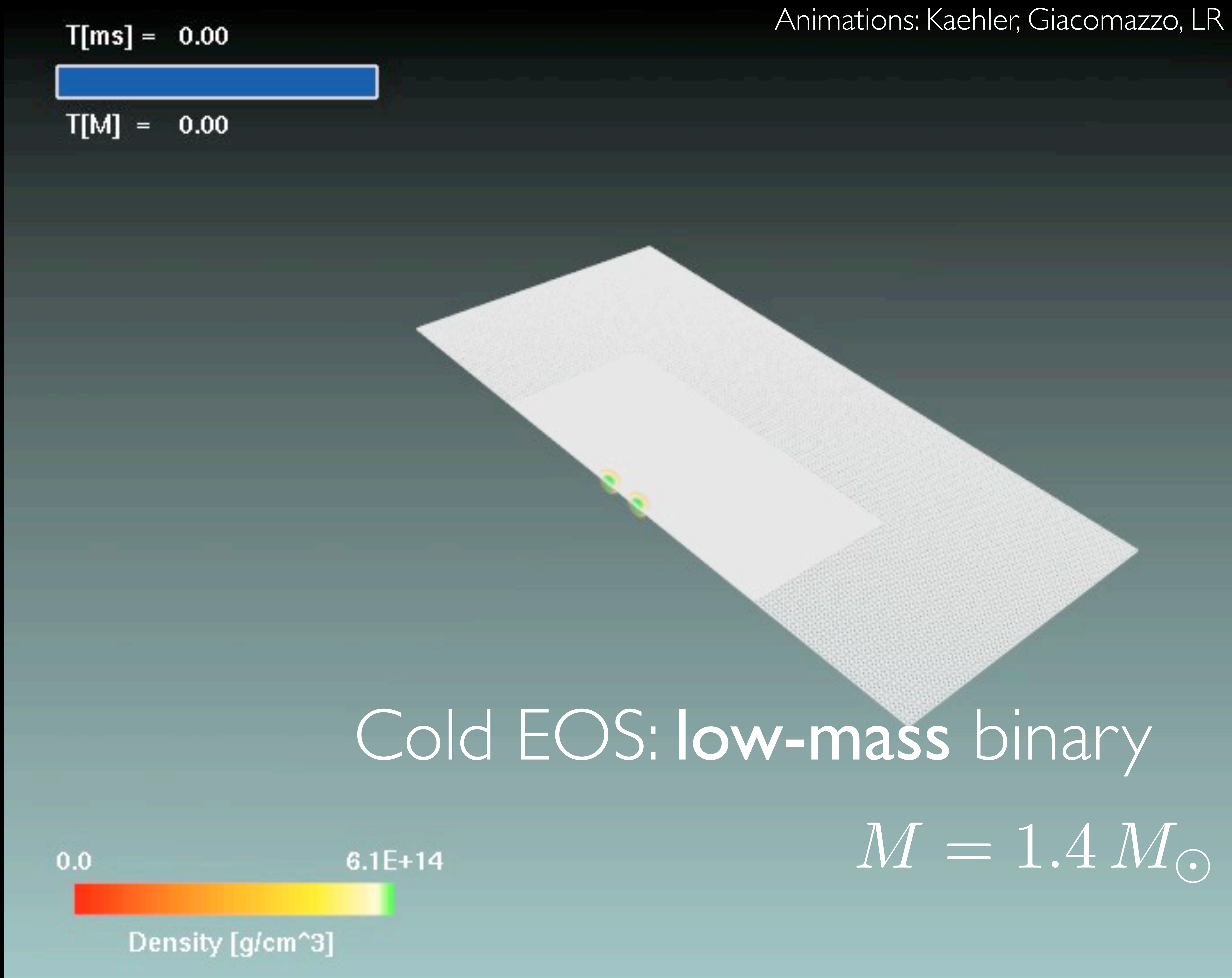
Baiotti, Giacomazzo, LR (PRD 2008, CQG 2008)



Cold EOS: **high-mass** binary

$$M = 1.6\, M_{\odot}$$

Animations: Kaehler, Giacomazzo, LR



Cold EOS: **low-mass** binary

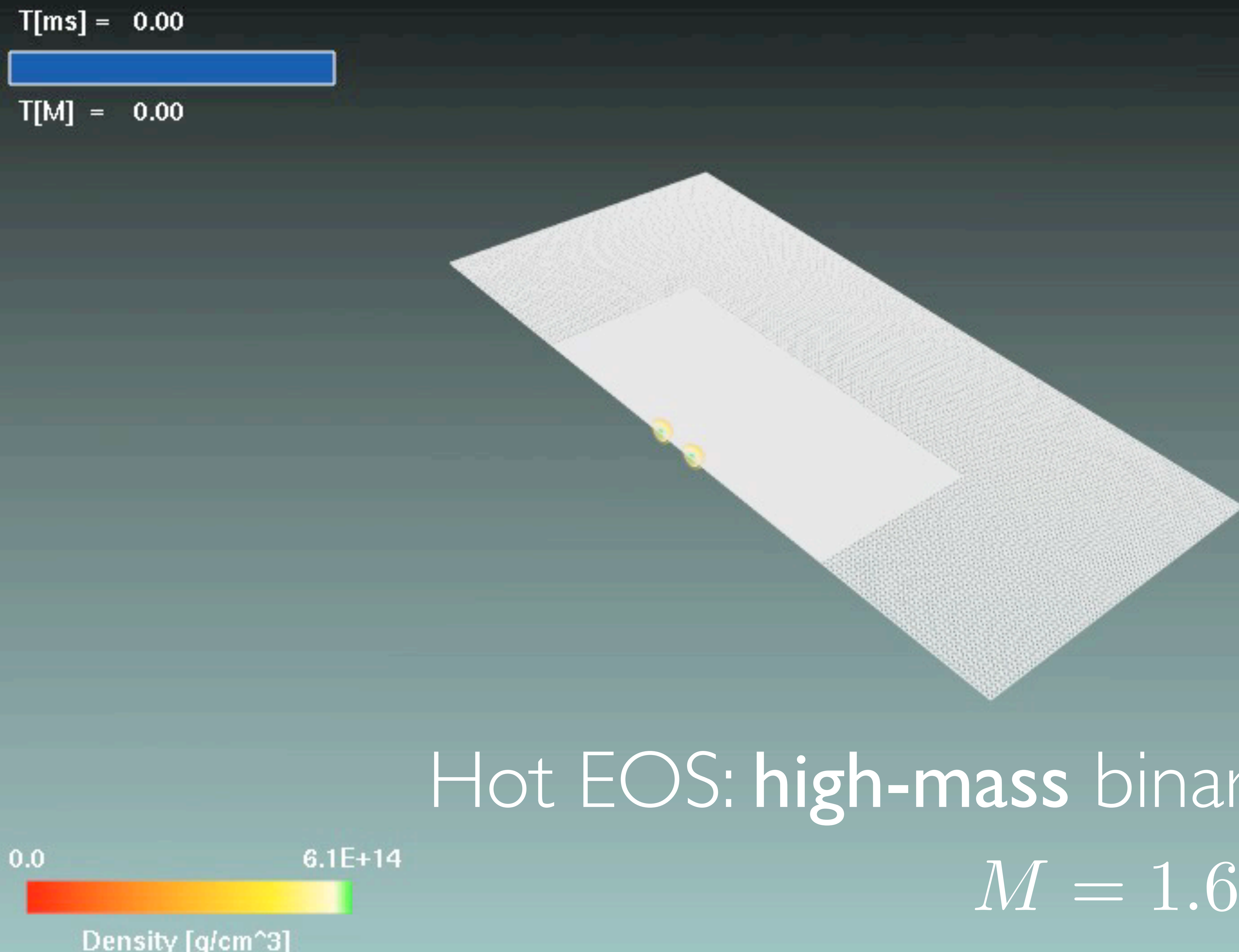
$M = 1.4\,M_\odot$

“merger → HMNS → BH + torus”

Quantitative differences are produced by:

- **differences induced by the gravitational MASS:**
  a binary with smaller mass will produce a HMNS further away from the stability threshold and will collapse at a later time
- **differences induced by the EOS (“cold” or “hot”):**
  a binary with an EOS with large thermal capacity (ie hotter after merger) will have more pressure support and collapse later
- **differences induced by MASS ASYMMETRIES:**
  tidal disruption before merger; may lead to prompt BH
- **differences induced by MAGNETIC FIELDS:**
  the angular momentum redistribution via magnetic braking or MRI can increase/decrease time to collapse
- **differences induced by RADIATIVE PROCESSES:**
  radiative losses will alter the equilibrium of the HMNS

