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# R-modes instability, gravitational waves and Einstein Telescope science

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**Dr. Leone B. Bosi**  
*INFN Perugia*

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# Way R-modes could be interesting

Sá and Tomé model (2005).

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- R-modes can produce Gravitational waves
- The GW signals may carry information on:
  - gravitational physics
  - nuclear physics (that can't be done in other ways).

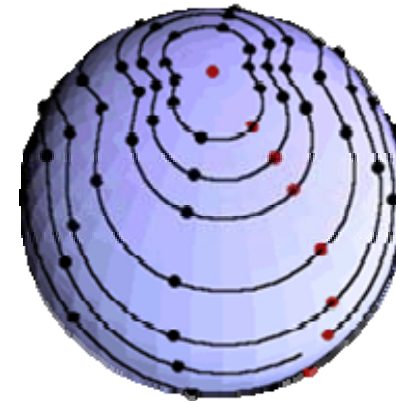
# R-modes intro

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- After the supernova event, the newborn neutron star may spin down during up to one year due to R-modes instability (first investigated by Owen et al.[1998])
- *R*-modes are non-radial pulsation modes of rotating stars that have the Coriolis force as restoring force.
- The characteristic frequency is comparable to the rotation speed of the star.
- These modes are driven unstable by gravitational radiation, inducing a differential rotation at second order in mode's amplitude
- Which leads the non linear evolution of the r-mode instability, making it mode difficult the GW observation.



A young neutron star  
(PSR J1846-0258 pulsar)



# Evolution of the *r*-mode instability: model hypothesis

Sá and Tomé model (2005).

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- Model initial conditions:
  - Initial angular velocity ( $\Omega_0 \approx 5.6\text{kHz}$ )
  - Initial temperature ( $T_0 = 10^{11}\text{K}$ )
  - Initial *r*-modes amplitude ( $\alpha_0 = 10^{-6}$ )
  - Amount of differential rotation associated to the *R*-modes ( $-5/4 < K < 10^{13}$ )
  - NS polytropic Equation of state  $\rightarrow \mathbf{p} = \mathbf{k}\rho^2$

# Evolution of the *r*-mode instability:

Sá and Tomé model (2005).

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- Initially the R-mode amplitude grows exponentially and after few hundred seconds saturates at:

$$\alpha_{\text{sat}} = \alpha_{\text{sat}}(K) \rightarrow (K+2)^{-0.5}$$

- The star angular velocity decreases

$$\Omega(t_f) = (0.065 - 0.067) \Omega_0$$

- The R-mode instability is active:

- From  $t \approx 0$
- To  $t_f = (3.6 \div 7.1) 10^6 \text{ s}$

## R-modes Instability as gravitational waves source

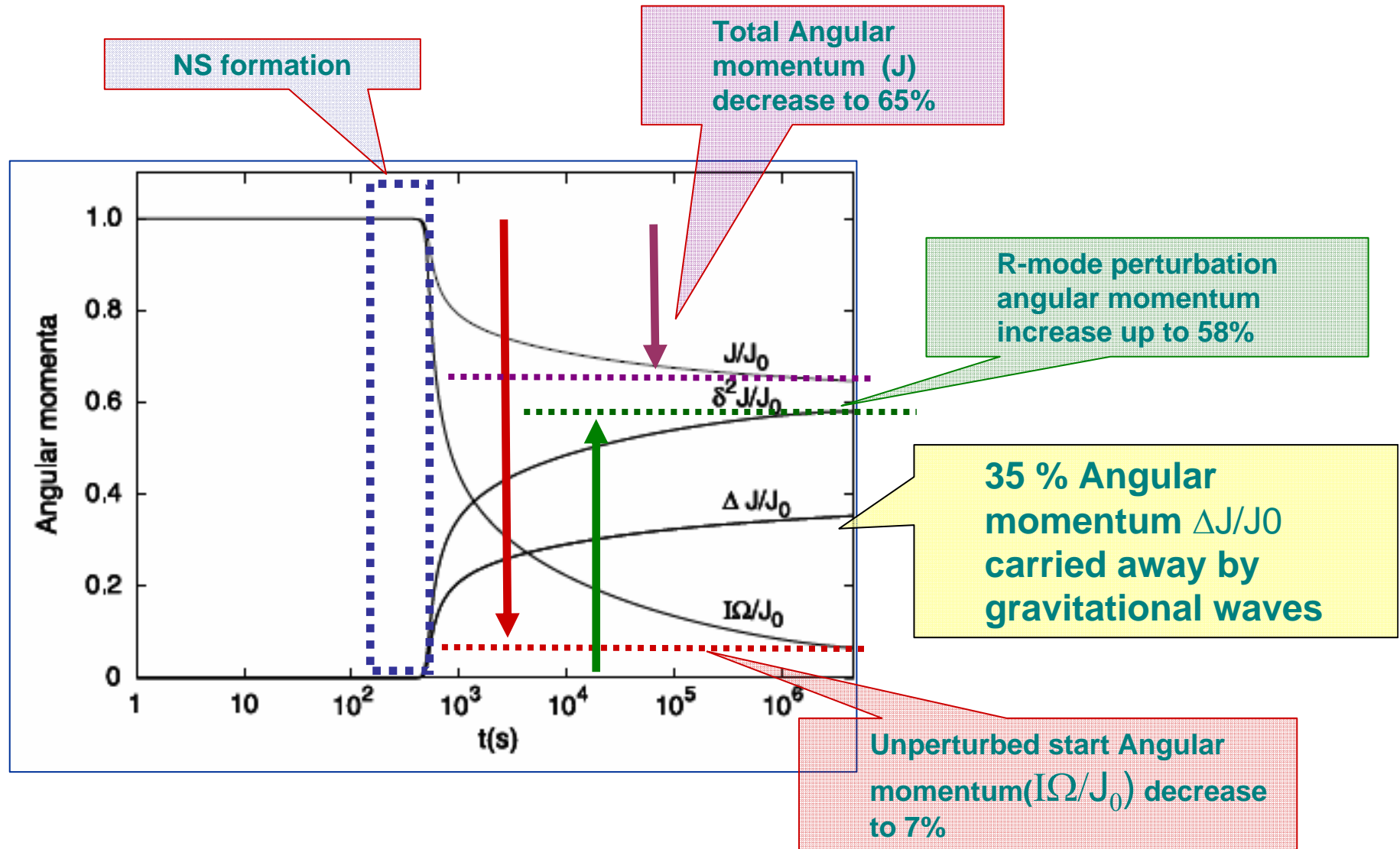
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- Gravitational waves from r-mode depend on the angular momentum carried away by gravitational waves

