Parametric instability of Fabry-Perot cavities in Advanced LIGO, LCGT, and ET

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0. Abstract

I would like to show not details but outline

to evaluate parametric instability in ET interferometer.

Cavities in baselines without power recycling,

signal recycling, resonant sideband extraction

Advanced LIGO (U.S.A.) : Serious problem

LCGT (Japan) : Not serious problem

Einstein Telescope (Europe) : ?

Contents

1. Introduction

Advanced LIGO (U.S.A.), LCGT (Japan)

Second generation interferometric gravitational wave detector

Einstein Telescope (Europe)

Third generation interferometric gravitational wave detector

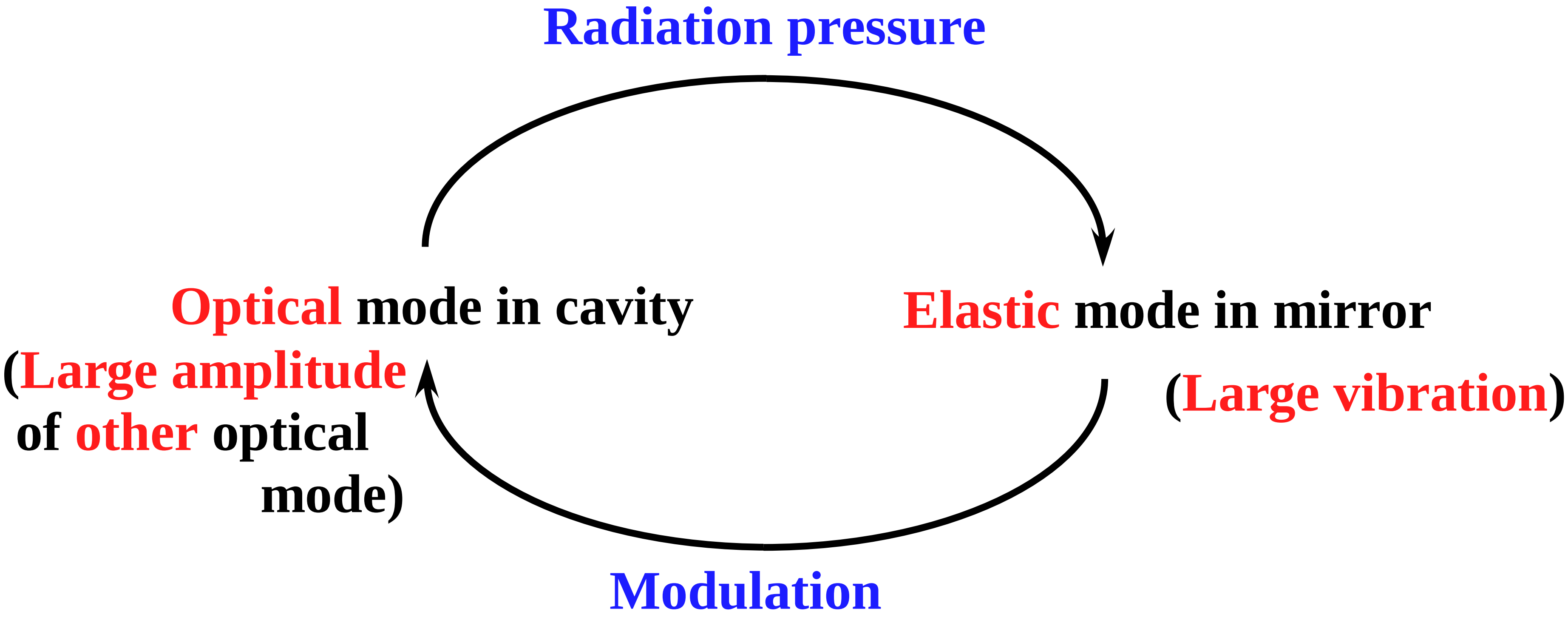
Long Fabry-Perot cavity : > 3 km

→ Interval of optical mode in cavity : < 10 kHz

Interval of elastic mode in mirror : ~ 10 kHz

Parametric instability

Phys. Lett. A 287 (2001) 331.