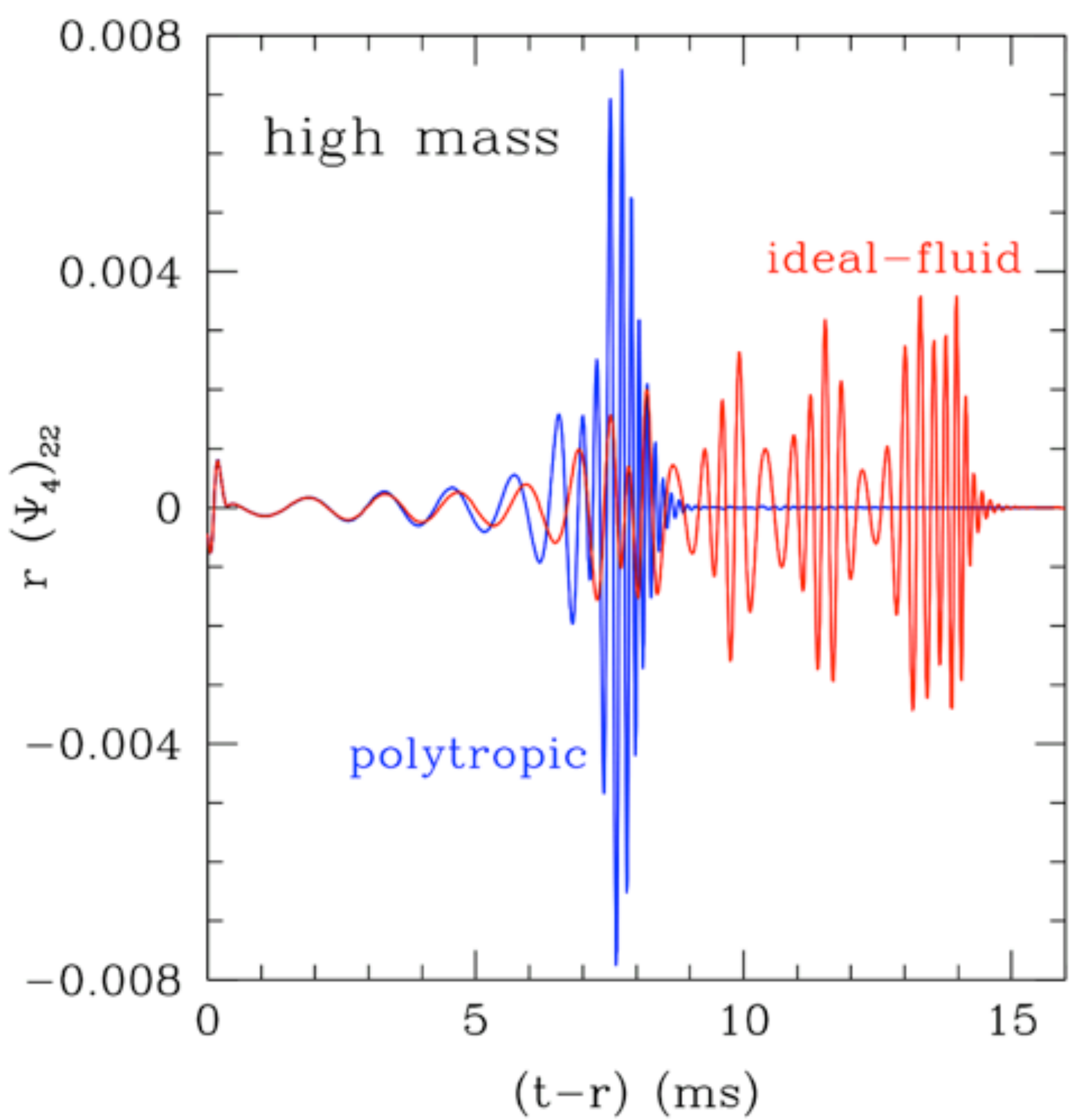
Animations: Kaehler, Giacomazzo, Rezzolla

