

Interferometry with Laguerre- Gauss Modes

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Overview

- The three beams that were compared
- Thermal noise reduction using Laguerre-Gauss (LG) modes
- Longitudinal error signals using Pound-Drever-Hall method
- Tilt to longitudinal phase coupling of a FP cavity
- Alignment analysis of an arm cavity
- Differential arm cavity misalignment coupling to output
- The next step: experimental verification of results

Prospects of higher-order Laguerre Gauss modes in future gravitational wave detectors

Simon Chelkowski,¹ Stefan Hild,¹ and Andreas Freise¹

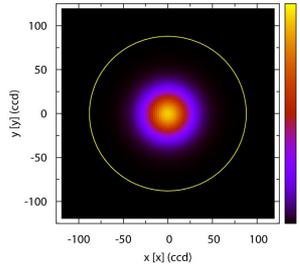
¹*School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham B15 2TT, UK*

(Dated: June 8, 2009)

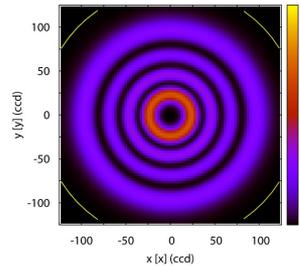
The application of higher-order Laguerre Gauss (LG) modes in large-scale gravitational wave detectors has recently been proposed. In comparison to the fundamental mode, some higher-order



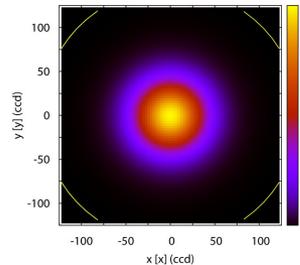
The three beams that were compared



- $LG_{00\text{small}}$: The 'reference' configuration
 $R_c=1910\text{m}$, $w=35.2\text{mm}$, $w_0=16.3\text{mm}$



- LG_{33} : LG_{33} with same parameters as $LG_{00\text{small}}$
 $R_c=1910\text{m}$, $w=35.2\text{mm}$, $w_0=16.3\text{mm}$

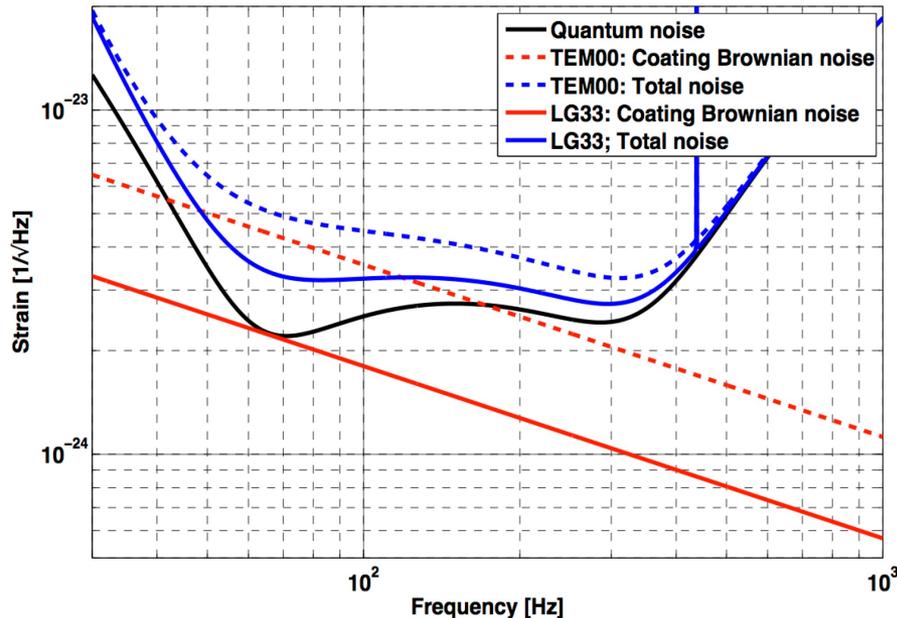


- $LG_{00\text{large}}$: LG_{00} mode on same mirror as LG_{33}
 $R_c=1536.7\text{m}$, $w=57.7\text{mm}$, $w_0=8.9\text{mm}$



Thermal noise benefits of LG modes

- Can reduce coating and substrate thermal noise
- See Franc's talk for more on this!

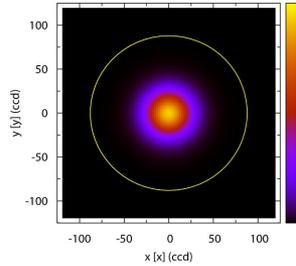


Advanced Virgo Improvement

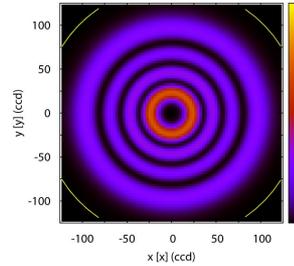
	LG00	LG33
SR detuning [Hz]	300	300
Beam size [cm]	6	~ 4
NS/NS inspiral range [Mpc]	145	191



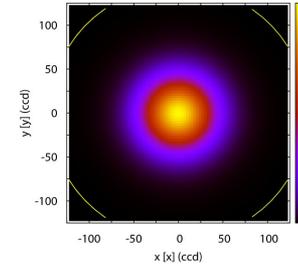
Longitudinal error signals using Pound-Drever-Hall method



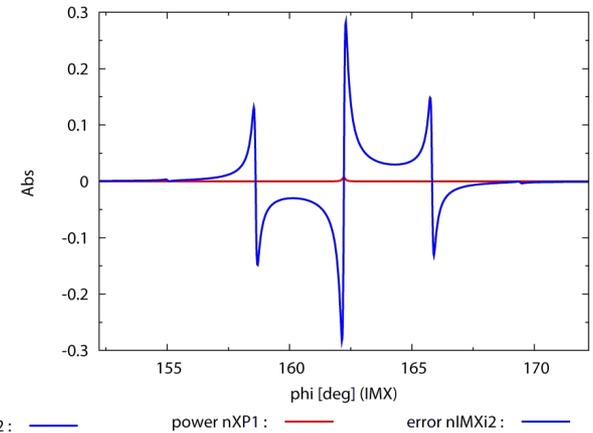
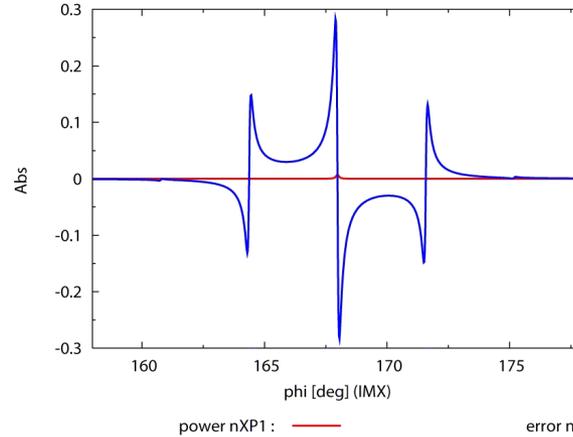
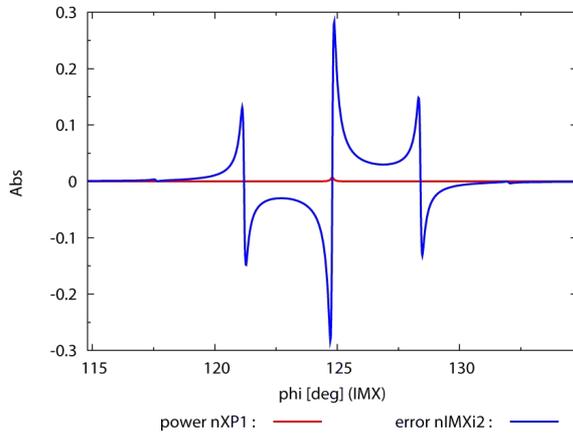
LG00 small



