Testing Einstein with Einstein Telescope

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BBH Signals as Testbeds for GR

- Gravity gets ultra-strong during a BBH merger compared to any observations in the solar system or in binary pulsars
  - In the solar system: $\varphi/c^2 \sim 10^{-6}$
  - In a binary pulsar it is still very small: $\varphi/c^2 \sim 10^{-4}$
  - Near a black hole $\varphi/c^2 \sim 1$
  - Merging binary black holes are the best systems for strong-field tests of GR
- Dissipative predictions of gravity are not even tested at the 1PN level
  - In binary black holes even $(v/c)^7$ PN terms might not be adequate for high-SNR (~100) events

Qualitative Tests

- Polarization states
  - Are there polarizations other than those predicted by GR
    - No concrete proposals yet but some work within the LV
    - No evaluation in the context of ET
- Quasi-normal modes
  - Is the inspiral phase followed by a quasi-normal mode?
    - No concrete evaluations yet
  - Are the different quasi-normal modes consistent with each other?
    - Berti, Cardosa, Will: In the context of LISA
- Is the geometry of the merged object that of a Kerr black hole? (Ryan)
  - Many evaluations in the context of LISA none in the case of ET

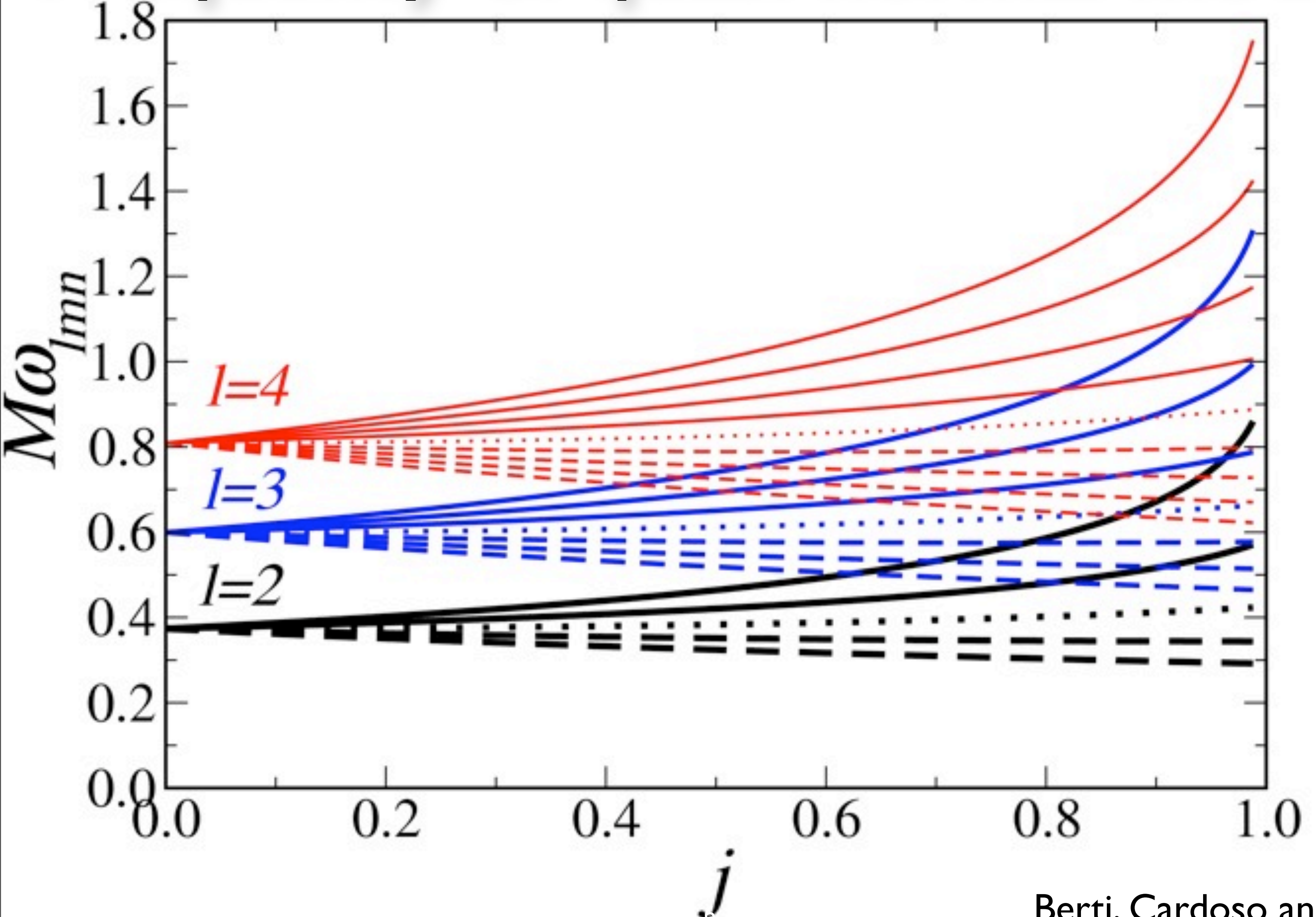
Quantitative Tests

- Is the phasing of the waveform consistent with General Relativity
  - Can we measure the different post-Newtonian terms and to what accuracy?
    - Detailed study in the case of non-spinning BBH on a quasi-circular orbit (Mishra et al)
    - Effect of spin is important: Neglecting them could lead to erroneous conclusion that GR is wrong while it is not
- Is the signal from the merger phase consistent with the predictions of numerical relativity simulations?
  - Are the parameters of the system from the inspiral, merger and ringdown phases consistent with one another?

Black hole quasi-normal modes

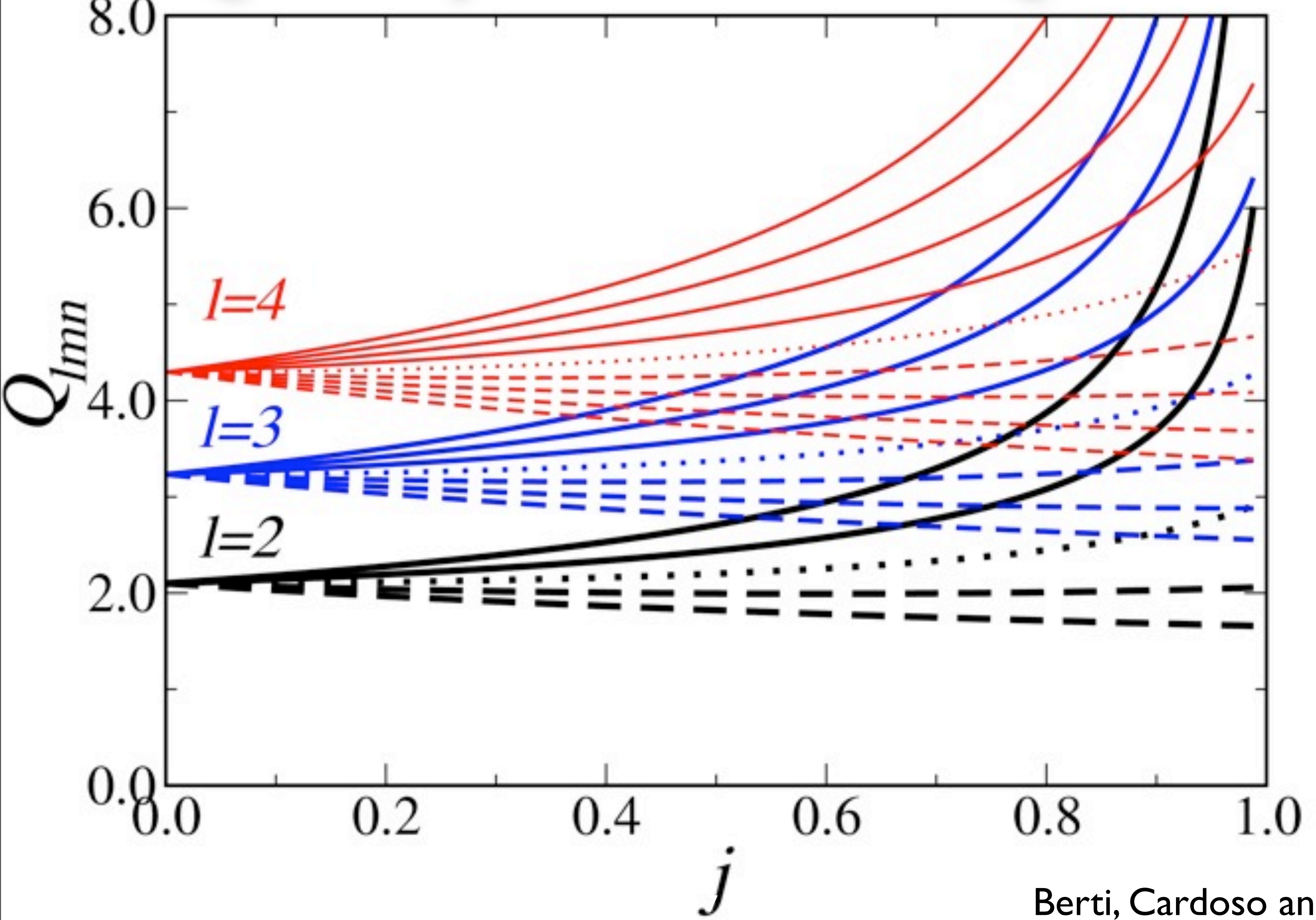
- Damped sinusoids with characteristic frequencies and decay times
  - In general relativity frequencies f_{lmn} and decay times t_{lmn} all depend only on the mass M and spin q of the black hole
- Measuring two or modes unambiguously, would severely constrain general relativity
  - If modes depend on other parameters (e.g., the structure of the central object), then test of the consistency between different mode frequencies and damping times would fail
- ET could observe formation of black holes out to red-shifts of several