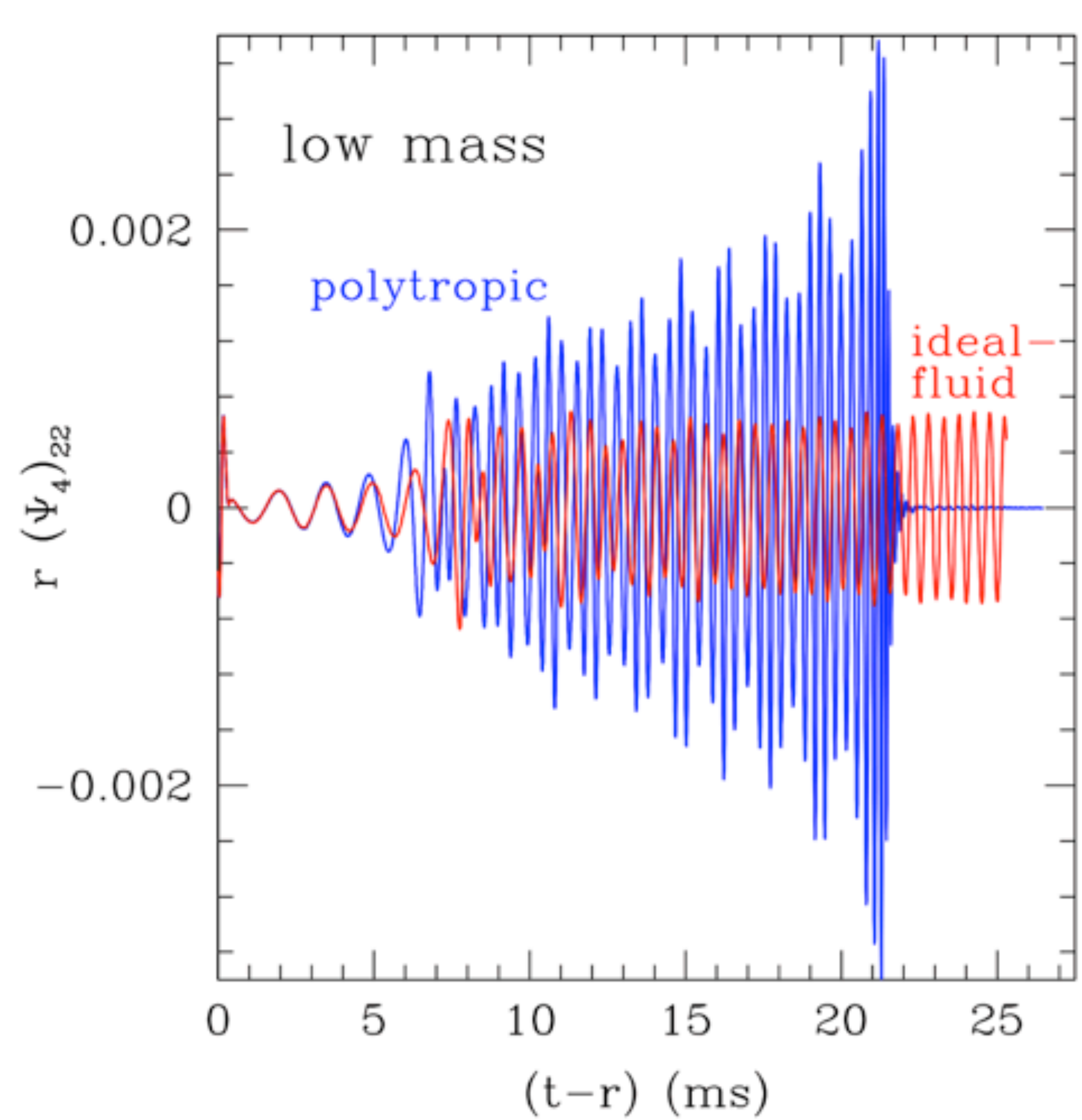


Hot EOS: **high-mass** binary

$$M = 1.6\, M_{\odot}$$

Imprint of the EOS: **hot** vs **cold**

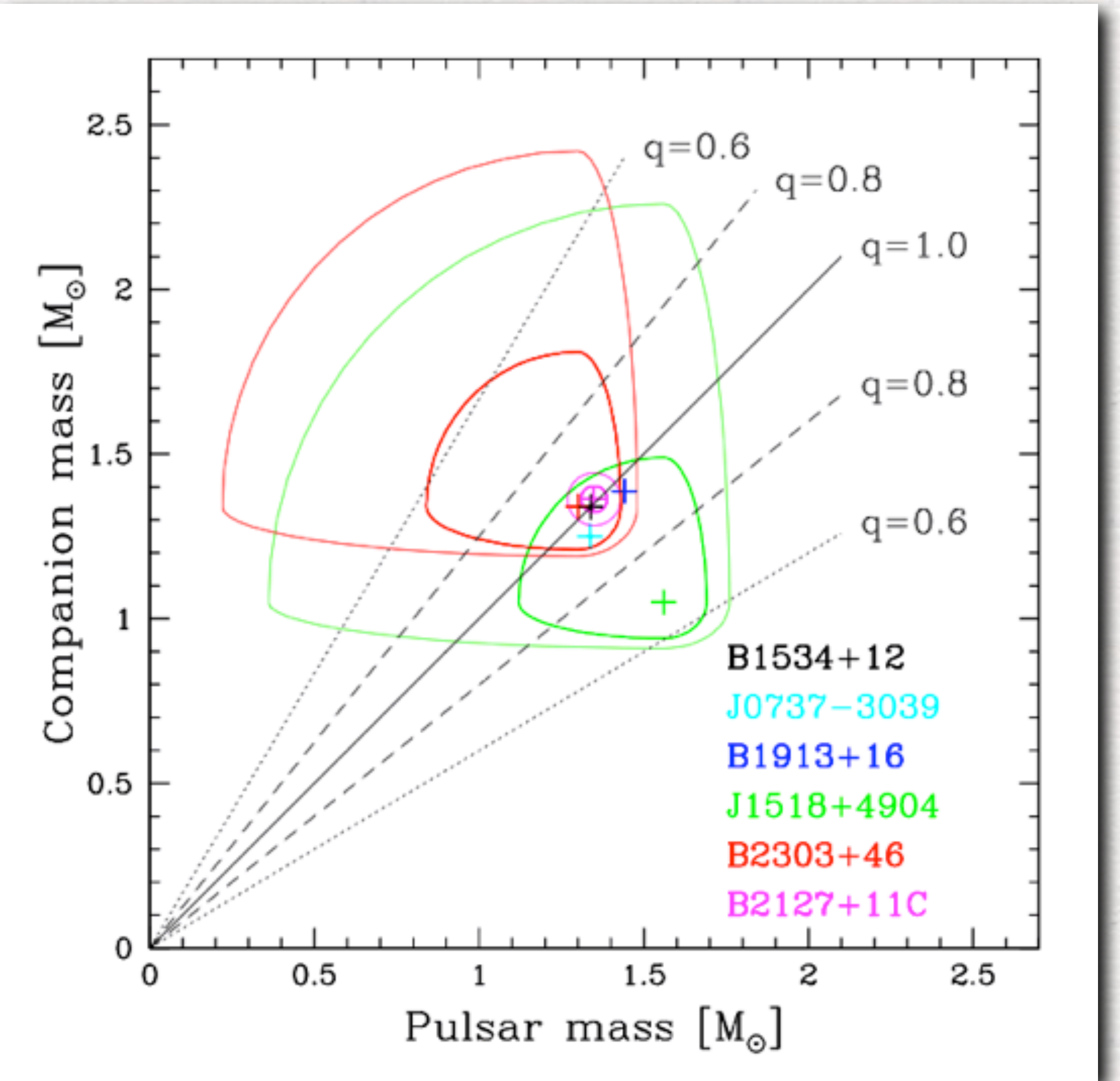




After the merger a BH is produced over a timescale comparable with the dynamical one

After the merger a BH is produced over a timescale larger or much larger than the dynamical one

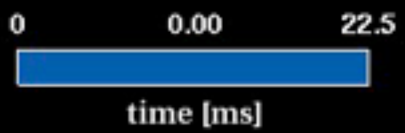
In contrast to binary black holes, binary neutron stars do not large variations in the mass ratio but it surely not exactly one.



D. Gondek-Rosinska (2009)

M_1	M_2	
1.44	1.38	B1913+16
1.33	1.34	B1534+12
1.33	1.25	J0737-3039
1.40	1.18	J1756-2251
1.36	1.35	B2127+11C
1.35	1.26	J1906+0746
1.62	1.11	J1811-1736
1.56	1.05	J1518+4904
1.14	1.36	J1829+2456

Are these small (!) mass asymmetries important? For black holes they would hardly matter

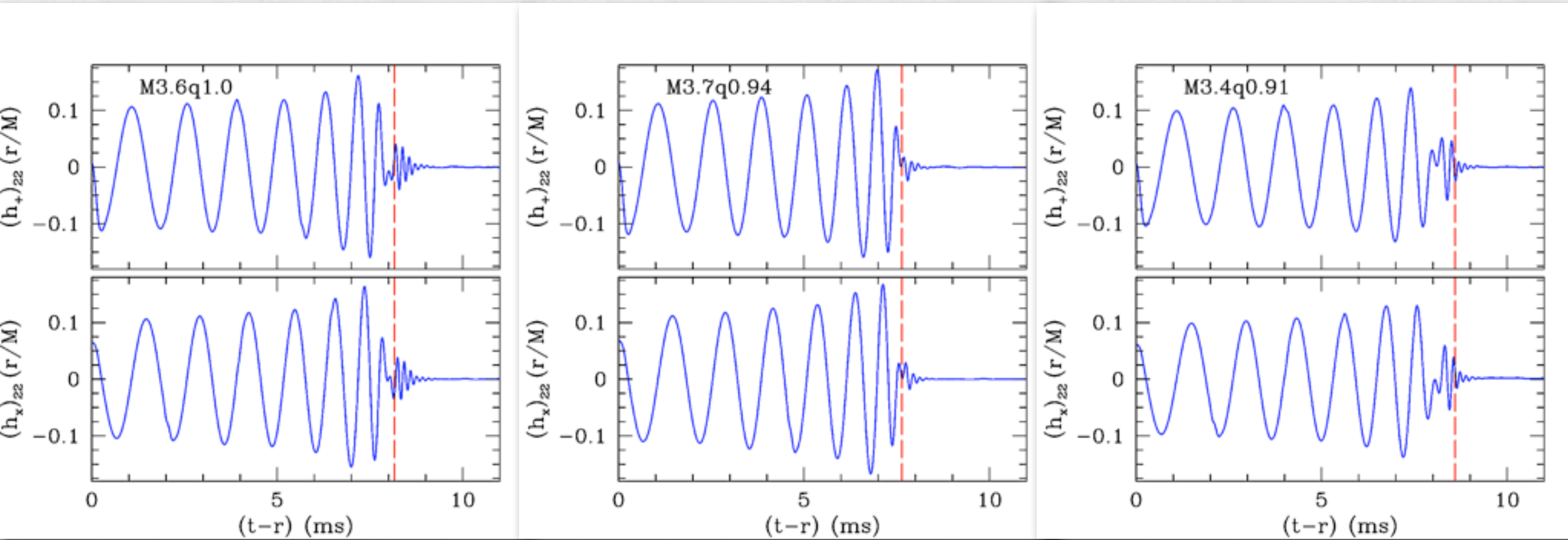


Animations: Giacomazzo, Koppitz, LR

Total mass : $3.37\,M_{\odot}$; mass ratio :0.80;

- the torii are generically more massive
- the torii are generically more extended
- the torii tend to stable quasi-Keplerian configurations
- overall unequal-mass systems have all the ingredients needed to create a GRB

Gravitational waveforms



- Note the waveforms are very simple with moderate modulation induced by mass asymmetry.
- Furthermore, no HMNS is produced and the QNM ringing (shown by dashed vertical line) is choked by the intense mass accretion rate (the BH cannot ringdown...)

