WG4 assumptions regarding ET and astrophysics

To arrive at preliminary estimates as to what ET might be capable of in terms of astrophysics and cosmology, some assumptions have to be made about the design of ET. This document provides one possible set of assumptions for the noise curve and particularly the topology which new members of WG4 can use as a starting point. However, it is important to note that (a) the noise curve is very provisional and will presumably get more sophisticated through the course of the design study; and (b) the final topology of ET is yet to be decided on.

I. NOISE CURVE

ET will probably consist of several different interferometers, with opening angles of either 60^o or 90^o, and as yet unspecified arm lengths. The power spectral density (PSD) we are about to specify is for a single interferometer with 90^o opening angle and 10 km arm length. To compute signal-to-noise ratios or Fisher matrices for a combination of interferometers from this PSD, the opening angles and arm lengths of the various interferometers will need to be folded in appropriately; this will be discussed in the next section.

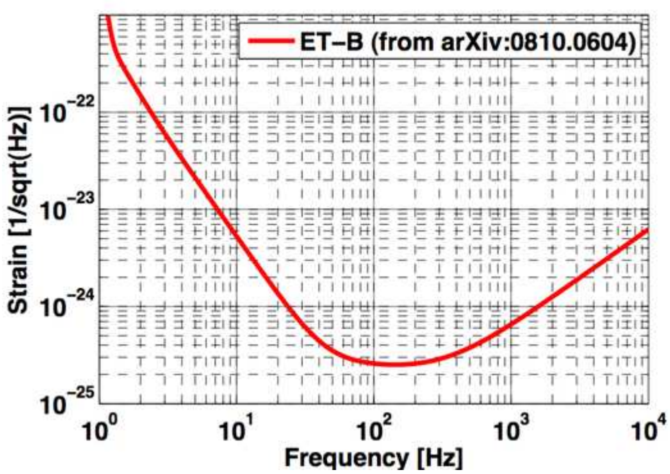

Figure 1: The ET-B strain sensitivity.

Fig. 1 shows the so-called ET-B strain sensitivity curve. For a single L-shaped interferometer with 90^o opening angle and 10 km arms, $S_h(f)$ has been fitted as

$$S_h(f) = 10^{-50}\left[2.39 \times 10^{-27} x^{-15.64} + 0.349 x^{-2.145} + 1.76 x^{-0.12} + 0.409 x^{1.1}\right]^2 \text{ Hz}^{-1}, \quad (1.1)$$

with $x = f/(100\,\text{Hz})$. We also need to specify a lower cut-off frequency, f_{lower}; commonly used values are 1 Hz, 5 Hz, and 10 Hz.

II. SENSITIVITY LOBES OF VARIOUS DETECTOR DESIGNS

One possible set-up for ET would be a triangular tube with 10 km edges containing three interferometers (ifos) with 60 degree opening angles. We first consider the combined signal-to-noise ratio (SNR) for the three ifos together. We then show that the triangular, three ifo design produces the same combined SNR as a combination of two appropriately scaled ifos with 90 degree opening angles.

A. Triangle with three interferometers

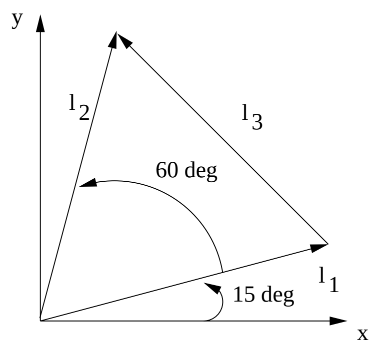

Figure 2: The unit vectors defining the detector tensors for the triangular ET.

Consider three ifos with 60 degree opening angles, arranged in an equilateral triangle. Let $\hat{l}_i$, $i = 1, 2, 3$ be unit vectors tangent to the edges of the triangle as in Fig. 2. These can be expressed in terms of unit vectors $(\hat{x}, \hat{y}, \hat{z})$ defining a Cartesian coordinate system, where $(\hat{x}, \hat{y})$ are in the detector plane:

$$\hat{l}_i = \cos(\alpha_i)\,\hat{x} + \sin(\alpha_i)\,\hat{y}, \tag{2.1}$$

with

$$\alpha_i = \frac{\pi}{12} + (i-1)\frac{\pi}{3}. \tag{2.2}$$

The three ifos inside the triangular tube have detector tensors

$$\begin{aligned} d_1^{ab} &= \frac{1}{2}(\hat{l}_1^a \hat{l}_1^b - \hat{l}_2^a \hat{l}_2^b), \\ d_2^{ab} &= \frac{1}{2}(\hat{l}_2^a \hat{l}_2^b - \hat{l}_3^a \hat{l}_3^b), \\ d_3^{ab} &= \frac{1}{2}(\hat{l}_3^a \hat{l}_3^b - \hat{l}_1^a \hat{l}_1^b), \end{aligned} \tag{2.3}$$

where $a = 1, 2, 3$ are spatial indices. Using (2.3) we can compute the responses of the three interferometers. Let $h_{ab}(t)$ be the metric perturbation in transverse-traceless gauge; then the strain h_K in the K-th interferometer is

$$h_K(t) = h_{ab}(t)\, d_K^{ab}, \tag{2.4}$$

where summation over repeated indices is assumed. In terms of the usual 'plus' and 'cross' polarizations, this leads to

$$h_K = F_+^K h_+ + F_\times^K h_\times, \tag{2.5}$$

with

$$\begin{aligned}
F_+^1(\theta,\phi,\psi) &= \frac{\sqrt{3}}{2}\left[\frac{1}{2}(1+\cos^2(\theta))\,\cos(2\phi)\,\cos(2\psi) - \cos(\theta)\,\sin(2\phi)\,\sin(2\psi)\right],\\
F_\times^1(\theta,\phi,\psi) &= \frac{\sqrt{3}}{2}\left[\frac{1}{2}(1+\cos^2(\theta))\,\cos(2\phi)\,\sin(2\psi) + \cos(\theta)\,\sin(2\phi)\,\cos(2\psi)\right],\\
F_{+,\times}^2(\theta,\phi,\psi) &= F_{+,\times}^1(\theta,\phi+2\pi/3,\psi),\\
F_{+,\times}^3(\theta,\phi,\psi) &= F_{+,\times}^1(\theta,\phi+4\pi/3,\psi),
\end{aligned} \tag{2.6}$$

where (θ, ϕ) denote the sky position and ψ is the polarization angle.

Now suppose we are looking for the signal in the outputs of the interferometers by means of matched filtering. Assuming that the noise in the three ifos is completely uncorrelated, their outputs can be coherently combined. If ρ_K, $K = 1, 2, 3$ are the signal-to-noise ratios (SNRs) in the individual ifos, the combined SNR ρ is given by

$$\rho = (\rho_1^2 + \rho_2^2 + \rho_3^2)^{1/2} \tag{2.7}$$

If the templates and the signals are from the same waveform families then

$$\rho_K^2 = 4\int_{f_{lower}}^{f_{upper}} \frac{|\tilde{h}_K(f)|^2}{S_h(f)}df, \tag{2.8}$$

for some lower and upper cut-off frequencies f_{lower}, f_{upper}, and $\tilde{h}_K(f)$ are the Fourier transforms of the ifo responses $h_K(t)$. Hence

$$\rho = \left(4\int_{f_{lower}}^{f_{upper}} \frac{\sum_{K=1}^{3}|\tilde{h}_K(f)|^2}{S_h(f)}df\right)^{1/2}. \tag{2.9}$$

With some algebra, one can show that

$$\sum_{K=1}^{3}|\tilde{h}_K|^2 = \frac{9}{32}\left(4\tilde{h}_{xy}^2 + (\tilde{h}_{xx} - \tilde{h}_{yy})^2\right), \tag{2.10}$$

where $\tilde{h}_{xx}(f) = \tilde{h}_{ab}(f)\hat{x}^a\hat{x}^b$, $\tilde{h}_{yy}(f) = \tilde{h}_{ab}(f)\hat{y}^a\hat{y}^b$, and $\tilde{h}_{xy}(f) = \tilde{h}_{ab}(f)\hat{x}^a\hat{y}^b$.

