

Precision Cosmology with the Einstein Telescope

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Measuring the geometry of the Universe

- How does one measure the geometry?
 - Measure the space-time metric
- Use clocks and rods?
 - This is a difficult job in cosmology
- Project observations onto a cosmological model?
 - Test predictions of various models

Predictions of a cosmological model

- Number counts of sources vs red-shift
- The age of the universe in units of the Hubble time H^{-1}
- Angular size as a function of red-shift
- Magnitude as a function of red-shift
 - The Hubble diagram
- Number counts of sources versus magnitude
- Gravitational lensing of high red-shift sources
- Spectrum of a primordial stochastic background
- ...

Testing cosmological models

- Each of these predictions could, in principle, be tested with gravitational-wave observations
 - the key to this is the existence of a standard siren
 - The ability to accurately identify and measure the properties of the siren
 - Ability to measure cosmological backgrounds

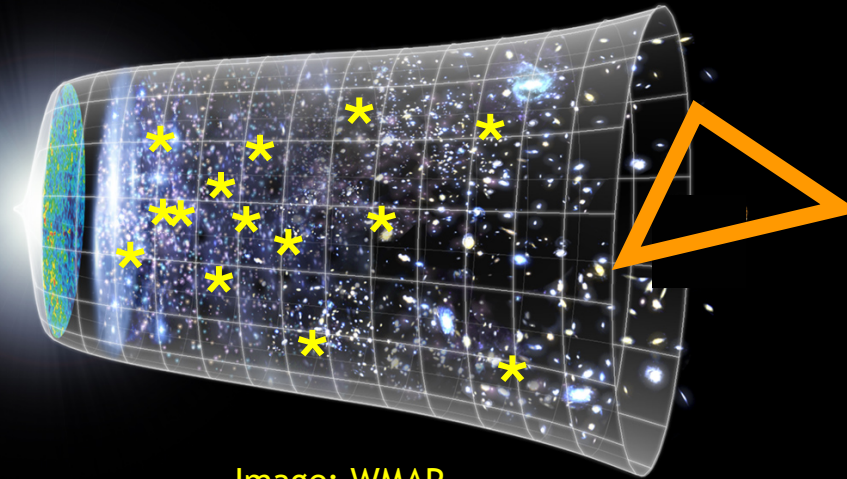
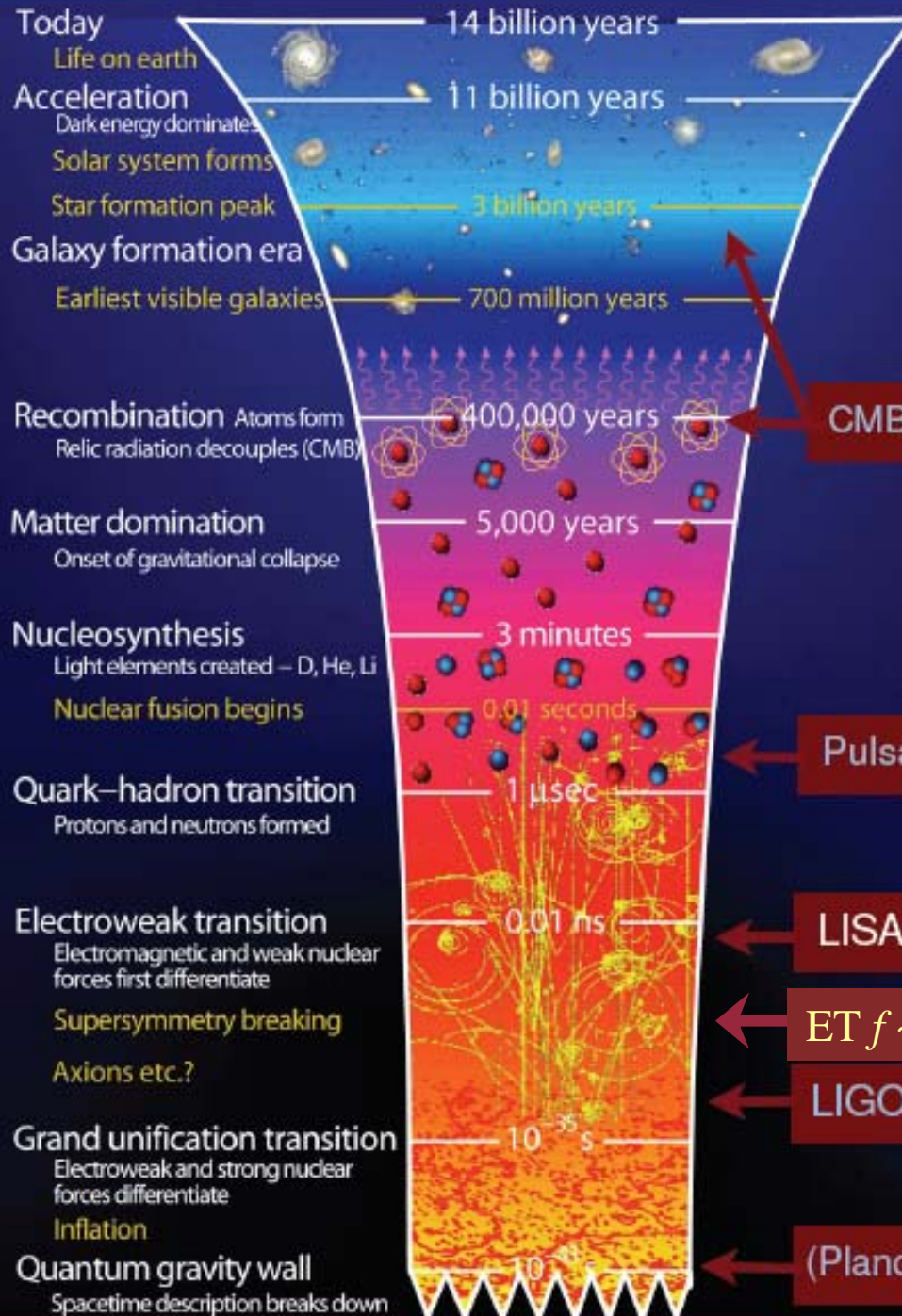