LG33



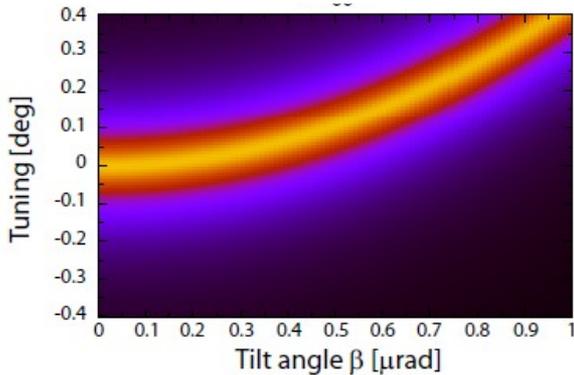
LG00 large





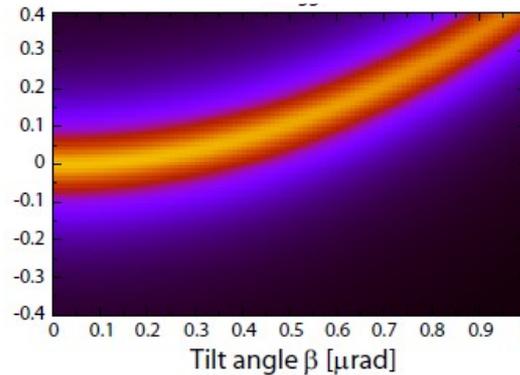
Tilt to longitudinal phase coupling of a Fabry-Perot cavity

LG00 small



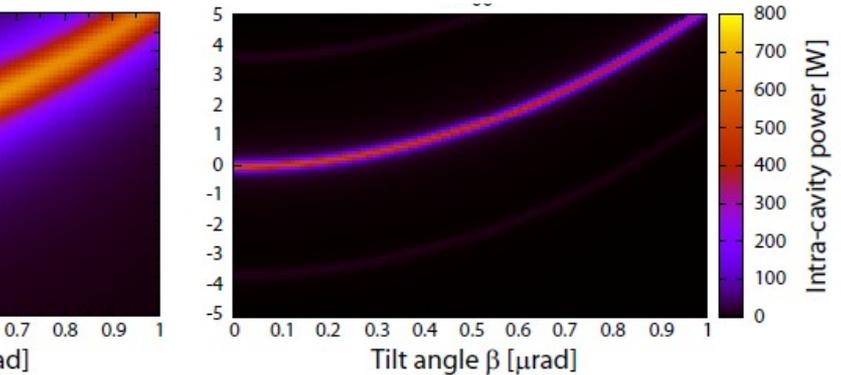
$$\Delta\phi \approx 0.4^\circ$$

LG33



$$\Delta\phi \approx 0.4^\circ$$

LG00 large

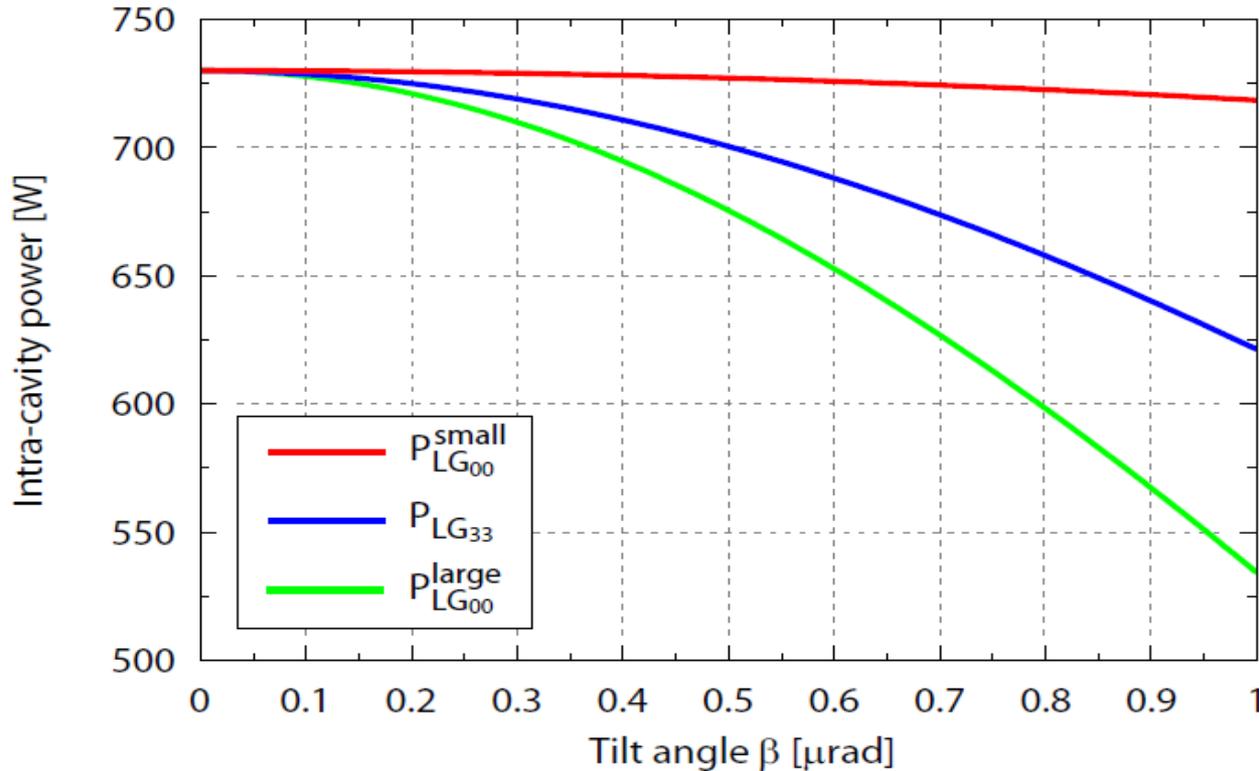


$$\Delta\phi \approx 5.0^\circ$$

- LG33 outperforms LG00_{large} in this area
- LG33 performance is similar to LG00_{small}
- Beam parameters are the dominant factor