B. 'Double L' configuration

The triangular set-up we just described is equivalent to having two co-located interferometers with 90 degree opening angles, but rotated 45 degrees with respect to each other (Fig. 3), and with appropriately scaled arm lengths. Define

$$\hat{x}'^a = \frac{\sqrt{2}}{2}(\hat{x}^a - \hat{y}^a) \qquad \hat{y}'^a = \frac{\sqrt{2}}{2}(\hat{x}^a + \hat{y}^a). \tag{2.11}$$

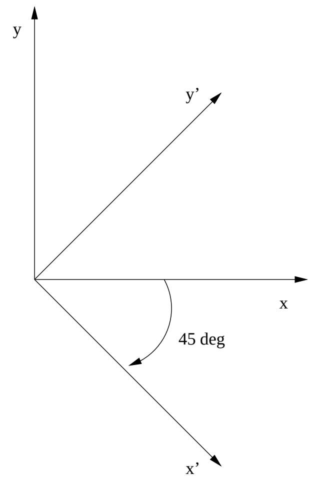

Figure 3: The unit vectors defining the detector tensors for two ifos with 90 degree opening angles at 45 degrees from each other.

Then the detector tensors associated with the two 'L-shaped' ifos are

$$\begin{aligned} D_1^{ab} &= \frac{1}{2}(\hat{x}^a\hat{x}^b - \hat{y}^a\hat{y}^b), \\ D_2^{ab} &= \frac{1}{2}(\hat{x}'^a\hat{x}'^b - \hat{y}'^a\hat{y}'^b). \end{aligned} \tag{2.12}$$

The responses of the individual ifos are

$$H_A(t) = H_{ab}(t)\, D_A^{ab} \tag{2.13}$$

for $A = 1, 2$, and the combined SNR is given by

$$\rho = \left(4\int_{f_{lower}}^{f_{upper}} \frac{\sum_{A=1}^{2}|\tilde{H}_A(f)|^2}{S_h(f)}df\right)^{1/2}. \tag{2.14}$$

It is not difficult to show that

$$\sum_{A=1}^{2}|\tilde{H}_A(f)|^2 = \frac{1}{4}\left(4\tilde{h}_{xy}^2 + (\tilde{h}_{xx} - \tilde{h}_{yy})^2\right). \tag{2.15}$$

Comparing this with (2.10), we see that the three 10 km V-shaped ifos in a triangle are equivalent to two L-shaped ifos at 45 degrees to each other and with arm lengths of $3/(2\sqrt{2}) \times 10$ km.

Note that this is *also* equivalent to having two L-shaped tubes at 45 degrees to each other and with $(3/4) \times 10$ km arm length, but with *two ifos in each of the two tubes* ($\sqrt{2} \times 3/4 = 3/(2\sqrt{2})$). In this configuration the total tube length is again 30 km, as with the triangle configuration, but now we would have a total of four L-shaped ifos with 7.5 km arm length.

Instead of multiplying $S_h(f)$ by a factor of $(2\sqrt{2}/3)^2$, it is convenient to multiply the responses (2.13) by $3/(2\sqrt{2})$:

$$H'_A(t) = \frac{3}{2\sqrt{2}}H_A(t). \tag{2.16}$$

The explicit expressions for these in terms of the gravitational wave polarizations h_+, $h_\times$ and as functions of sky position (θ, ϕ) and polarization angle ψ are simply

$$\begin{aligned} H_1'(\theta,\phi,\psi;t) &= \frac{3}{2\sqrt{2}}\left(F_+(\theta,\phi,\psi)h_+(t) + F_\times(\theta,\phi,\psi)h_\times(t)\right), \\ H_2'(\theta,\phi,\psi;t) &= \frac{3}{2\sqrt{2}}\left(F_+(\theta,\phi+\pi/4,\psi)h_+(t) + F_\times(\theta,\phi+\pi/4,\psi)h_\times(t)\right), \end{aligned} \tag{2.17}$$

where

$$\begin{aligned} F_+(\theta,\phi,\psi) &= \frac{1}{2}\left(1+\cos^2(\theta)\right)\cos(2\phi)\cos(2\psi) - \cos(\theta)\sin(2\phi)\sin(2\psi), \\ F_\times(\theta,\phi,\psi) &= \frac{1}{2}\left(1+\cos^2(\theta)\right)\cos(2\phi)\sin(2\psi) + \cos(\theta)\sin(2\phi)\cos(2\psi). \end{aligned} \tag{2.18}$$

The combined SNR for the rescaled detectors is just

$$\rho' = \left(4\int_{f_{lower}}^{f_{upper}} \frac{\sum_{A=1}^{2}|\tilde{H}_A'(f)|^2}{S_h(f)}df\right)^{1/2}. \tag{2.19}$$