$h \rightarrow \Delta J = J_0 - J(t)$  where  $J_0 \approx I\Omega_0$  (NS Initial angular momentum)

- For this model the star total angular momentum is function of only two variables:  $(\Omega, \alpha)$

# Total Angular Momentum (J) evolution



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- Unperturbed star angular momentum is reduced to **7%**
- **58%** of the initial angular momentum is transferred to the r-mode
- About **35%** of the initial angular momentum is carried away **by gravitational waves**
- This is for **K=0**
- **For K >100** the initial angular momentum carried away decreases:  **$\Delta J/J_0 < 1\%$**



## R-mode Gravitational waves characterization

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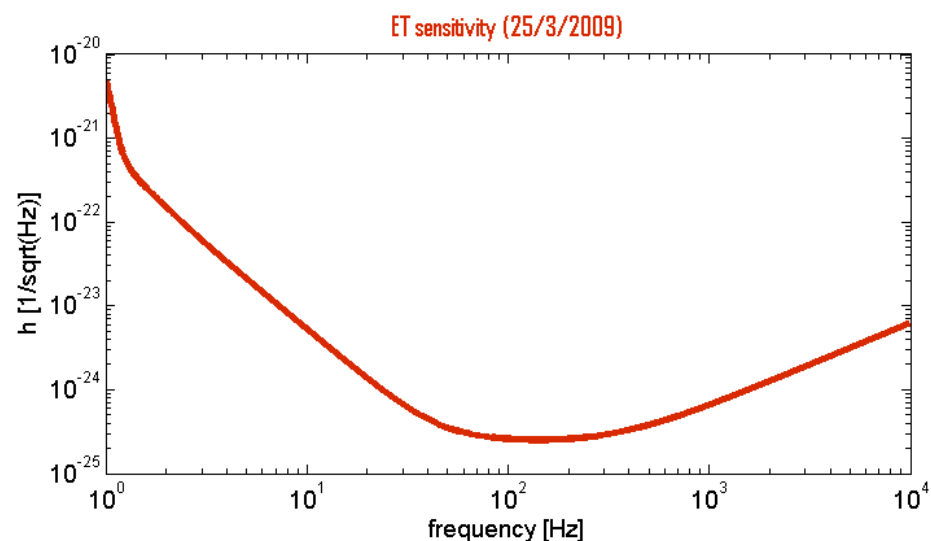
- The frequency of these waves is related to the angular velocity by:  
$$f = 2\Omega / (3\pi)$$
- The frequency bound is given:
  - $f_{\max} \approx \mathbf{1200 \text{ Hz}}$ , it depends on the initial value of the angular velocity  $\Omega_0$
  - $f_{\min} \approx \mathbf{[77 \div 80] \text{ Hz}}$ , it depends on the final value of the angular velocity  $\Omega(t_f)$  and  $K$
- The GW duration we recall that is roughly:
  - $t_f = \mathbf{(3.6 \div 7.1) 10^6 \text{ s}}$