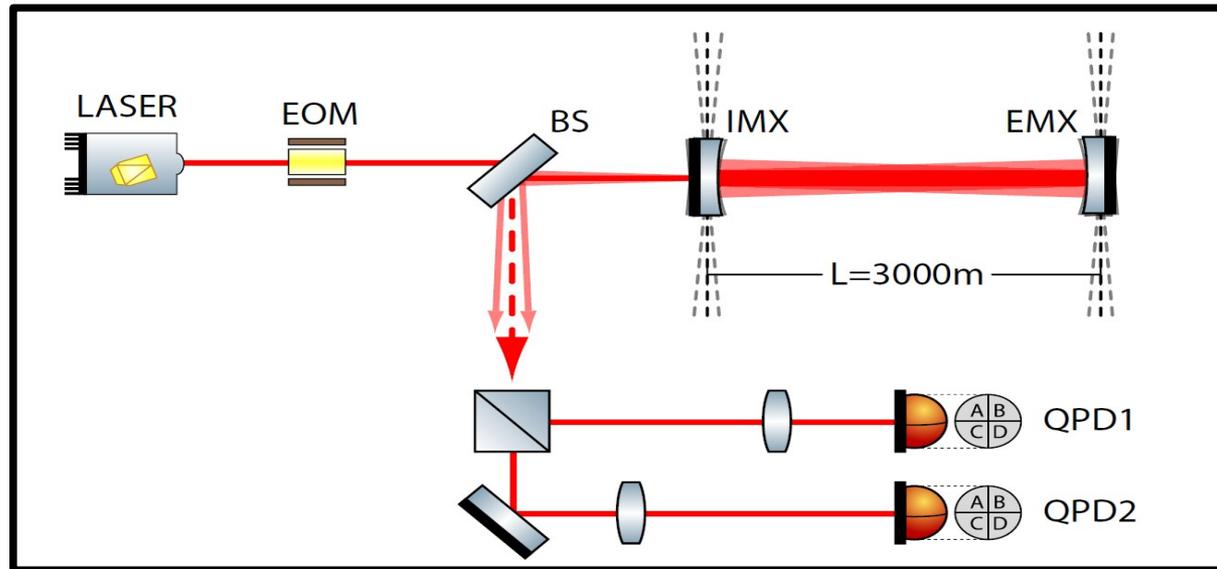


Misalignment: intra-cavity power loss



- LG_{33} outperforms $\text{LG}_{00}^{\text{large}}$ again

Alignment analysis of an arm cavity



$$C_{\text{configuration}} = \begin{pmatrix} \sigma_{\text{QPD1}}^{\text{IMX}} & \sigma_{\text{QPD1}}^{\text{EMX}} \\ \sigma_{\text{QPD2}}^{\text{IMX}} & \sigma_{\text{QPD2}}^{\text{EMX}} \end{pmatrix}$$

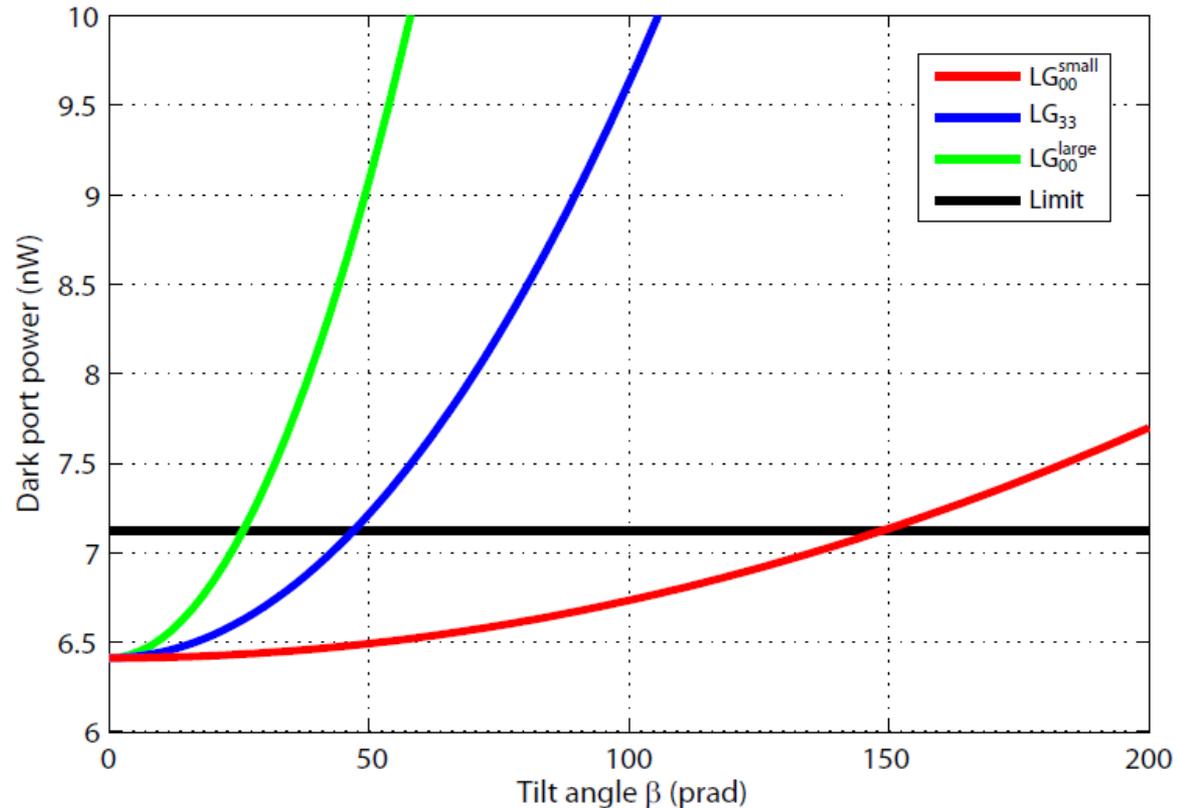
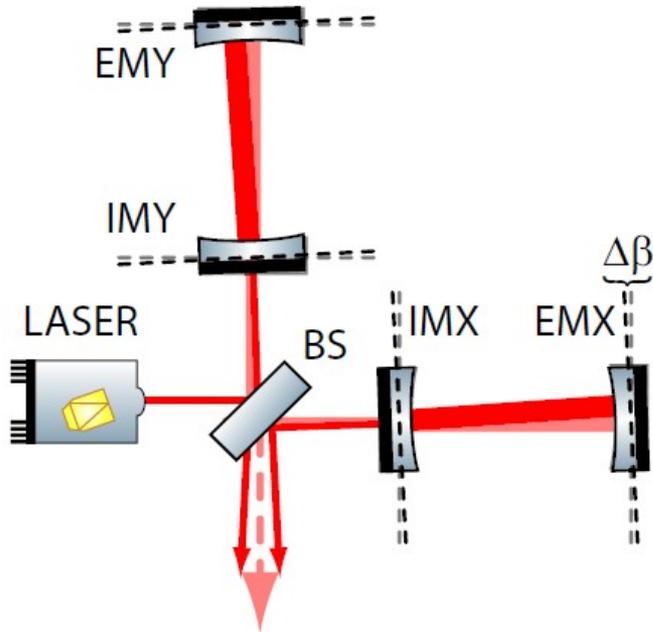
$$C_{\text{LG}_{00}^{\text{small}}} = 5.6152 \begin{pmatrix} 1 & 0.009 \\ 0.385 & 0.639 \end{pmatrix}$$

$$C_{\text{LG}_{33}} = 7.444 \begin{pmatrix} 1 & 0.003 \\ 0.368 & 0.641 \end{pmatrix}$$

$$C_{\text{LG}_{00}^{\text{large}}} = 17.774 \begin{pmatrix} 1 & 0.862 \\ 0.645 & 0.153 \end{pmatrix}$$