Frequency of quasi normal modes



Berti, Cardoso and Will

Quality Factor of QNMs



Berti, Cardoso and Will

Status of tests with QNM

- Studying QNM from NR simulations at various mass ratios: 1:1, 1:2, 1:4, 1:8, final spins from -0.8 to +0.8
  - It is not too difficult to generate the QNM only part of the merger signal
  - Can carry out a wide exploration of the parameter space
- What is the relative energy in the various ringdown modes?
  - Are there at least two modes containing enough energy so that their damping times and frequencies can be measured with good (i.e. at least 10% accuracy)?
  - 33 seems to contain contain enough energy compared to 22 modes; should be possible to extract the total mass and spin magnitude
  - Measuring the relative amplitudes of the different modes can shed light on the binary progenitor, namely the total mass and its mass ratio
  - Polarization of ringdown modes can measure the spin axis of merged BH

Inspiralling compact binaries and testing general relativity

Adiabatic inspiral phase of a compact binary coalescence is well modelled using post-Newtonian (PN) formalism.

- Determination of coefficients in phasing formula can lead to meaningful tests
  - Detectability of tails [Blanchet & Sathyaprakash, 1994].
  - Measuring the dipolar content of the gravitational wave and test scalar-tensor theories [Will, 1994; Krolak et al, 1995, Damour & Esposito-Farése, 1998].
  - Parametrizing the 1PN coefficient of the phasing formula capturing the compton wavelength of the massive graviton and bounding its value from GW observations [Will, 1998].

The question

Can these tests be generalized, without having to know a priori the parameters of the underlying theory of gravity?

Parametrized test of PN theory

Phasing formula in the restricted waveform approximation

$$\tilde{h}(f) = \frac{1}{\sqrt{30}\,\pi^{2/3}} \frac{\mathcal{M}^{5/6}}{D_L} f^{-7/6} e^{i\psi(f)},$$

and to 3.5PN order the phase of the Fourier domain waveform is given by

$$\psi(f) \;=\; 2\pi f t_c - \phi_c - \frac{\pi}{4} + \sum_{k=0}^{7} (\psi_k + \psi_{kl} \ln f)\, f^{\frac{k-5}{3}},$$