# R-mode Gravitational waves and Einstein Telescope sensitivity

- Gravitational waves strain  $h(f)$  generated by R-mode (Sà and Tomé model) is:

$$|\tilde{h}(f)| = \frac{4.6 \times 10^{-25}}{\sqrt{K+2}} \sqrt{\frac{f_{\max}}{f}} \frac{20 \text{ Mpc}}{D} \text{ Hz}^{-1}.$$

- The Einstein Telescope design sensitivity used in this talk:

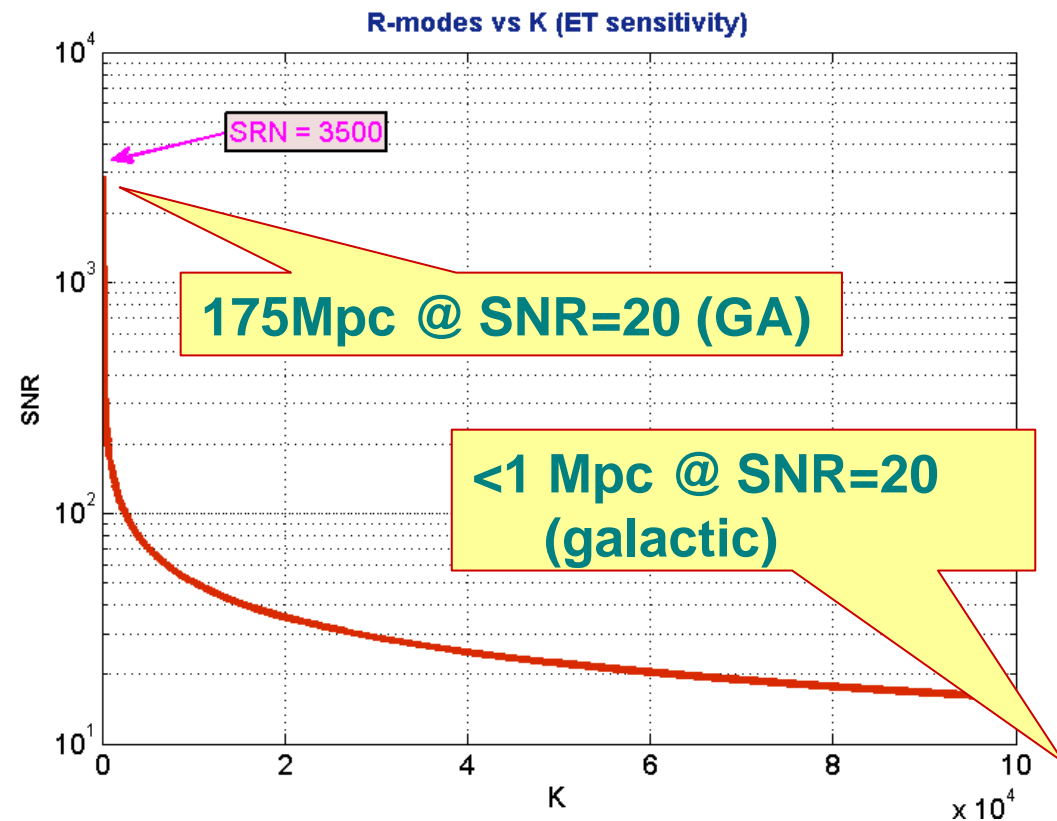


# Estimation of R-Mode GW Signal-to-Noise ratio for ET case (preliminary)

- The optimal Signal-to-Noise ratio for Einstein Telescope has been estimated, and it is given by the formula:

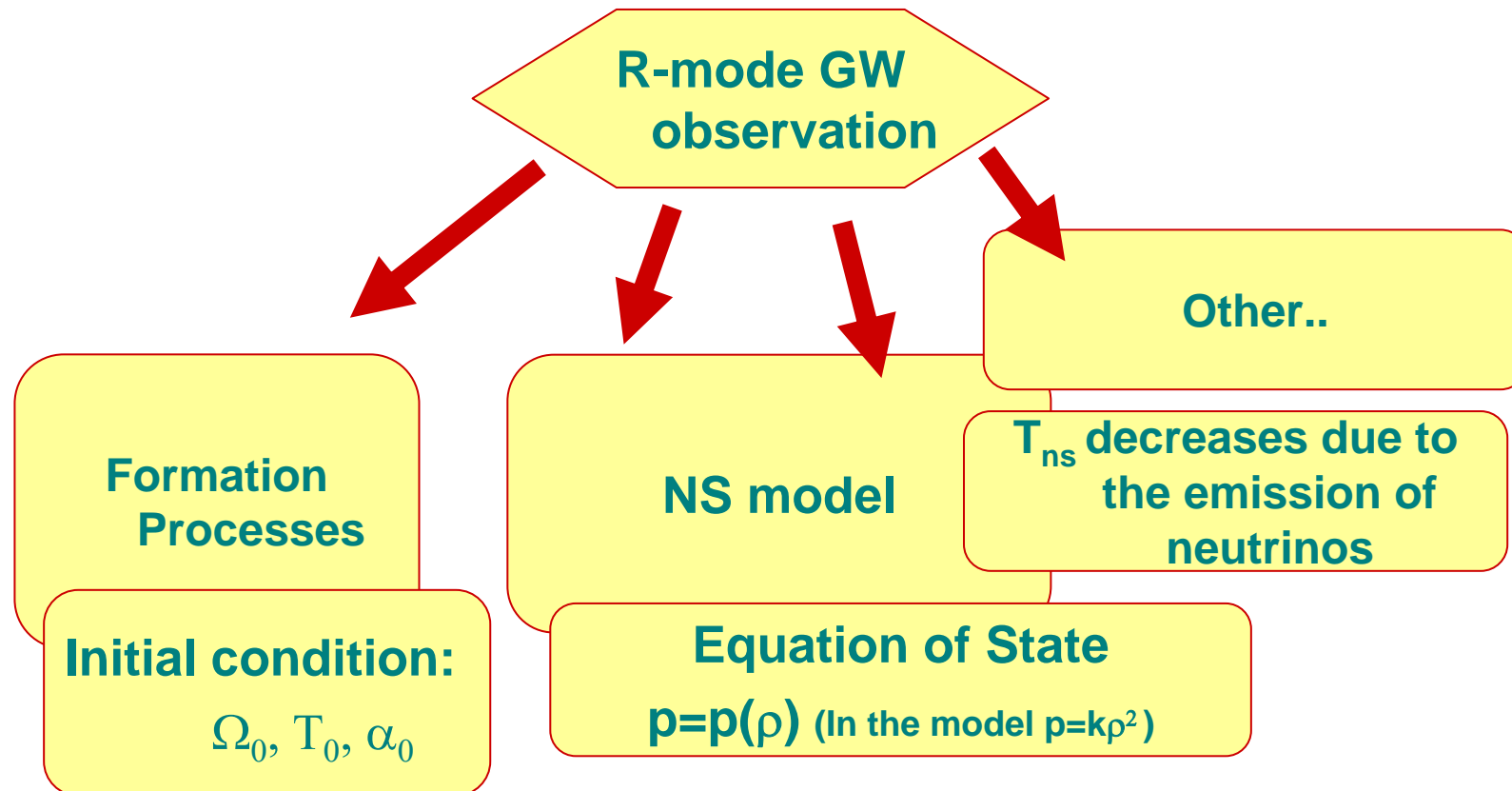
$$\frac{S}{N} = \frac{250}{\sqrt{2 + K}} \frac{20 \text{ Mpc}}{D}$$

- The SNR dependence on K is show in figure:
- Pessimistic case is for very high K, when NS born with substantial differential rotation:  
 $K \approx [10^5 \div 10^6] \rightarrow$   
 SNR is [5-15] @1Mpc



# R-modes GW, ET science case and nuclear physics

- R-mode gravitational wave observation can produce important and unique correlation with the nuclear physics of the neutron star and formation processes.



# Conclusions

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- The r-mode GW optimal Signal-to-Noise ratio for Einstein Telescope has been estimated
- r-mode GW could be observed by ET with  $\text{SNR} > 20$
- GW signals may carry information on:
  - gravitational physics
  - nuclear physics
- This signals could be an opportunity to know the NS nuclear physics, that maybe can't be done in other ways.

# What next

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- Correlate ET SNR with the Expected newborn Neutron
  - Our galaxy
  - Up to 150Mpc
- Take a look on the detection algorithms, parameters reconstruction and computational requirements for the sub-optimal case
- Compile the science opportunity list, given by this type of signal for ET

# References

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- **“Gravitational waves from *r*-modes” Paulo M. Sá · Brigitte Tomé** - *Astrophys Space Sci* (2007) 308: 557–561
- **“Gravitational waves from the *r*-modes of rapidly rotating neutron stars” Owen B.J.** - *Gravitational Waves, AIP Conference Proceedings, Vol. 523. New York: AIP Press, 2000., p.55*
- [http://www.phys.psu.edu/people/display/index.html?person\\_id=1484&mode=research&research\\_description\\_id=333&year=2004](http://www.phys.psu.edu/people/display/index.html?person_id=1484&mode=research&research_description_id=333&year=2004)