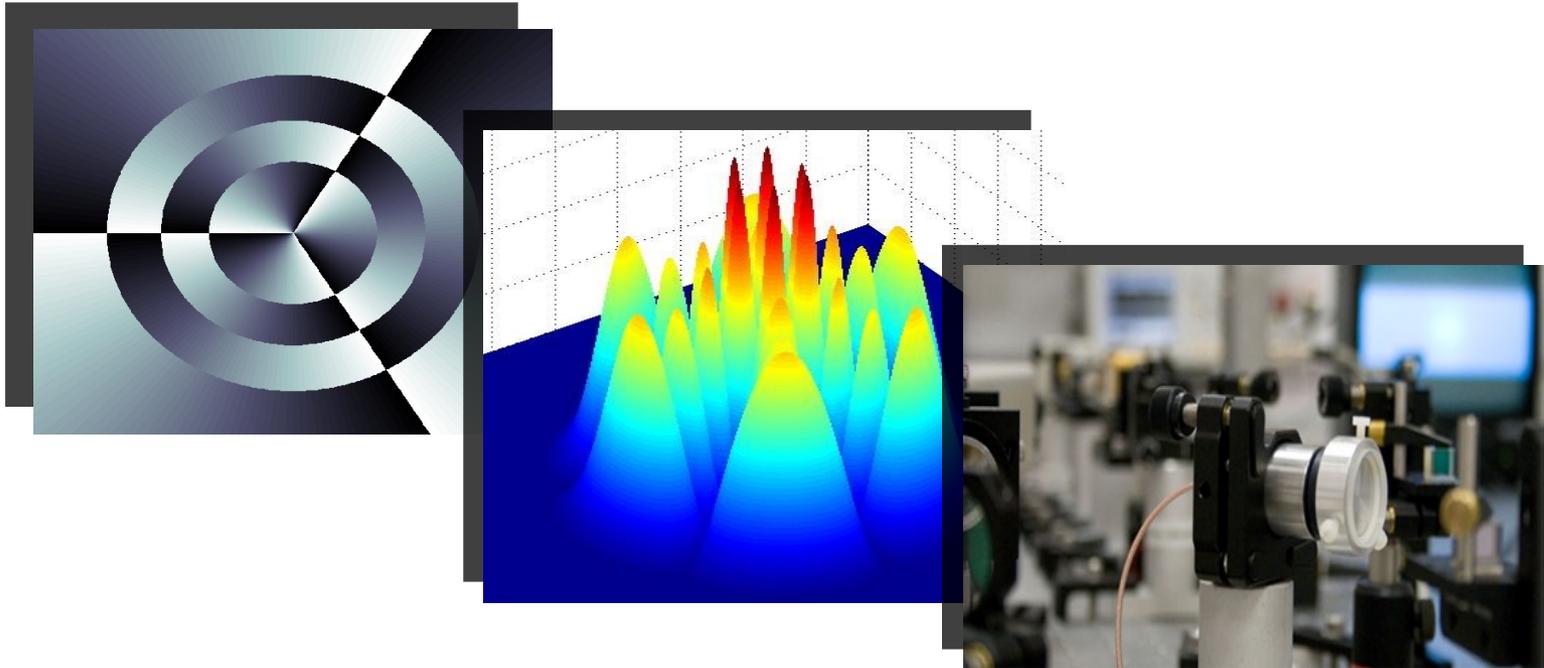


Differential arm cavity misalignment coupling to output port power





The next step: experimental verification

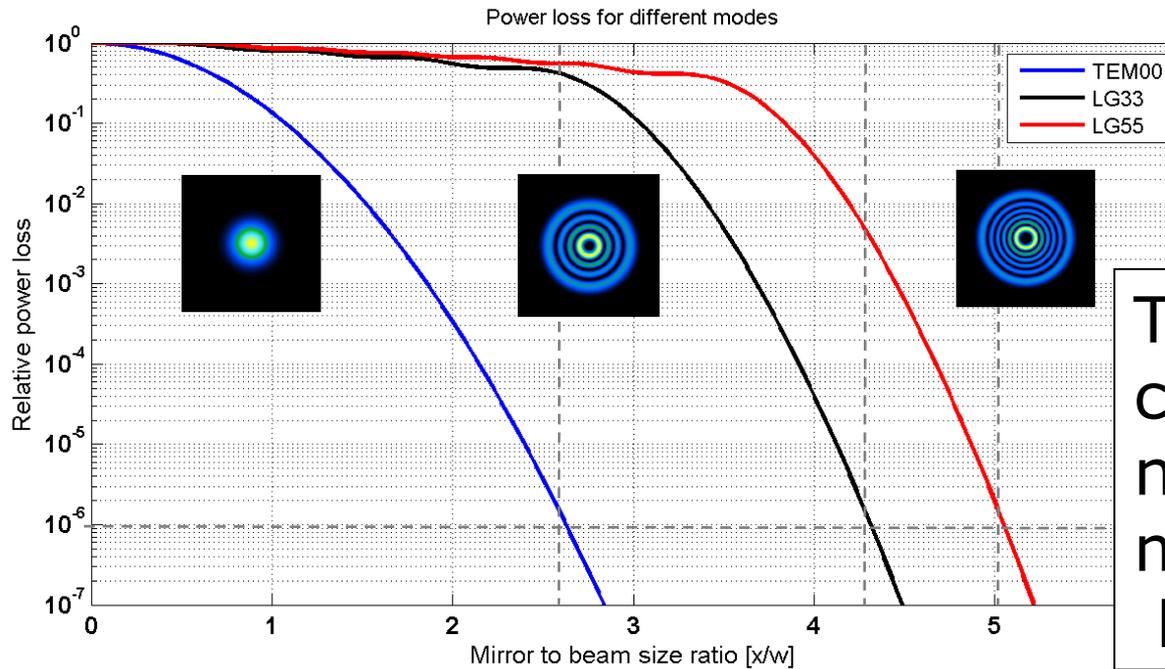


- Both configurations require lab demonstration
- Experience with LG mode interferometry will be crucial in order to reap the thermal noise benefits



...The end

Constraint: clipping loss



To get the same clipping loss we need to change the mirrors ROC (or use bigger mirrors).

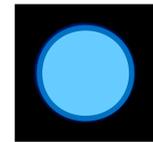
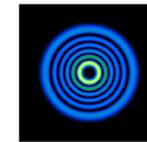
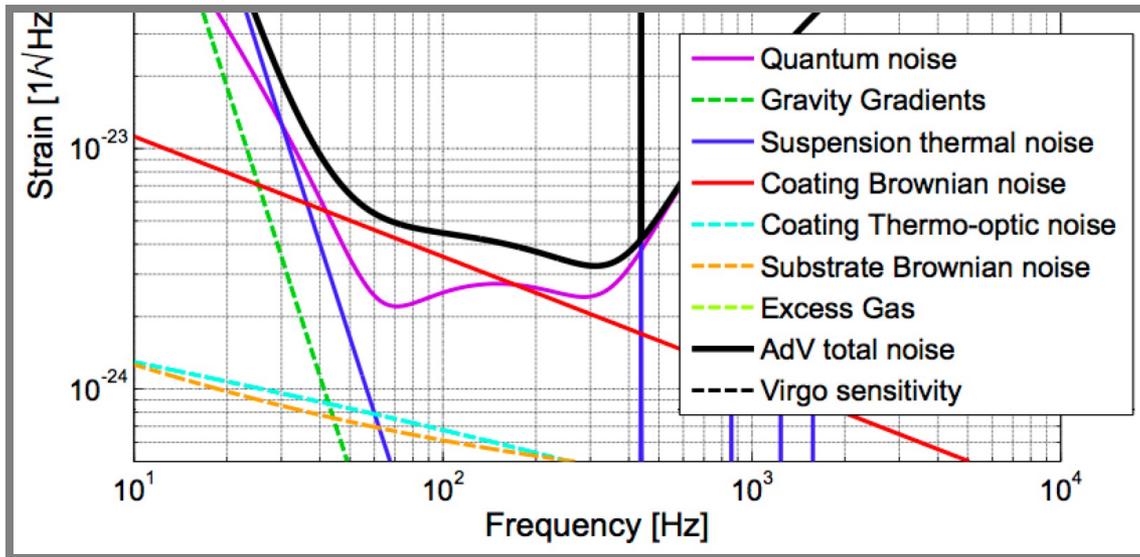
Mode scaling factors	LG ₀₀	LG ₃₃	LG ₅₅
Mirror size	1	1.64	1.92
Beam size	1	0.61	0.52