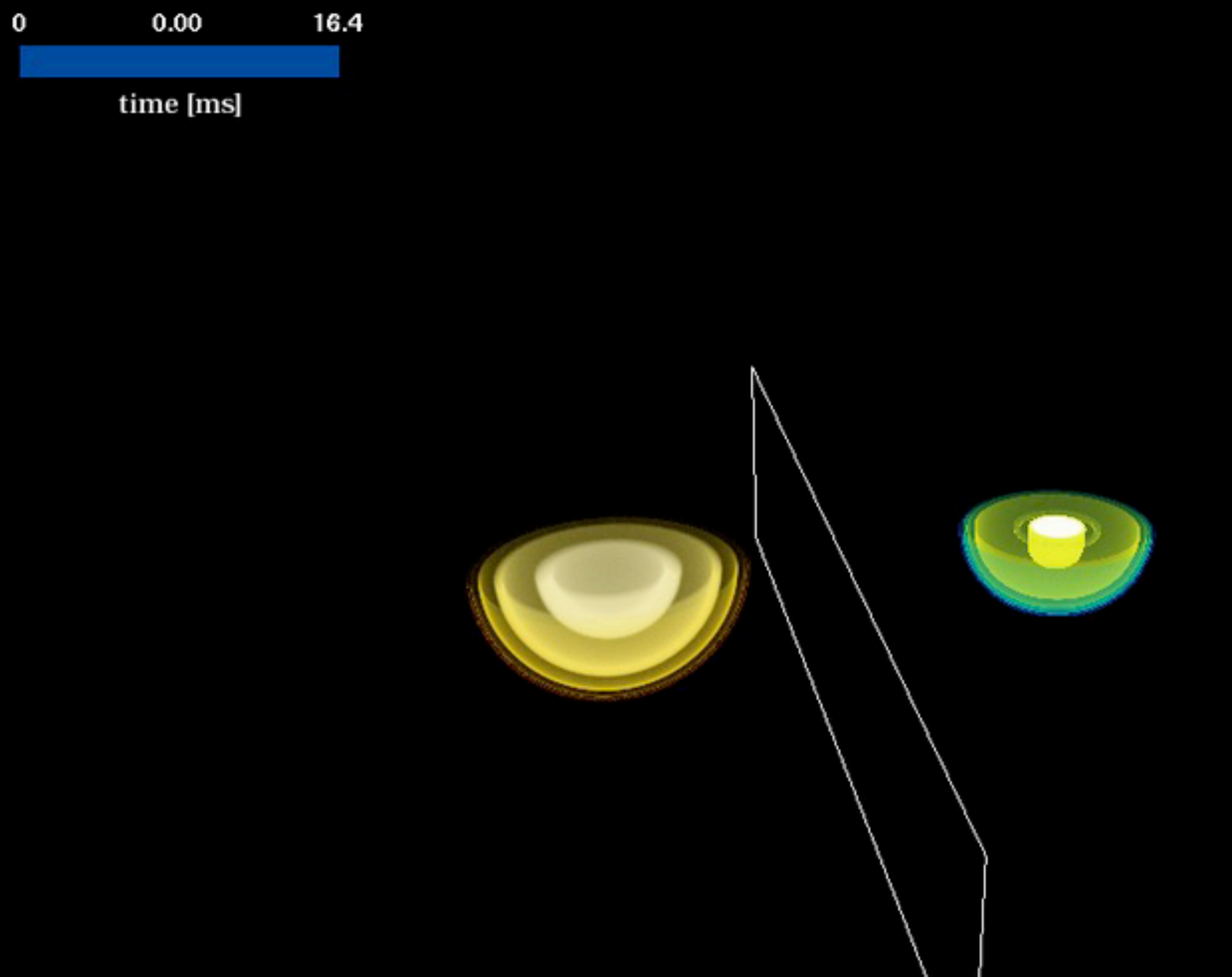
Extending the work to MHD

NSs have large magnetic fields but are routinely searched with PN waveforms which do not account for it. It is therefore natural to ask:

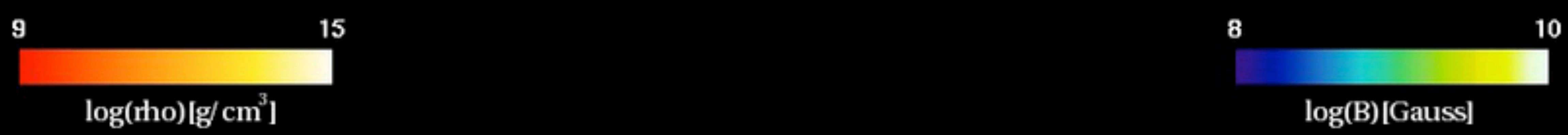
- can we detect magnetic fields during the inspiral?
- can we detect magnetic fields after the merger?
- how do magnetic fields influence the dynamics of the tori around the BH?

This is not easy to do can be done: relativistic hydrodynamics is extended to *ideal-MHD* (infinite conductivities). The magnetic fields are initially contained inside the stars: ie no magnetospheric effects. Overall we have considered 8 binaries (low/high mass) with MFs: $\mathrm{B}{=}0,\ 10^{12},\ 10^{14},\ 10^{17}\ \mathrm{G}$