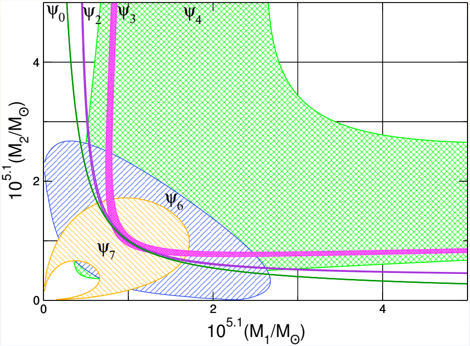
Log terms in the PN expansion

- Phasing coefficients are functions of component masses of the binary: $\psi_k(m_1, m_2)$ & $\psi_{kl}(m_1, m_2)$ [Spins negligible]
- Independent determination of 3 or more of the phasing coefficients ⇒ Tests of PN theory[KGA, Iyer, Qusailah & Sathyaprakash, 2006].

Basic Idea

- Parametrize the phasing formula in terms of various phasing coefficients where all of them are treated as independent.
- See how well can different parameters be extracted.
- Those which are well estimated, plot them (ψ_k & ψ_{kl}) in the $m_1 - m_2$ plane (similar to binary pulsar tests) with the widths of various curves proportional to $1 - \sigma$ error bars.

[KGA, Iyer, Qusailah, Sathyaprakash, 2006a]



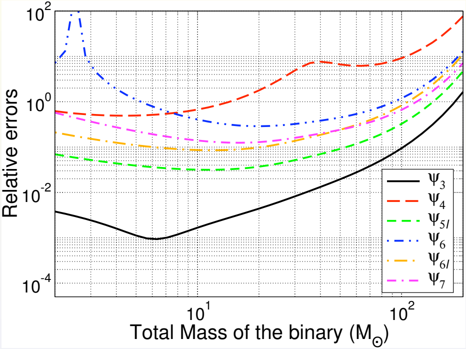
Issues

Highly correlated parameters ⇒ Ill-conditioned Fisher matrix for a large parameter space.

Alternative Proposal

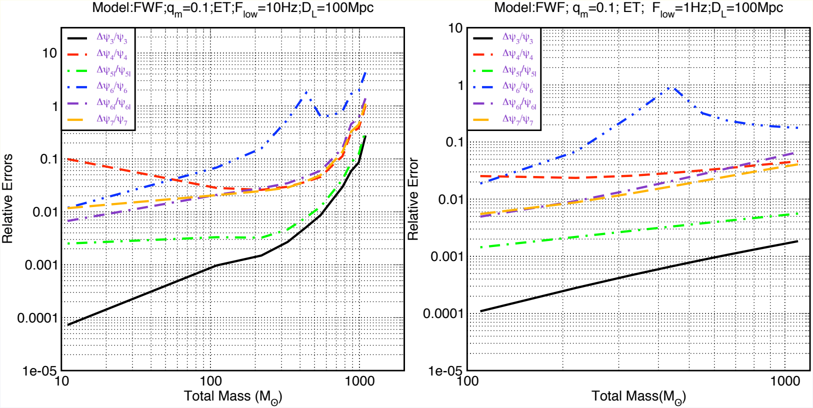
[KGA, Iyer, Qusailah & Sathyaprakash, 2006b]

- Treat two parameters as basic variables in terms of which one can parametrize all other parameters EXCEPT one which is the *test* parameter.
- This way, dimensionality of the parameter space is considerably reduced.
- Thus, one will have ${}^8C_3$ tests, not all of them independent.
- The best choice to be used as basic variables are the leading two coefficients at 0PN & 1PN, which are the best determined ones.
- Then one will have 6 tests.



- Used an earlier EGO noise PSD (similar to one of the ET noise PSDs).
- All parameters except ψ_4 determined quite well over a large range of masses.