Expected thermal noise improvements

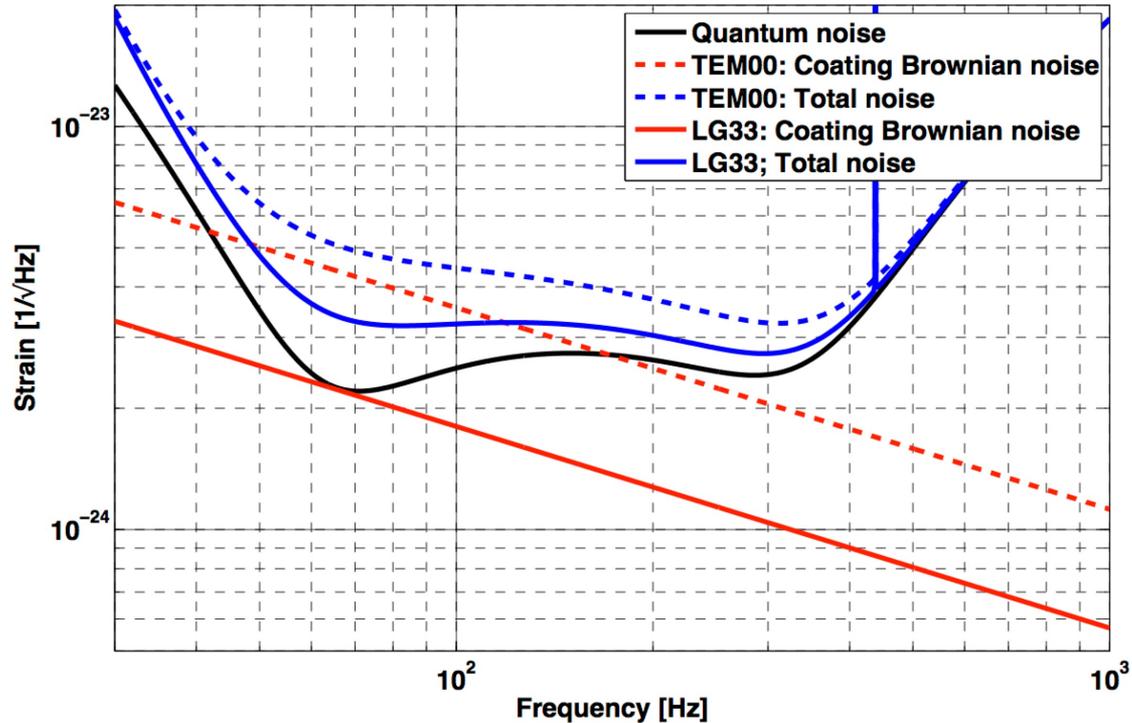
Reduction factors of thermal noise	LG00	LG33	LG55	Mesa
Coating thermal noise	1	~ 1.9	~ 2.1	~ 1.5
Substrate thermal noise	1	~ 2.1	~ 2.5	~ 1.8
Thermoelastic noise	1	~ 0.6	~ 0.4	~ 1.8

Reference: personal communication J.-Y. Vinet



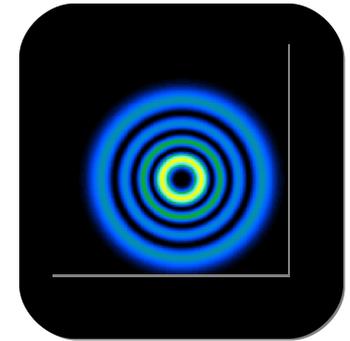
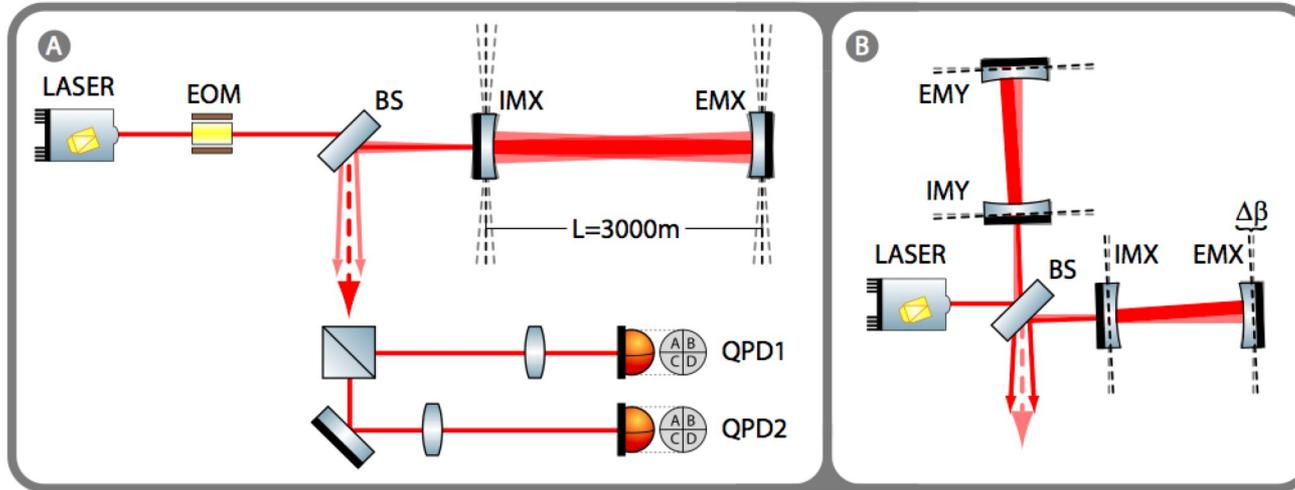


Advanced Virgo: inspiral range improvements



	LG00	LG33 (not optimised)
SR detuning [Hz]	300	300
Beam size [cm]	6	~ 4
NS/NS inspiral range [Mpc]	145	191