Image: WMAP

A brief history of the Universe



CMB $f < 3 \times 10^{-17} \text{ Hz}$ probes $300,000 \text{ yrs} < t_e < 14 \text{ Gys}$

Pulsars $f \sim 10^{-8} \text{ Hz}$ probe $t_e \sim 10^{-4} \text{ s}$ ($T \sim 50 \text{ MeV}$)

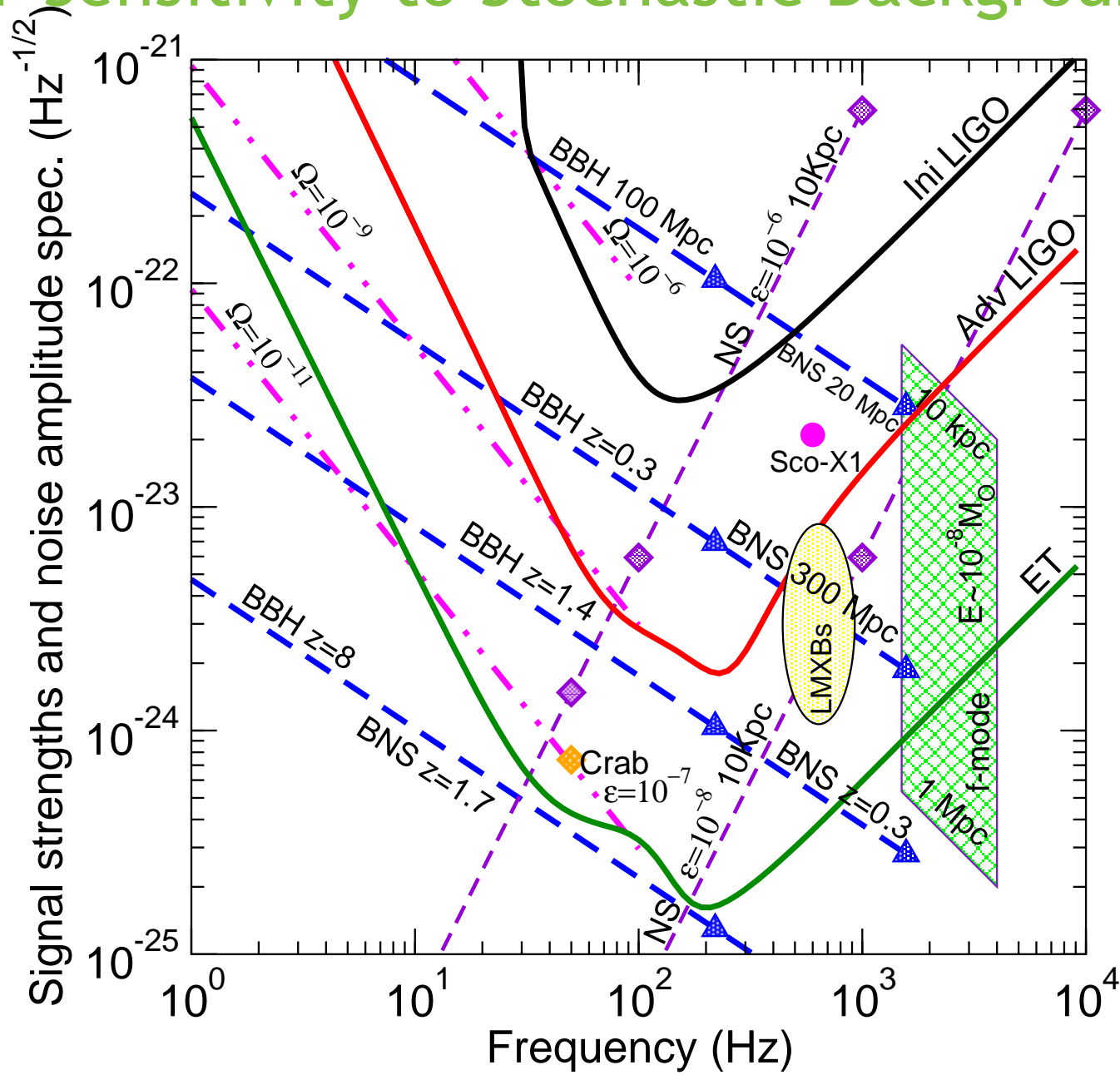
LISA $f \sim 10^{-3} \text{ Hz}$ probes $t_e \sim 10^{-14} \text{ s}$ ($T \sim 10 \text{ TeV}$)

ET $f \sim 10 \text{ Hz}$ probes $t_e \sim 10^{-20} \text{ s}$ ($T \sim 10^6 \text{ GeV}$)

LIGO $f \sim 100 \text{ Hz}$ probes $t_e \sim 10^{-24} \text{ s}$ ($T \sim 10^8 \text{ GeV}$)