Results, FWF: 10Hz Cut-off Vs 1Hz Cut-off



Features

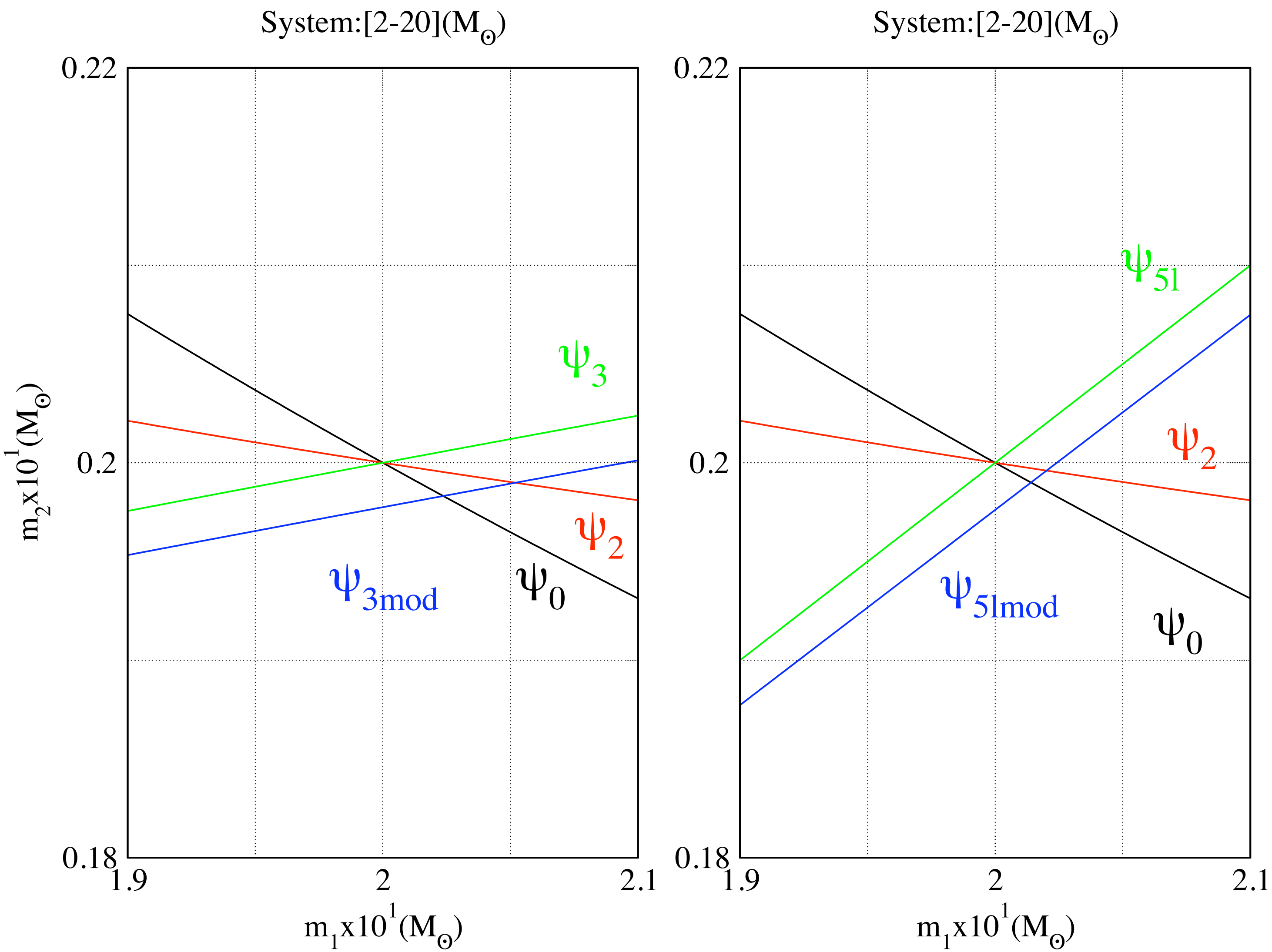
- Improvements are significant for masses $> 250 M_{\odot}$.
- ψ_4 makes the best use of lowered seismic cut-off.
- Otherwise nothing very dramatic due to lower seismic cut-off.