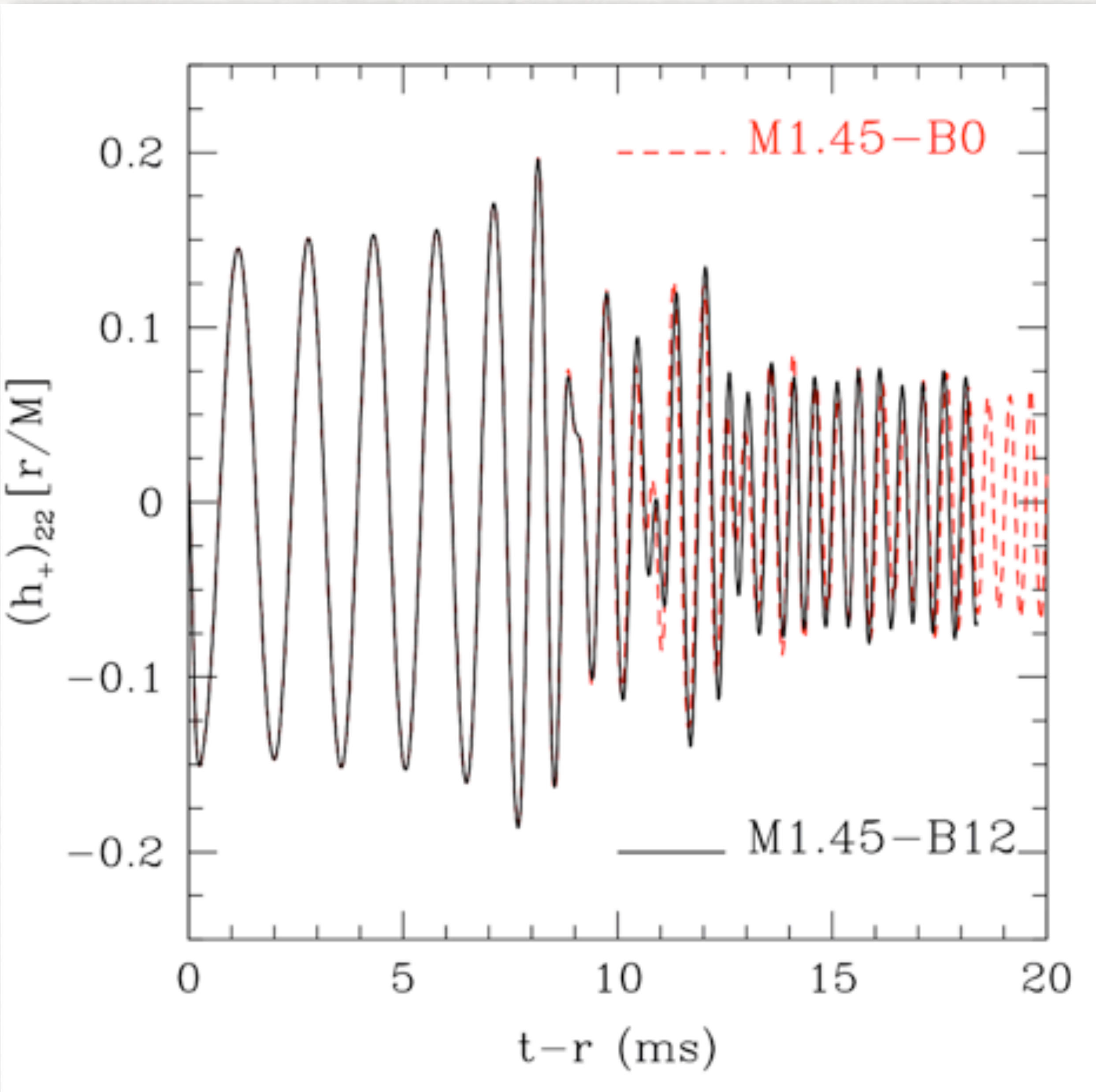
Animations: Koppitz, Giacomazzo, LR

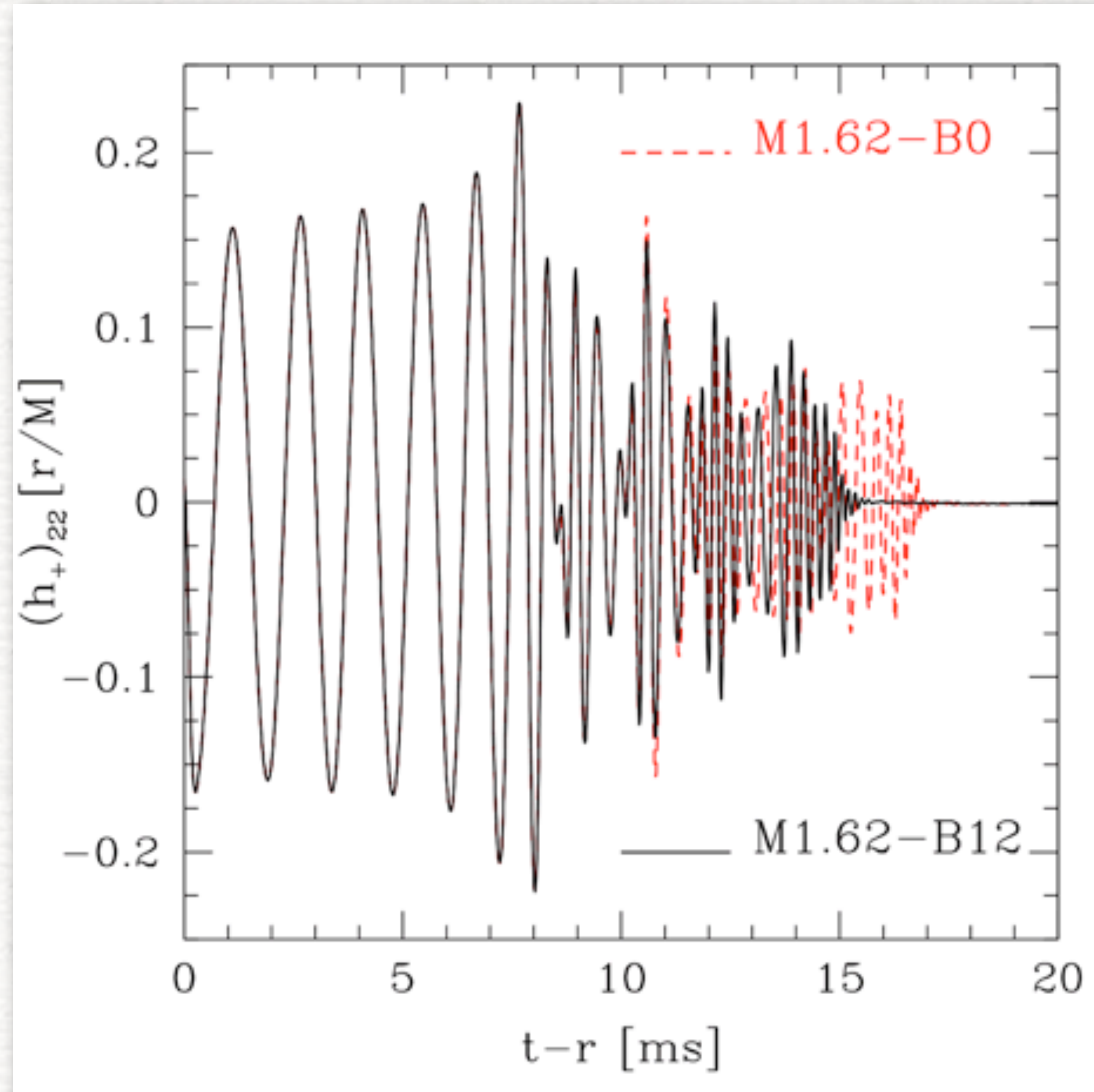


Typical evolution for a magnetized binary (hot EOS) $M = 1.65\, M_{\odot}, B = 10^{12}\,\mathrm{G}$