Radiation pressure

Optical mode in cavity

(Large amplitude of other optical mode)

Elastic mode in mirror

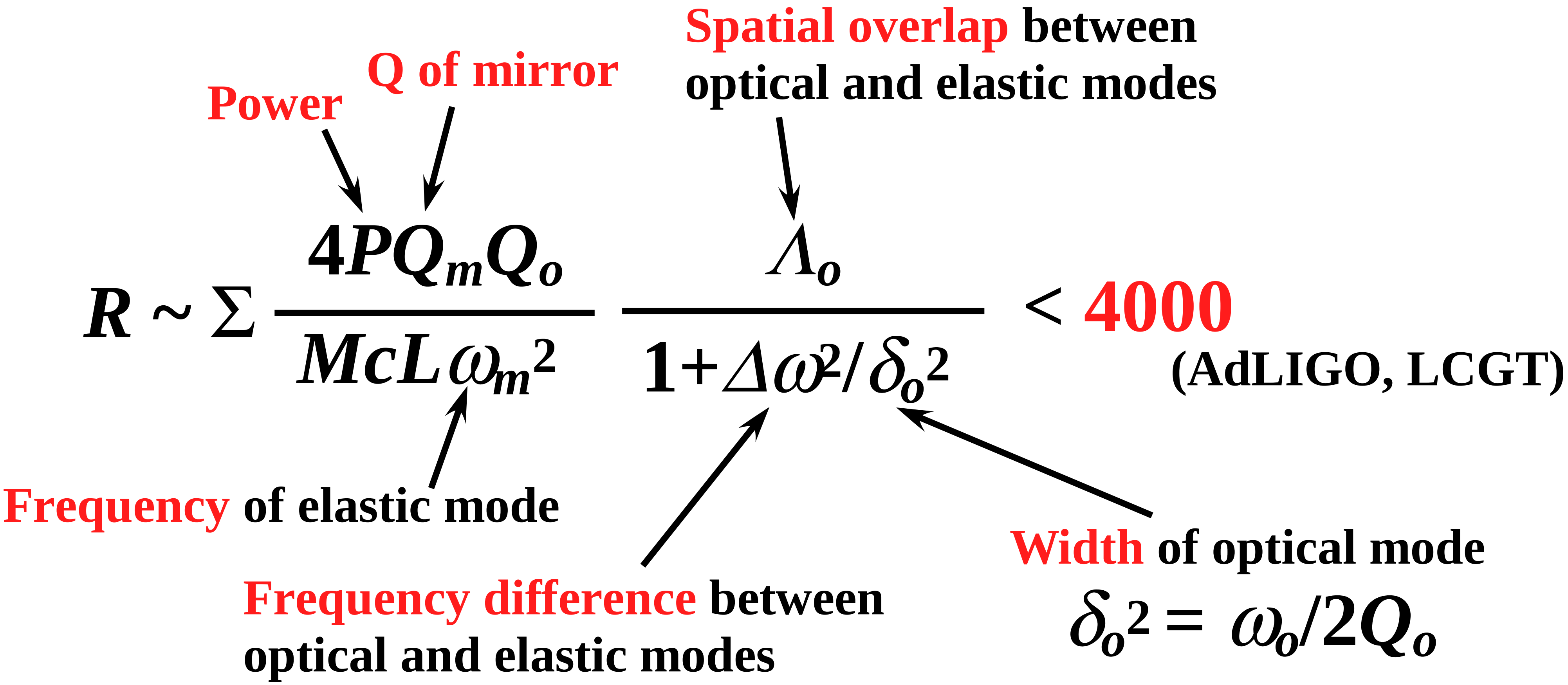
(Large vibration)

Modulation

Formula of parametric instability

Phys. Lett. A 287 (2001) 331.

$R > 1$: instable elastic mode



Power

Q of mirror

Spatial overlap between optical and elastic modes

$$R \sim \sum \frac{4PQ_mQ_o}{McL\omega_m^2}\,\frac{\Lambda_o}{1+\Delta\omega^2/\delta_o^2} < 4000$$

(AdLIGO, LCGT)

Frequency of elastic mode

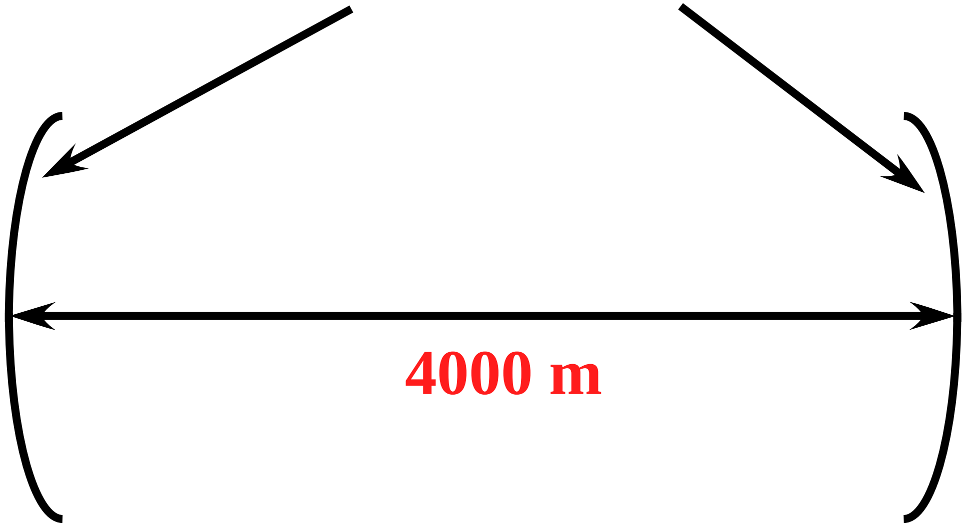
Frequency difference between optical and elastic modes