Power of a PN Test

- Suppose the GR k^{th} PN coefficient is $q_k(m_1,m_2)$ while the true k^{th} PN coefficient is $p_k(m_1,m_2)$
- The "measured value of the k^{th} PN coefficient is, say, p_0
- The curve $q_k(m_1,m_2)= p_0$ in the (m_1,m_2) plane will not pass through the masses determined from the other parameters

Power of the PPN test

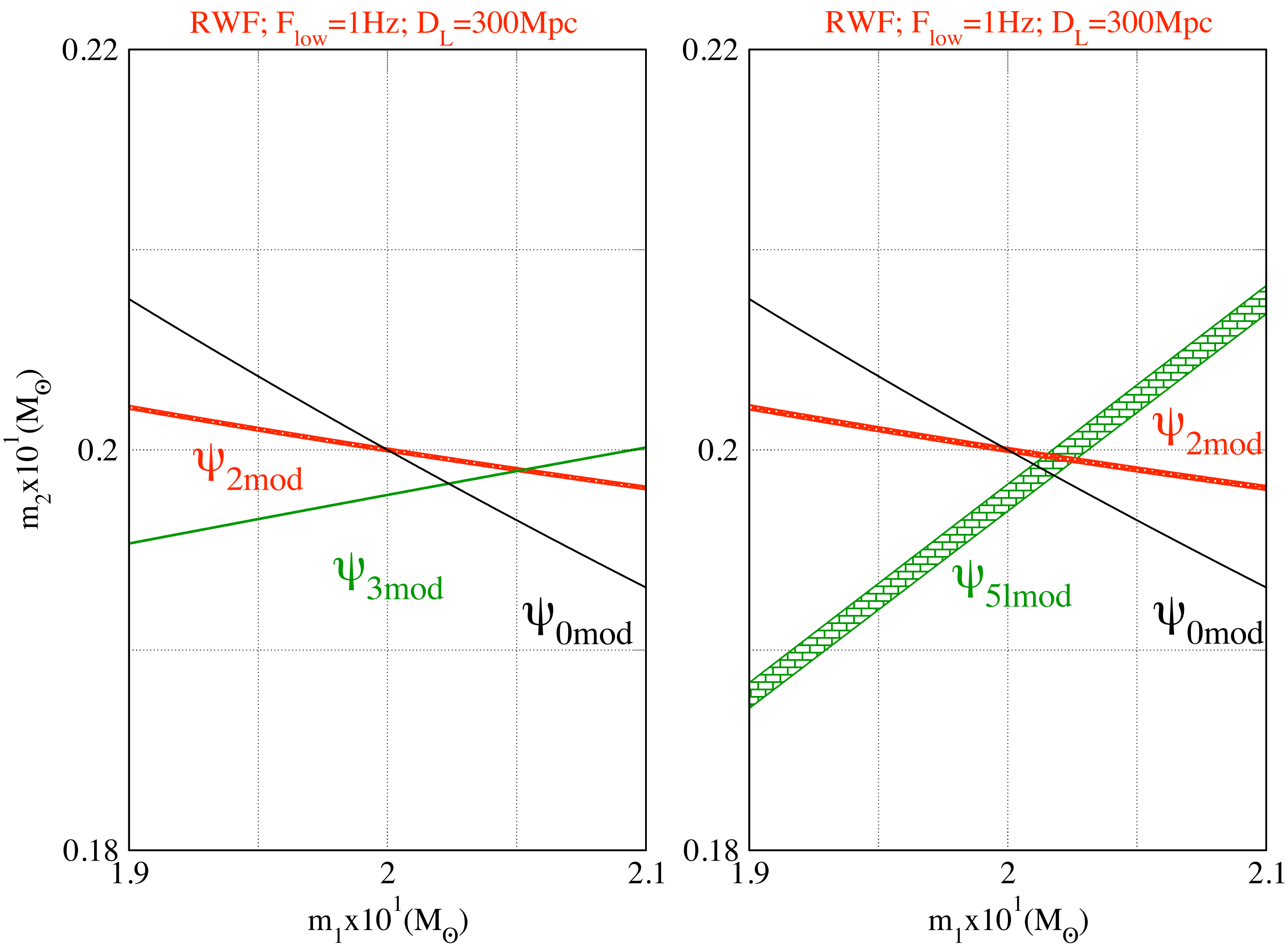
Effect of changing the coefficients ψ_3 and ψ_{51} by 1% on the test.



NOTE: Blue curve in the plot corresponds to the new ψ_k

Efficacy of the PPN Test

Effect of changing the coefficients ψ_3 and ψ_{51} by 1% on the test.



NOTE: Reference System: (2-20) ($M_\odot$)