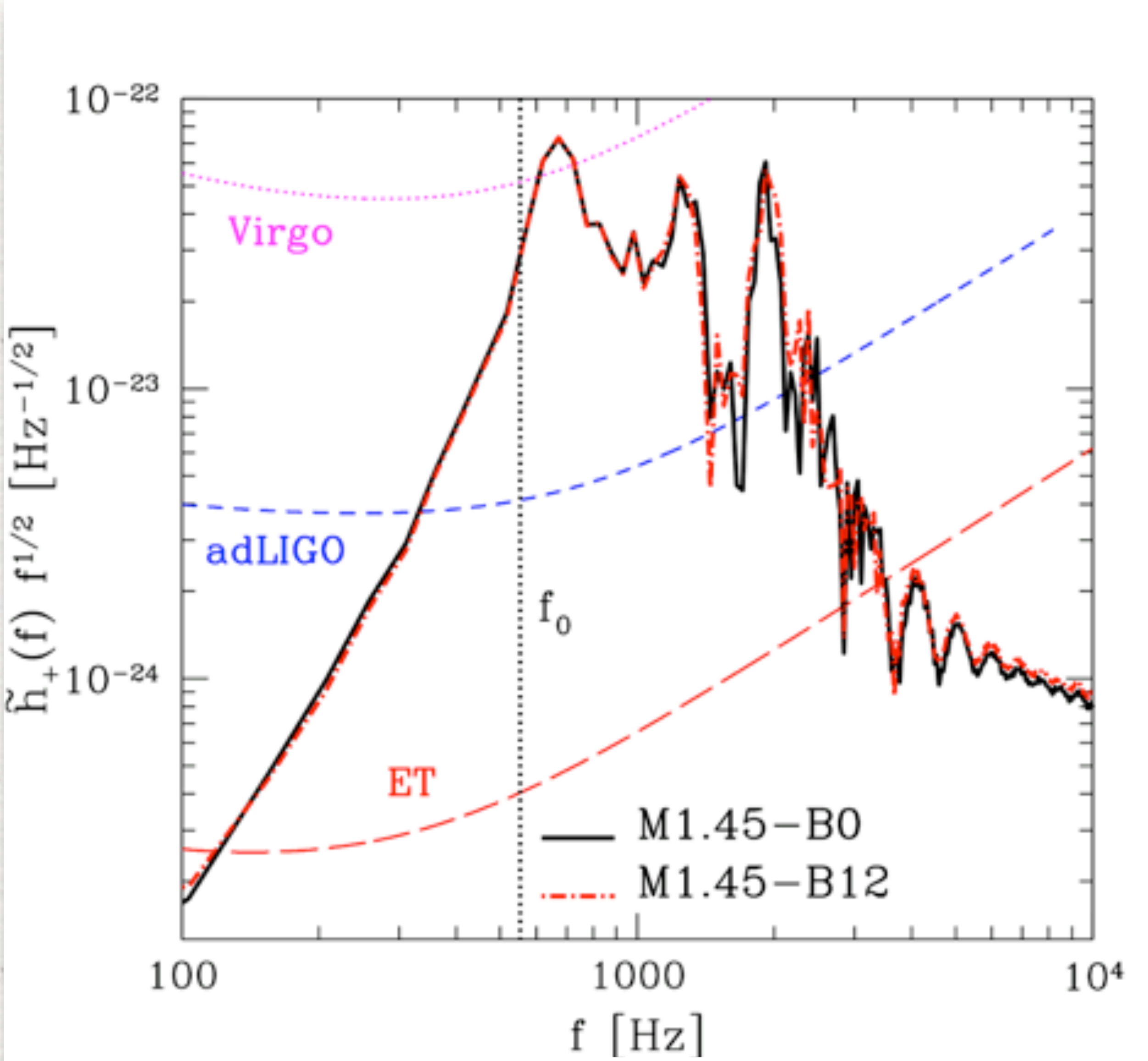


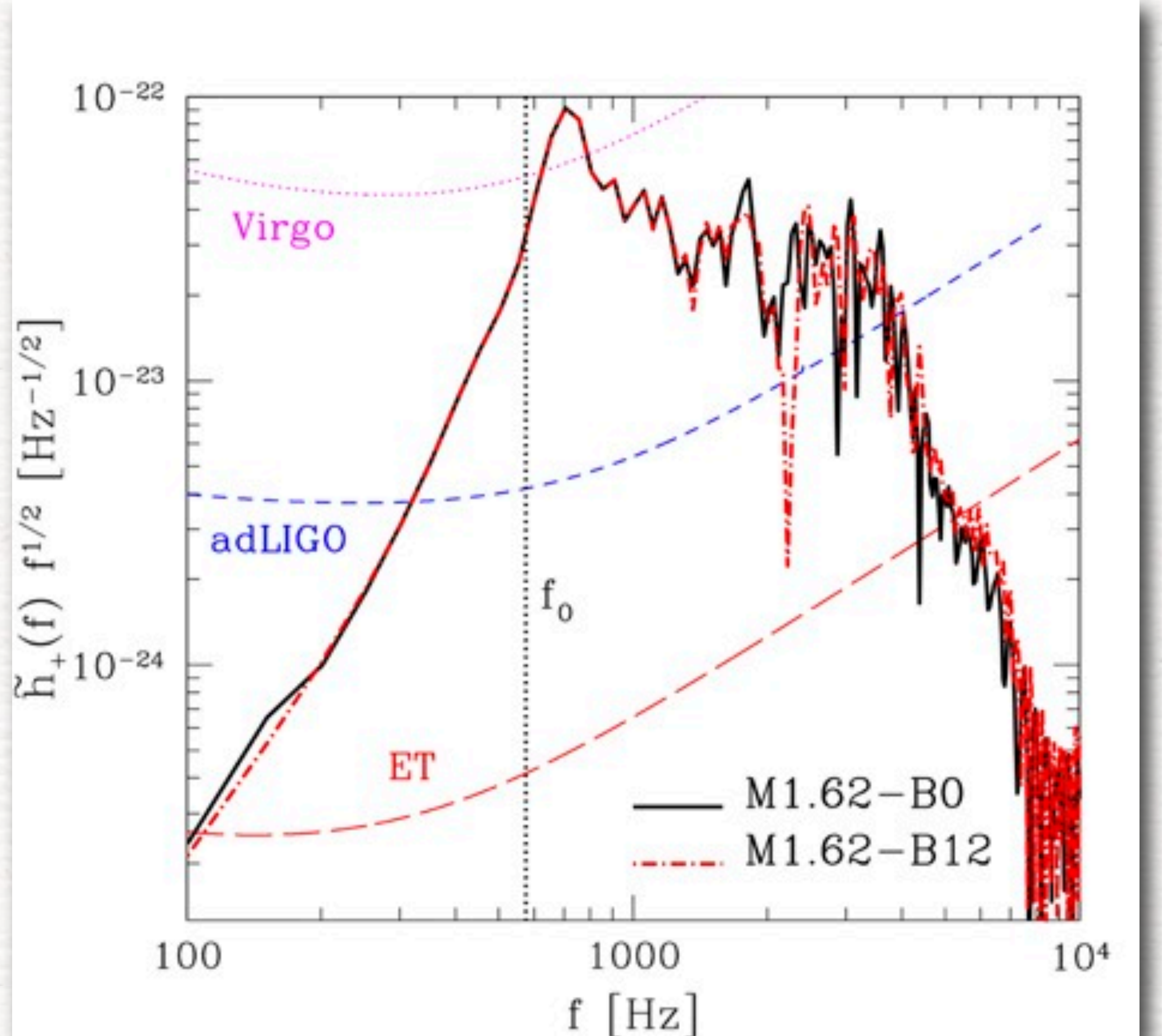
Some waveforms: time domain

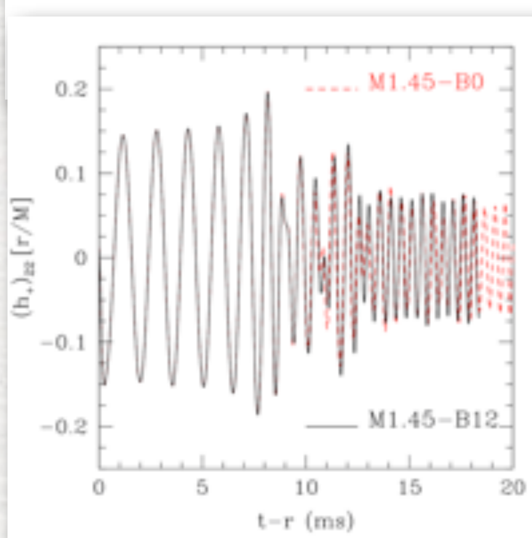




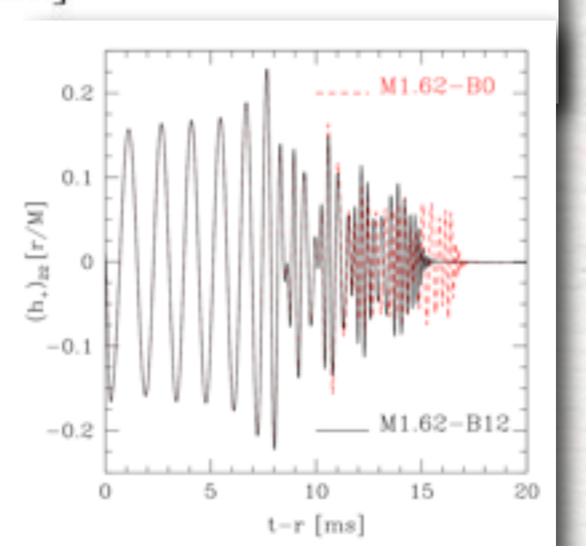
Some waveforms: frequency domain





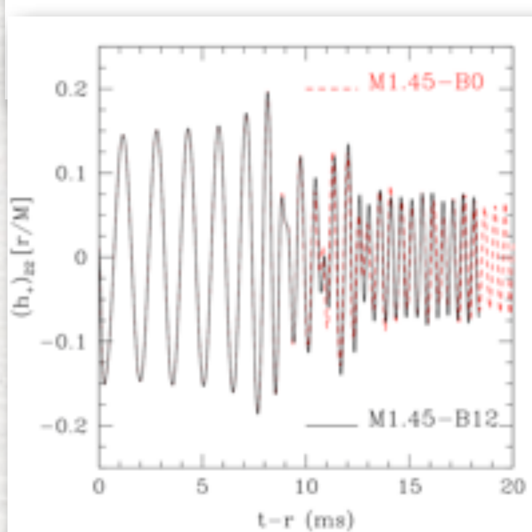


Binary	$\mathcal{O}$	$\mathcal{O}_{\rm insp}$	$\mathcal{O}_{\rm postm}$	SNR AdvVirgo	SNR AdvLIGO	SNR ET
M1.45-B0	1.000	1.000	1.000	1.94	2.11	38.90
M1.45-B8	0.997	0.999	0.926	1.94	2.10	38.72
M1.45-B10	0.996	0.999	0.934	1.94	2.11	38.82
M1.45-B12	0.996	0.999	0.899	1.94	2.11	39.01
M1.62-B0	1.000	1.000	1.000	2.00	2.24	42.57
M1.62-B8	0.998	1.000	0.938	2.00	2.24	42.59
M1.62-B10	0.993	1.000	0.724	2.00	2.23	42.48
M1.62-B12	0.997	1.000	0.893	2.00	2.24	42.49

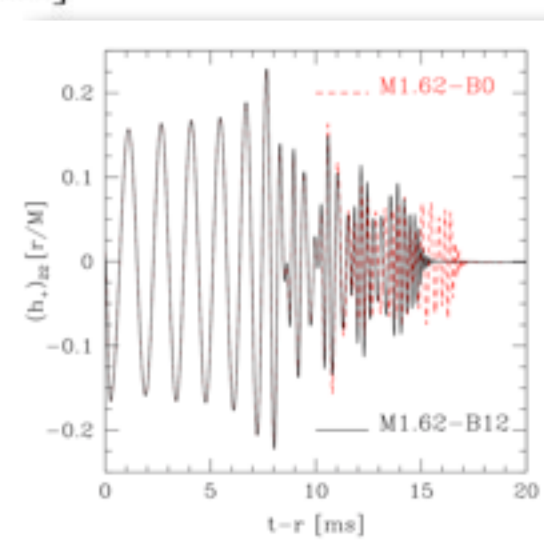


Some waveforms: frequency domain

These results show that not only ET may be the **only detector** able to reveal these processes but also that it is likely to be the **only one** which can determine the level of magnetization of the neutron-star matter