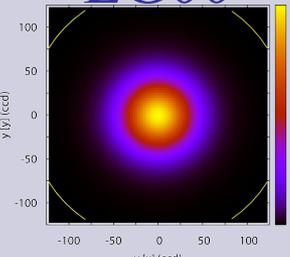
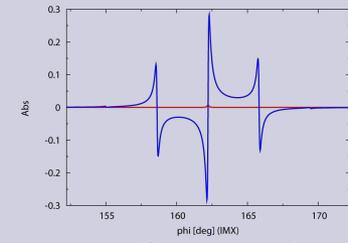
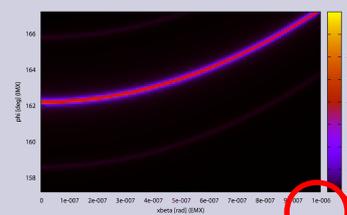
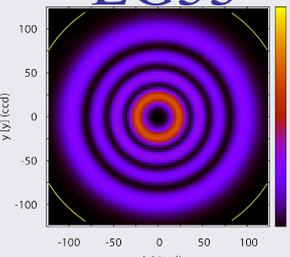
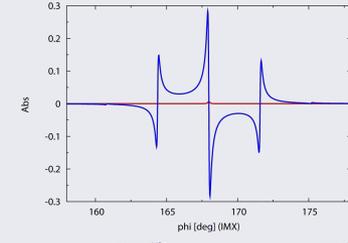
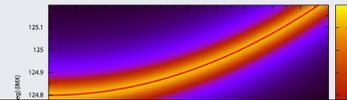
Comparison of length and alignment signals



It is important to compute also the beam jitter noise or coupling of alignment fluctuation into phase noise. Our first step: take a simple cavity, a simple Michelson to look for trouble.



Comparison of length and alignment signals

Transversal mode	Longitudinal error signal	Coupling tilt into longitudinal phase	Control Matrix for alignment signals
<p>LG00</p> 		 <p>$\Delta\phi \approx 5.0^\circ$</p> <p>1mrad</p>	$\begin{pmatrix} 1 & 0.862 \\ 0.645 & 0.153 \end{pmatrix}$ $\begin{pmatrix} \text{IMX to Qb} & \text{EMX to Qb} \\ \text{IMX to Qa} & \text{EMX to Qa} \end{pmatrix}$
<p>LG33</p> 			$\begin{pmatrix} 1 & 2.96e-3 \\ 0.368 & 0.641 \end{pmatrix}$

[Chelkowski et al (arxiv.org/abs/0901.4931)]

The performance regarding interferometric sensing and control of the LG33 mode is found to be similar, if not even better in all aspects of interest. Coupling of alignment into phase noise is comparable or better.



Summary

- Alternative beam shapes are an interesting (and in comparison rather simple) method for **reducing thermal effects** (thermal noise, thermal lensing)
- Thermal noise can be **reduced by factors >2** (linear spectral density)
- Generation of such beams seem to be feasible (information from other fields, to be verified)
- LG modes are **compatible with current optical designs**, it is easy to make a design for upgrading advanced detectors
- LG modes also seem to be compatible with other future technologies (QND, cooling, ...)



Thermal noise in mirrors

$$S_x(f) = \frac{4k_B T}{\pi f} \Phi U$$

↙ Loss angle
↖

Vinet CQG 22 (2005)

Levin Phys. Rev. D 57 659 (1998)

the strain energy stored in the test mass by a pressure normalized to 1 N, and having the same distribution as the light intensity in the readout beam

simplicity this and the following considers only Brownian substrate noise for substrates of infinite size. A lot of effort has gone into computing accurate numbers for coating noises, thermo-optic noise, both infinite and finite size mirrors. A good review of this topic will be published soon:

thermal issues in advanced Gravitational Wave Interferometric detectors

Vinet, *Living Reviews in Relativity*, to be published

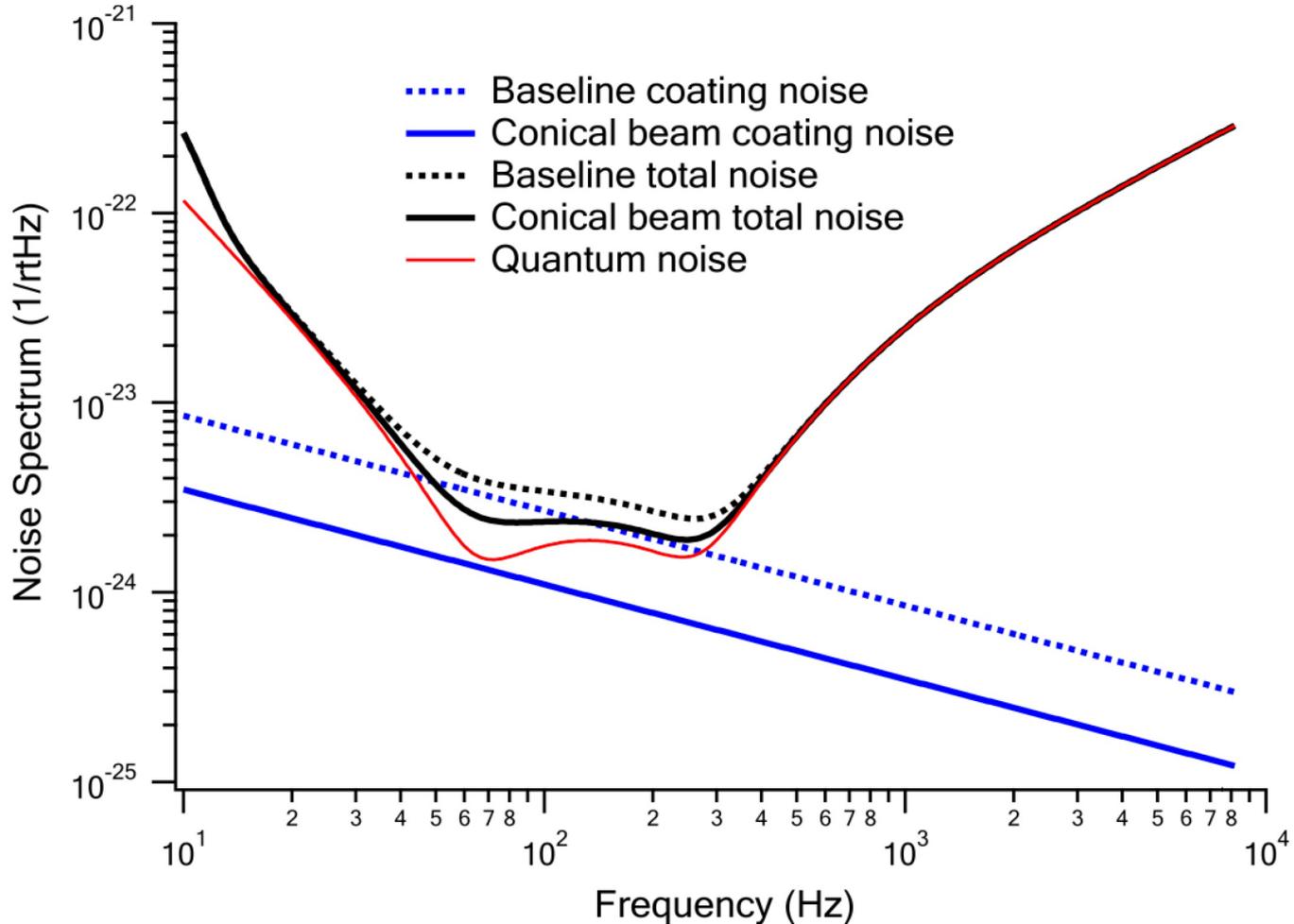


Readiness

- Thermal effect
 - Thermal noise calculations OK
 - Thermal lensing calculations OK
- Generation of LG modes
 - Conversion methods OK
 - Efficiency, mode purity in progress
 - Noise performance of LG converter to be done
- Interferometry with LG modes
 - Simulation of sensing and control OK
 - Table-top, prototype verification in progress
- Implementation into GW detectors
 - Core optics design OK