Width of optical mode

$$\delta_o^2 = \omega_o/2Q_o$$

2. Advanced LIGO

2-1. Specification



Curvature radius : 2076 m

Fused silica mirrors

4000 m

Gaussian beam

Power in cavities : 0.83 MW

Wavelength : 1064 nm

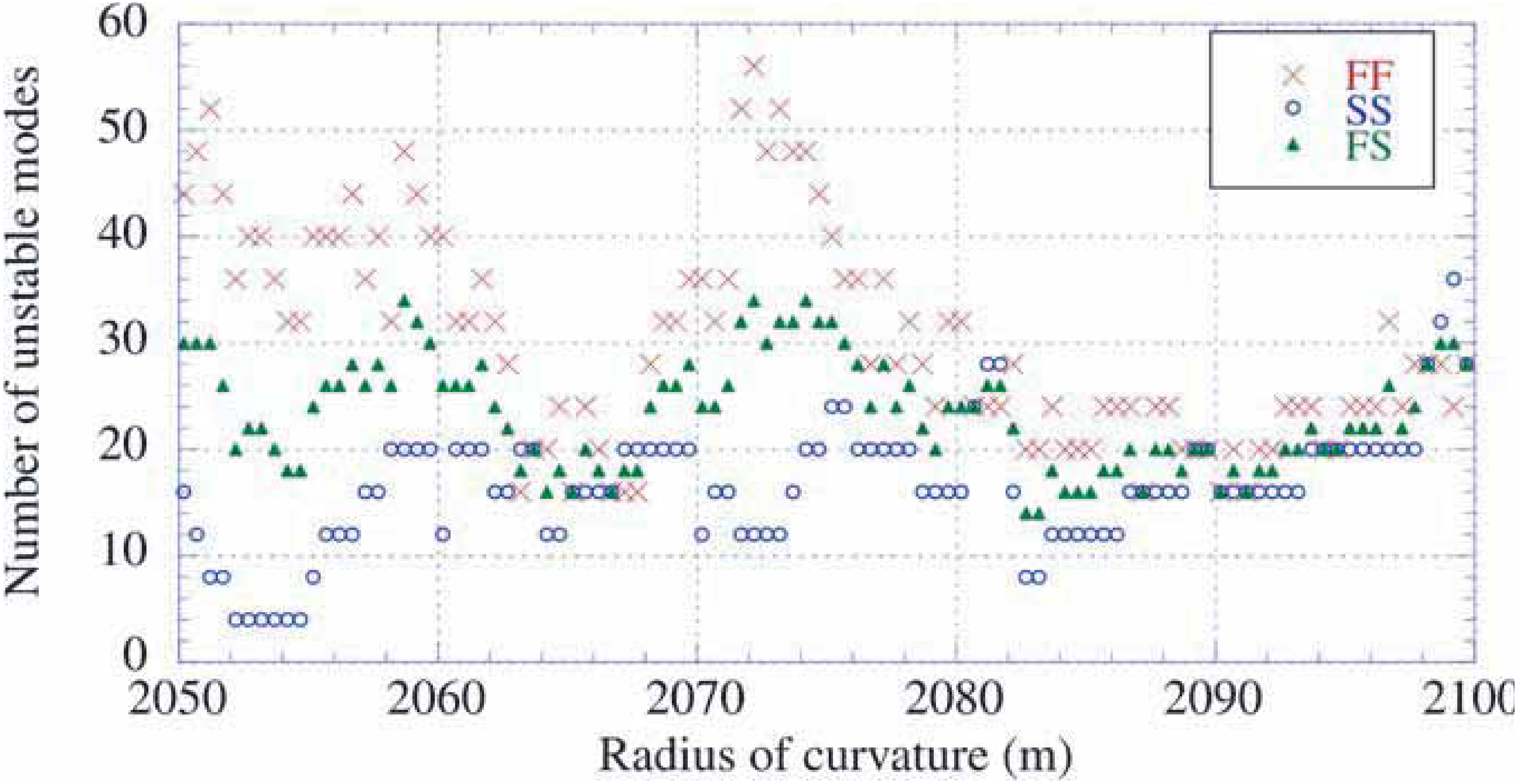
Study in University of Western Australia

Phys. Lett. A 354 (2006) 360.
Phys. Lett. A 355 (2006) 419.