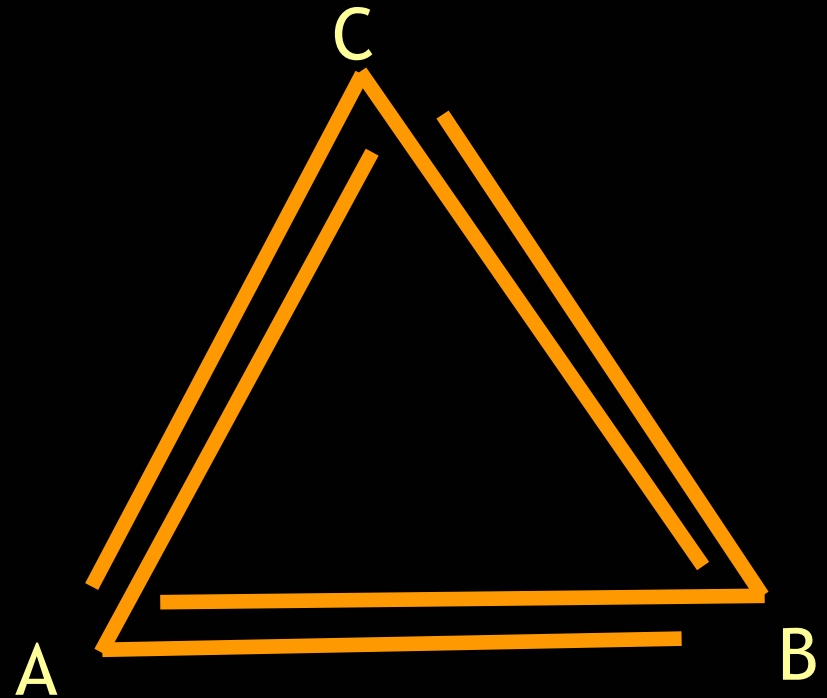
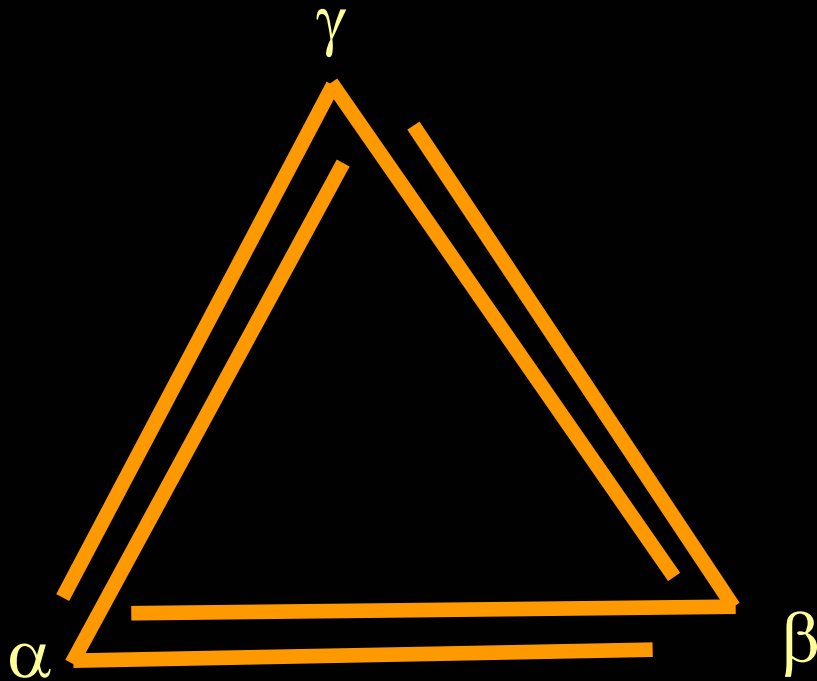
(Planck scale $f \sim 10^{11} \text{ Hz}$ has $t_e \sim 10^{-43} \text{ s}$ ($T \sim 10^{19} \text{ GeV}$))

ET Sensitivity to Stochastic Background



A Pair of ET's for Stochastic background

- Separated by less than 5,000 km will help build up the correlation at 10 Hz



Measuring the Stochastic Background

- Common noise at a site is a hindrance to detecting stochastic background
 - Especially bad in the case of ET since the stochastic signal has a big contribution from the low-frequency end of the sensitivity band
- Detectors at two locations allow to build-up nine-way cross correlation channels (four independent channels)
 - We could use the different channels for cancelling out the noise background
- Multi-band detectors are not a problem