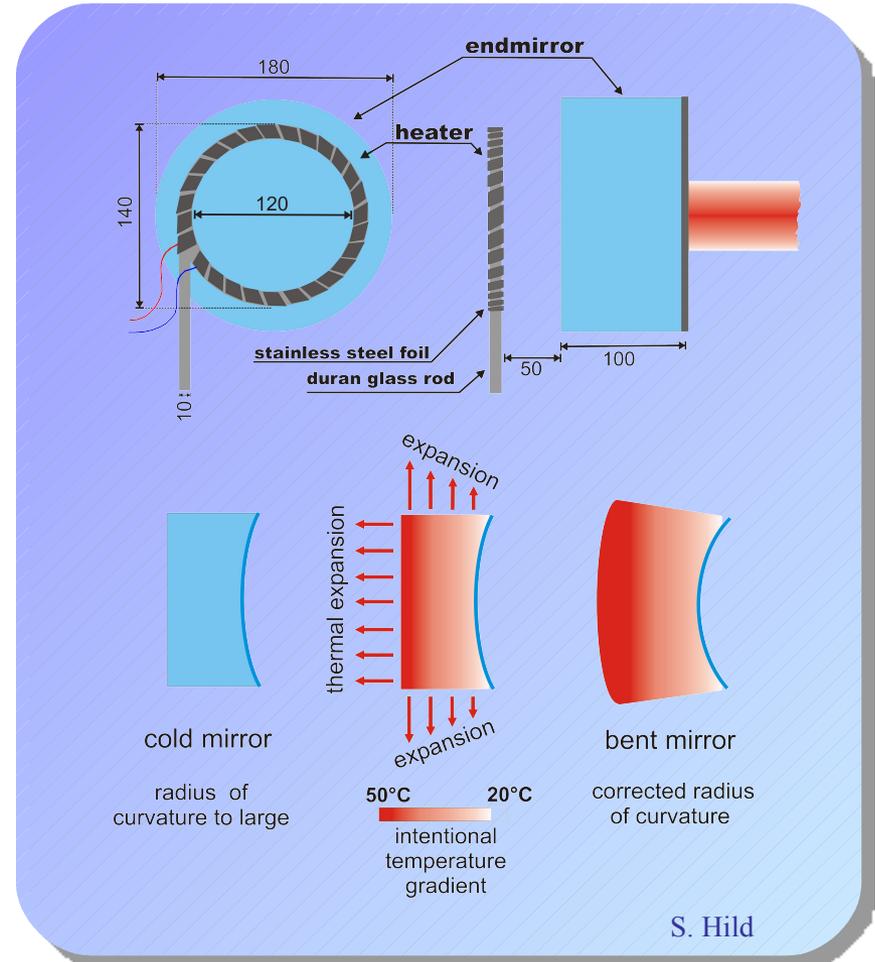
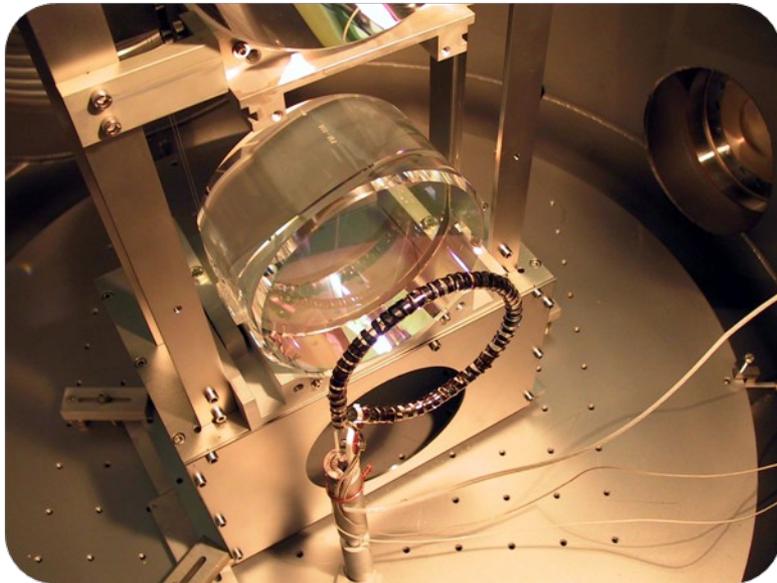
Optimised beams in AdLigo



[Bondarescu et al PRD 2008]

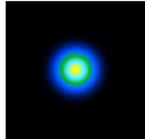


Use thermal compensation system to the change RoC





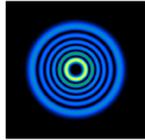
Thermal noise of Flat beams



LG₀₀ mode:

Bondu et al. Physics Letters A 246 (1998) 227

$$S_x(f) = \frac{4 k_B T}{\pi f} \frac{1}{Q} \frac{1 - \sigma^2}{2 \sqrt{\pi} Y w}$$



LG_{nm} modes:

Bondu et al. Physics Letters A 246 (1998) 227

$$S_x(f) = \frac{4 k_B T}{\pi f} \frac{1}{Q} \frac{1 - \sigma^2}{2 \sqrt{\pi} Y w} \alpha_n^m$$



Flat beams:

J.-Y. Vinet CQG 22 (2005) 1395

$$S_x(f) = \frac{4 k_B T}{\pi f} \frac{1}{Q} \frac{8 (1 - \sigma^2)}{3 \pi^2 Y b}$$

Reduction factors given in this talk are collected from various papers and refer to different examples (mirror size, clipping loss, coating parameters,...).