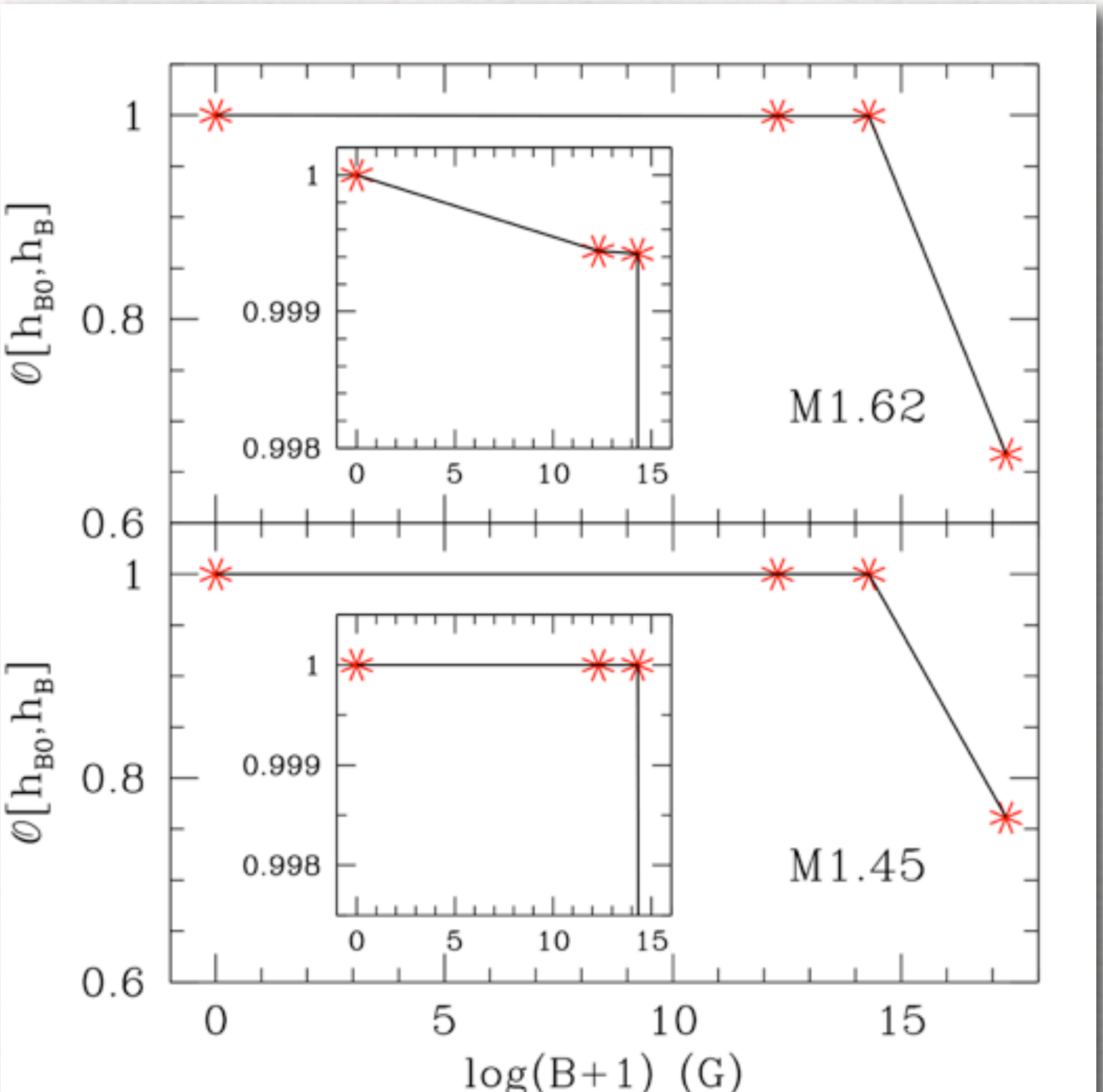


Binary	$\mathcal{O}$	$\mathcal{O}_{\rm insp}$	$\mathcal{O}_{\rm postm}$	SNR AdvVirgo	SNR AdvLIGO	SNR ET
M1.45-B0	1.000	1.000	1.000	1.94	2.11	38.90
M1.45-B8	0.997	0.999	0.926	1.94	2.10	38.72
M1.45-B10	0.996	0.999	0.934	1.94	2.11	38.82
M1.45-B12	0.996	0.999	0.899	1.94	2.11	39.01
M1.62-B0	1.000	1.000	1.000	2.00	2.24	42.57
M1.62-B8	0.998	1.000	0.938	2.00	2.24	42.59
M1.62-B10	0.993	1.000	0.724	2.00	2.23	42.48
M1.62-B12	0.997	1.000	0.893	2.00	2.24	42.49



Understanding the dependence on MF

To quantify the differences and determine whether detectors will see a difference in the inspiral, we calculate the overlap



$$\mathcal{O}[h_{B1}, h_{B2}] \equiv \frac{\langle h_{B1}|h_{B2}\rangle}{\sqrt{\langle h_{B1}|h_{B1}\rangle\langle h_{B2}|h_{B2}\rangle}}$$

where the scalar product is

$$\langle h_{B1}|h_{B2}\rangle \equiv 4\Re \int_0^\infty df \, \frac{\tilde{h}_{B1}(f)\tilde{h}^*_{B2}(f)}{S_h(f)}$$

In essence, at these res:

$$\mathcal{O}[h_{B0}, h_B] \gtrsim 0.999$$

for $\mathrm{B} \lesssim 10^{17}\ \mathrm{G}$

Because the match is even higher for lower masses, the influence of MFs on the inspiral is **unlikely to be detected!**

EOB and tidal corrections

- For binary BHs the point-particle approximation works well, but NSs are extended bodies for which deformations are important.
- Tidal effects become apparent already at low frequencies and thus considerably increase the detectability of BNSs (longer inspiral)
- Tidal effects are not present up to 5PN order (Damour, Soffel, Xu, 1992-94) and 5PN leading-order tidal correction were introduced recently, through the relativistic Love numbers (Flanagan & Hinderer 2008, Damour & Nagar 2009a, Binnington & Poisson 2009, Hinderer et al. 2009)
- More recently, Damour & Nagar 2009b have introduced next-to-leading order tidal corrections within the EOB approach
- These corrections have not yet been compared and constrained with full-GR, dynamical simulations

Incorporating tidal effects in the EOB Hamiltonian is straightforward:

$$A(r) = A_0(r) + A^{tidal(N)}(r)$$

where $A^{tidal(N)}(r)$ is computed in a Newtonian-like form (but with relativistic Love numbers)

$$A^{tidal(N)}(r) = \sum_{\ell \geq 2} A_\ell^{tidal(N)}(r) = \sum_{\ell \geq 2} \kappa_\ell^T u^{2\ell+2} \qquad u \equiv \frac{M}{r}$$

$$\kappa_\ell^T = 2 \frac{M_B M_A^{2\ell}}{M^{2\ell+1}} \frac{k_\ell^A}{c_A^{2\ell+1}} + 2 \frac{M_A M_B^{2\ell}}{M^{2\ell+1}} \frac{k_\ell^B}{c_B^{2\ell+1}} \qquad M \equiv M_A + M_B$$