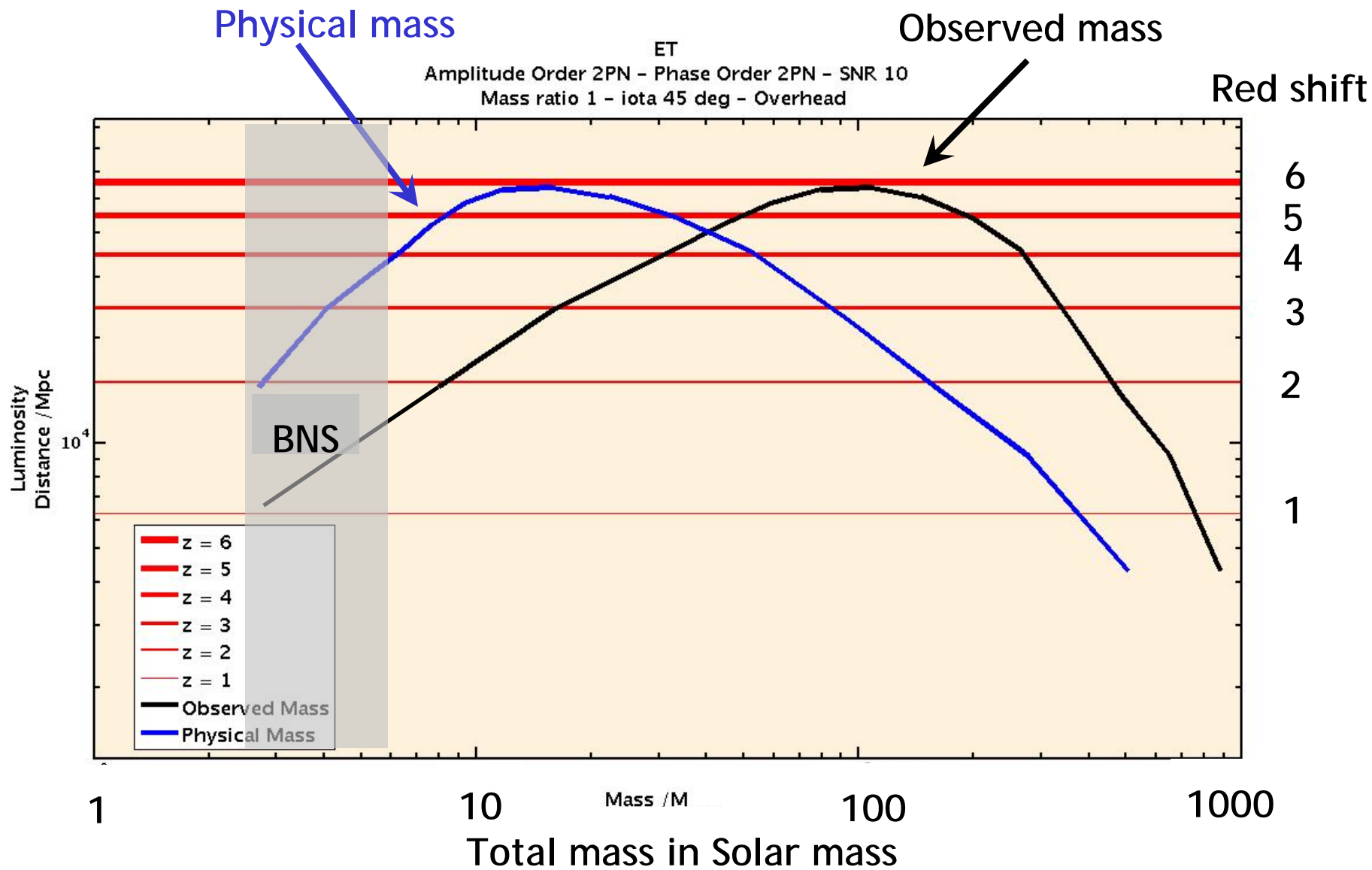
$$\begin{array}{l} \langle \alpha, A \rangle, \langle \alpha, B \rangle, \langle \alpha, C \rangle \\ \langle \beta, A \rangle, \langle \beta, B \rangle, \langle \beta, C \rangle \\ \langle \gamma, A \rangle, \langle \gamma, B \rangle, \langle \gamma, C \rangle \end{array}$$