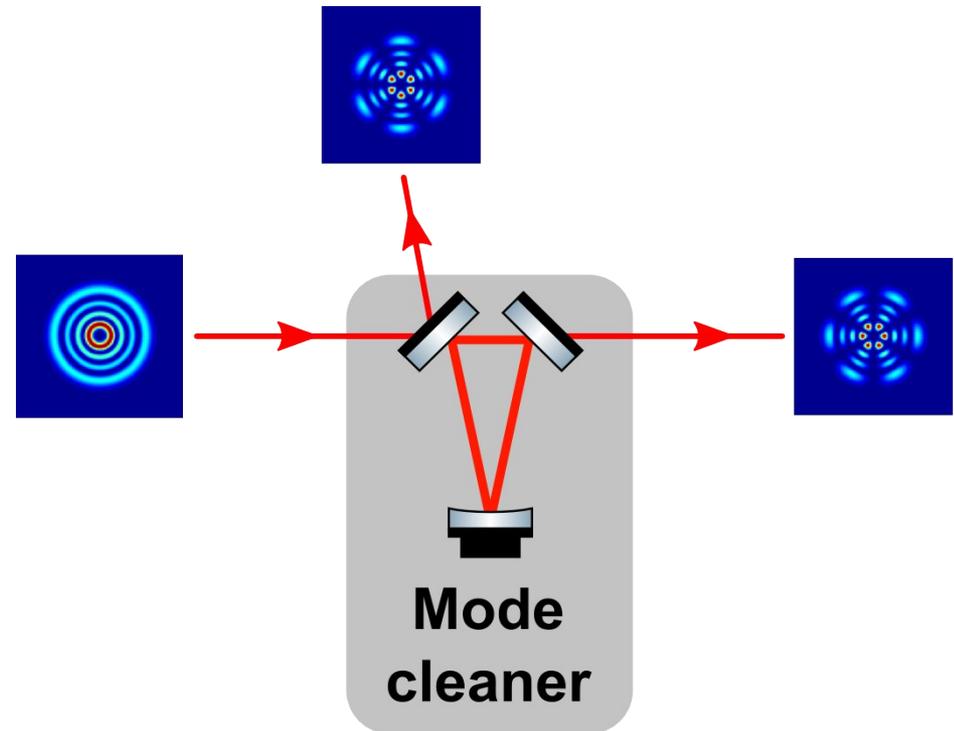
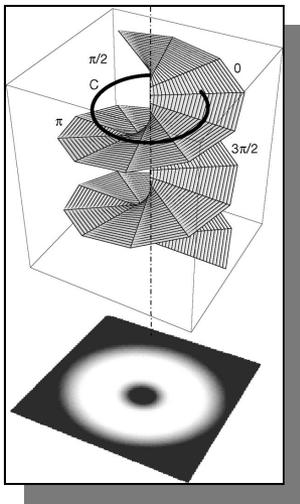
Equations to re-compute these factors properly can be found in (again):

'On thermal issues in advanced Gravitational Wave Interferometric detectors'
J. Y. Vinet, *Living Reviews in Relativity*, to be published

Helical LG modes versus triangular cavities

Helical LG modes

- Continuous ring structure
- Helical phase distribution

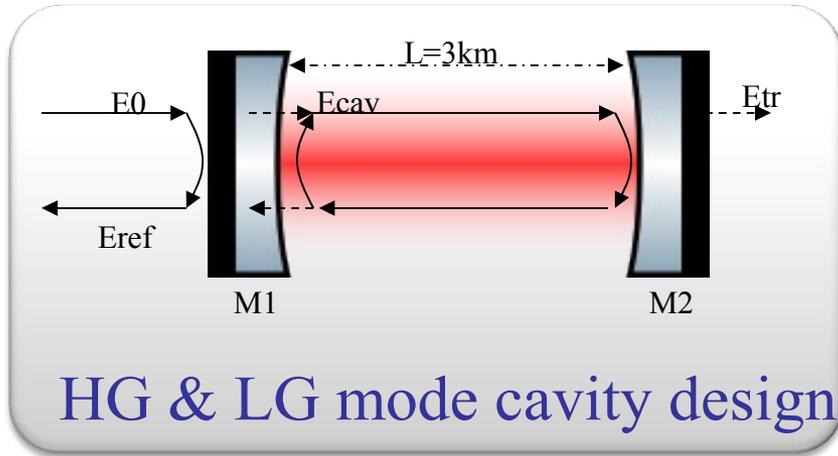


Two possible solutions for this problem:

- Do **not** use triangular cavities (e.g. use bow-tie configurations)
- Use sinusoidal LG modes (with slightly worse thermal noise reduction factors)

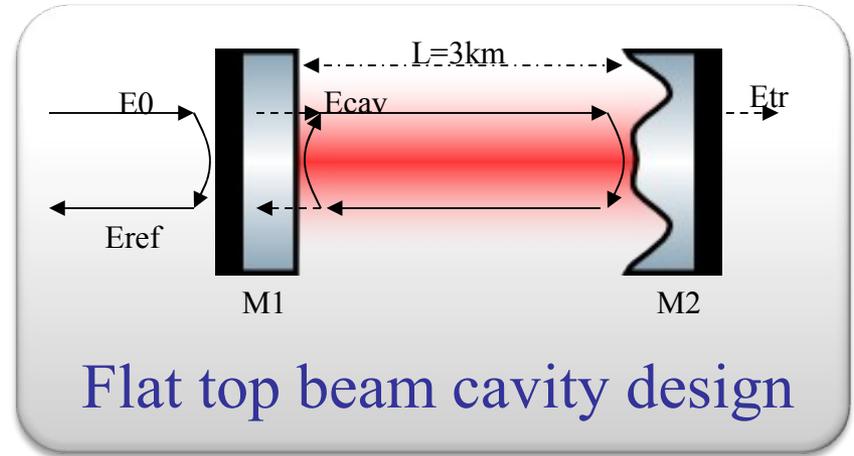


Why LG modes rather than flat top beams?



HG & LG mode cavity design

- Spherical phase fronts
- Compatible with current interferometers



Flat top beam cavity design

- Beam shape and phase fronts change on propagation
- Mirror surfaces are more complex

By M. Laval and J.-Y. Vinet

LG: LG modes are compatible with all current optics

LG? cavities resonant to higher order modes are resonant for several modes (of the same order)



Upgrade Advanced Virgo (or other future detectors) to use an LG33 mode

What we need to change:

- Add LG00 to LG33 converter on the laser table
- Change 3-mirror IMC to 4-mirror IMC
- Exchange core optics with mirrors of same size but different ROC
- Retune or replace mode matching optics

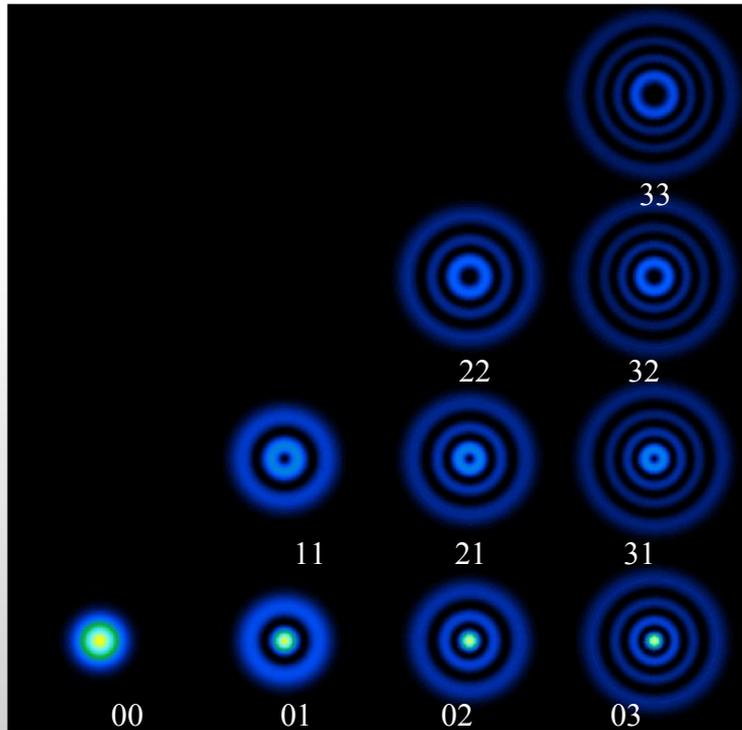
What we **don't** need to change:

- Input/output optics (EOMs, isolators, ...)
- Interferometer control systems (ISC/ASC)
- Vacuum system, suspension system, photodiodes, cameras, baffles, ...



Laguerre-Gauss modes

Helical Laguerre-Gauss modes



Sinusoidal Laguerre-Gauss modes