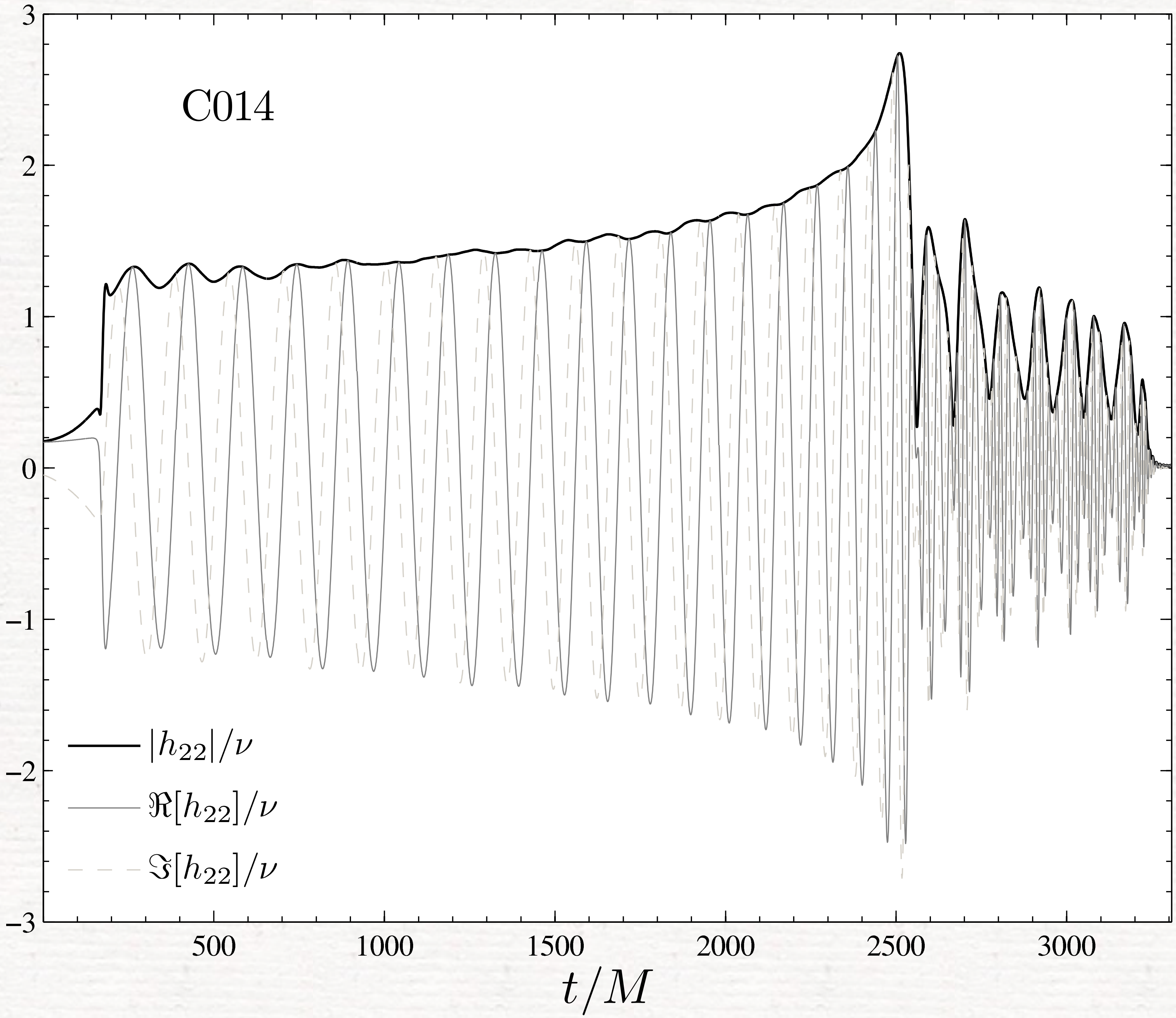
and higher (i.e. NLO) PN tidal corrections can be added as:

$$A_\ell^{tidal} = A_\ell^{tidal(N)} \hat{A}_\ell^{tidal}$$

$$\hat{A}_\ell^{tidal} = 1 + \bar{\alpha}_1 u + \bar{\alpha}_2 u^2 + \ldots$$

$$\kappa_\ell^{T,eff} = \kappa_\ell^T \hat{A}$$

where $\bar{\alpha}_1, \bar{\alpha}_2$ can be computed analytically (in principle) or estimated by comparing with numerical simulations (in practice).



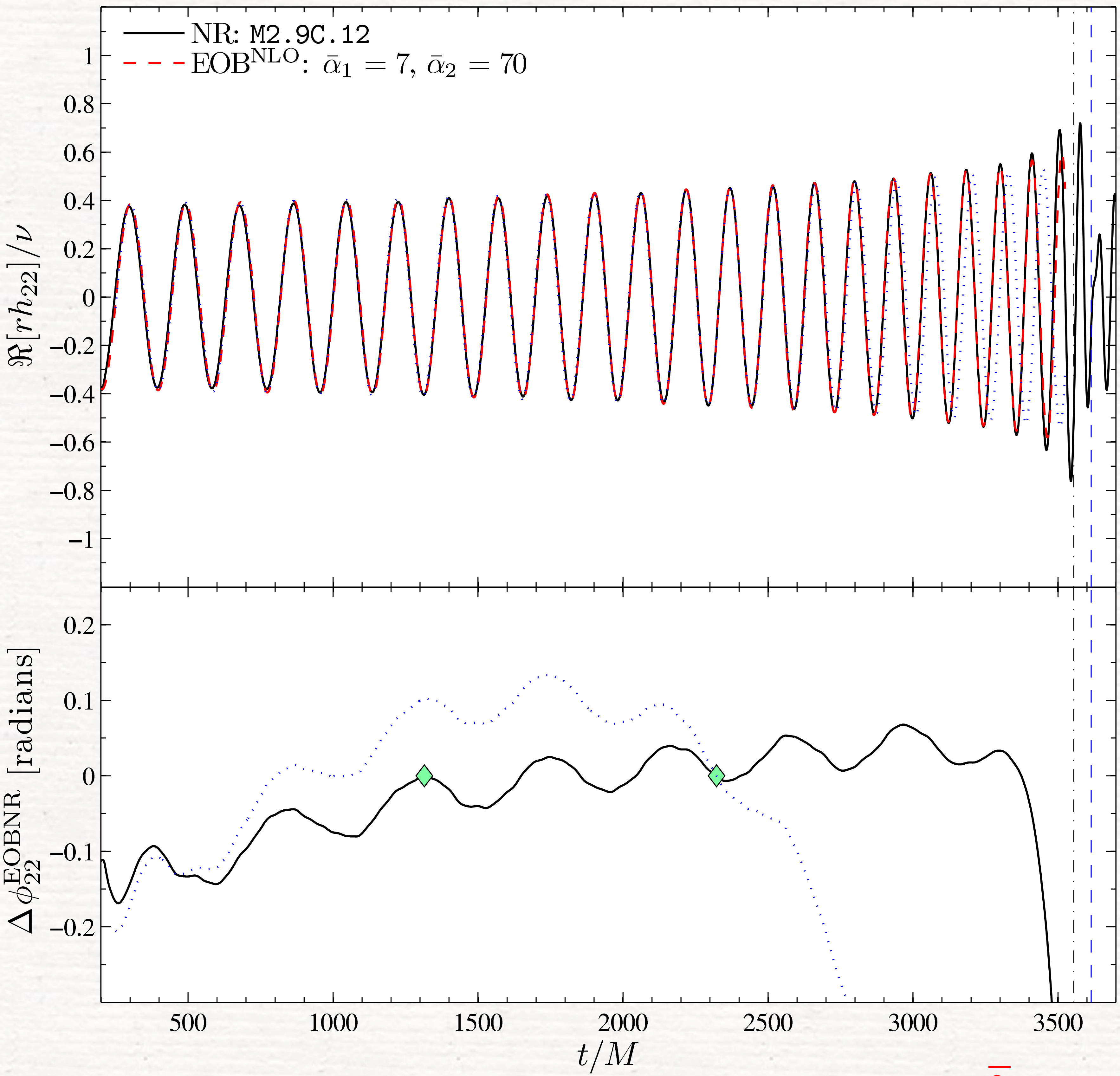
To compare and tune the tidal corrections we have produced long waveforms of equal-mass binaries with polytropic or ideal-fluid EOSs.

The binaries have mass/compactness:

$$M = 1.4,\ 1.6\, M_\odot$$

$$\mathcal{C} = 0.12,\ 0.14$$

These are the longest waveforms ever produced for binary neutron stars: 11 orbits or 22 cycles. The data shows an excellent level of consistency in the phase, amplitude and frequency evolution



$M = 1.4\,M_{\odot},\ \mathcal{C} = 0.12$

Low-mass, small compactness binary.

The agreement between the tidal-EOB waveform and the NR one is excellent and essentially up to the merger.

$$\Delta\phi \simeq \pm 0.24\,\mathrm{rad}\,,$$
$$\Delta\phi/\phi \simeq 0.2\%\,.$$

Tidal corrections are important to avoid large phase errors.

$$\bar{\alpha}_{1PN} = 7, \quad \bar{\alpha}_{2PN} = 70$$
$$\kappa_2^{T,eff} = \kappa_2^T\,\hat{A}^{tidal} \simeq 2.5\,\kappa_2^T$$

Conclusions

✴ Evolution BBHs is under control but need higher precision to hand small mass ratios and very high spins

✴ A lot of work is in progress to interface numerical relativity waveforms with DA pipelines (see NINJA, NRAR, etc)

✴Using simple EOSs have reached possibly the most complete description of BNSs from the inspiral, merger, collapse to BH. Can draw this picture with/without B-fields, equal and unequal masses.

✴ GWs from BNSs are much complex/richer than from BBHs: can be the Rosetta stone to decipher the NS interior.

✴Magnetic fields unlikely to be detected during the inspiral but important after the merger (amplified by dynamos or instabilities)

✴**In the ET-era NR will be a unique tool to bridge DA, physics, and astrophysics**