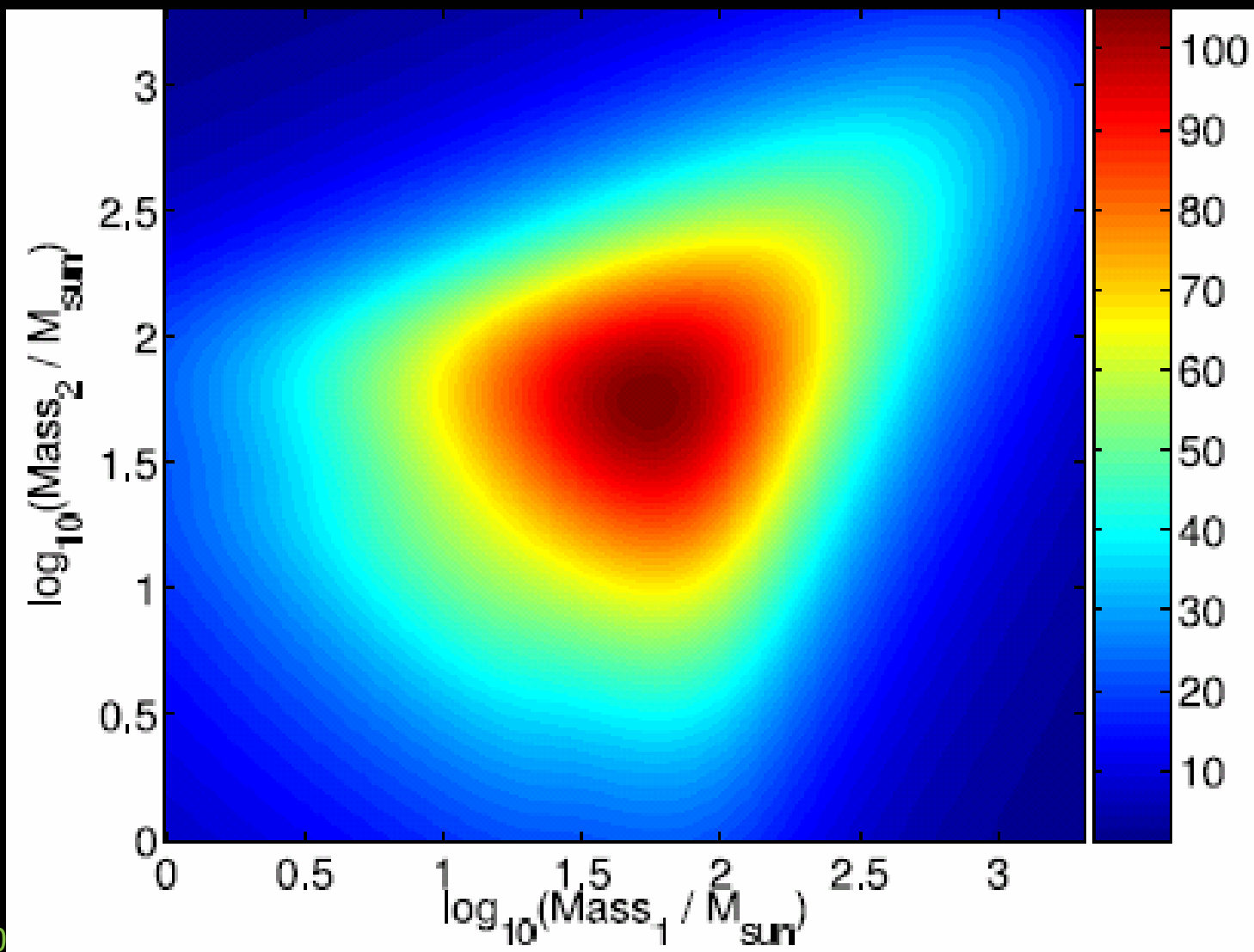
Standard sirens for precision cosmology

Compact binary coalescences in ET

Red shift makes a difference

McKechan et al, 2009



SNR for sources at $z=0.5$ 

Hubble Diagram

- Luminosity distance Vs. red shift depends on the cosmological parameters H_0 , Ω_M , Ω_b , Ω_Λ , w , etc.

$$D_L(z) = \frac{c(1+z)}{H_0} \int_0^z \frac{dz}{[\Omega_M(1+z)^3 + \Omega_\Lambda(1+z)^{3(1+w)}]^{1/2}}$$

- Einstein Telescope will detect 1000's of compact binary mergers for which the source can be identified (e.g. GRB) and red-shift measured.
- A fit to such observations can determine the cosmological parameters to better than a few percent.

How well can ET measure cosmological parameters

- We generated 500 random binary NS-BH detections
 - Uniformly distributed in comoving volume up to $z=4$
- For each detection ET measures the luminosity distance
 - Red-shift comes from EM identification and follow-up
- Assumed that the measured luminosity distances will be different from the true one by statistical errors determined by the Fisher matrix
 - Neglected the systematic errors such as weak gravitational lensing, waveform uncertainties, etc., which should be included in a future study
- Fitted the resulting (D_L, z) to a model

Results

- True values of the cosmological parameters

$$H_0 = 0.70, \Omega_M = 0.27, \Omega_\Lambda = 0.73, w = -1$$

- Measured values

- Measuring w as a function of red-shift (CVDB's talk)

- Two unknown parameters

$$w = -0.999 \pm 0.015, \quad \Omega_\Lambda = 0.733 \pm 0.0067$$

- Three unknown parameters

$$w = -0.96 \pm 0.041, \Omega_M = 0.255 \pm 0.014, \Omega_\Lambda = 0.741 \pm 0.012$$

- Four unknown parameters

- Errors are too large to be interesting

Future work

- Include systematic effects
 - Weak gravitational lensing and waveform uncertainty
- Allow cosmological evolution of standard sirens
 - Changes in rates of compact binary coalescences
- Examine how good is the number of sources Vs red-shift test