2-2. Number of unstable modes

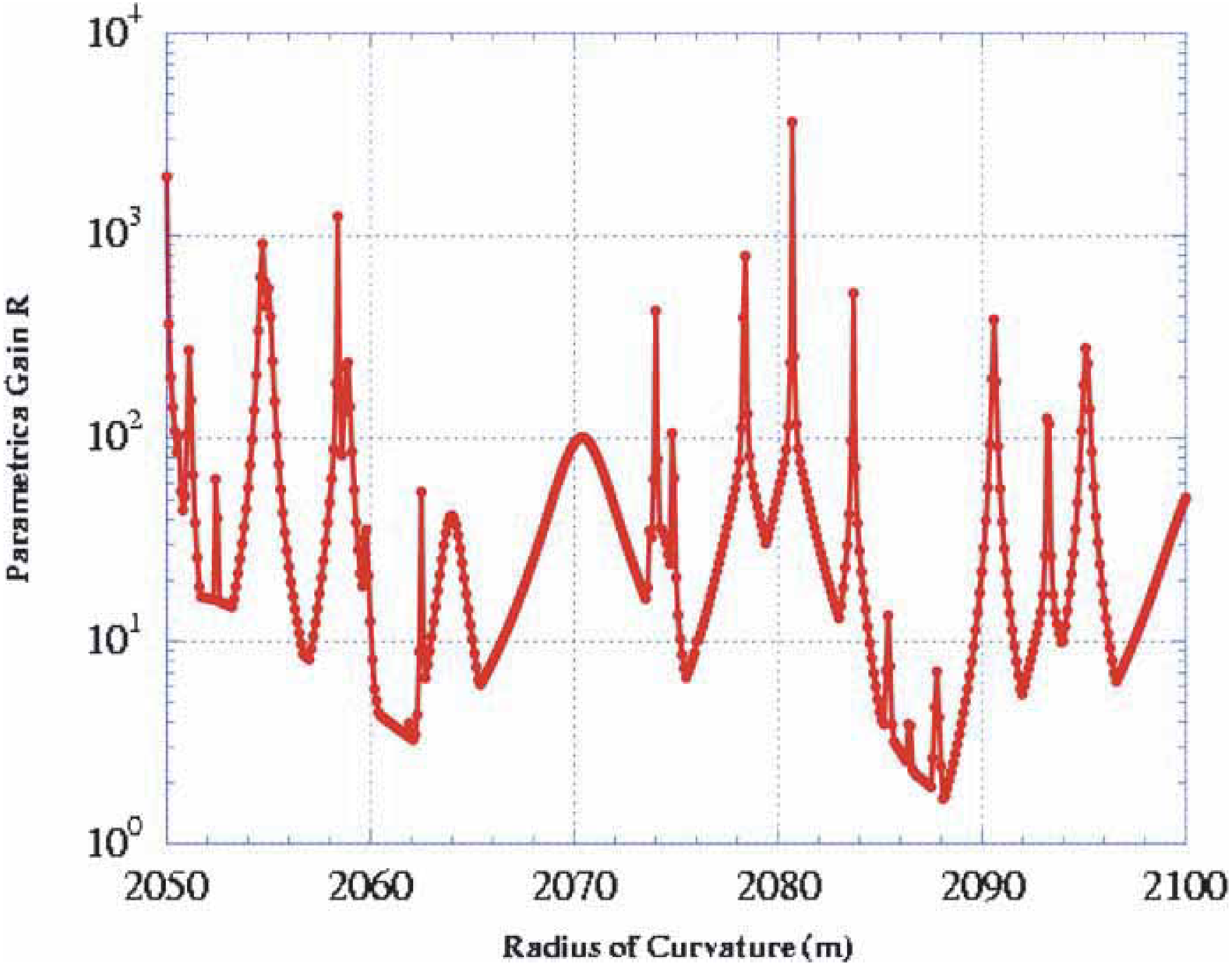
Phys. Lett. A 355 (2006) 419.

FF : Fused silica - Fused silica



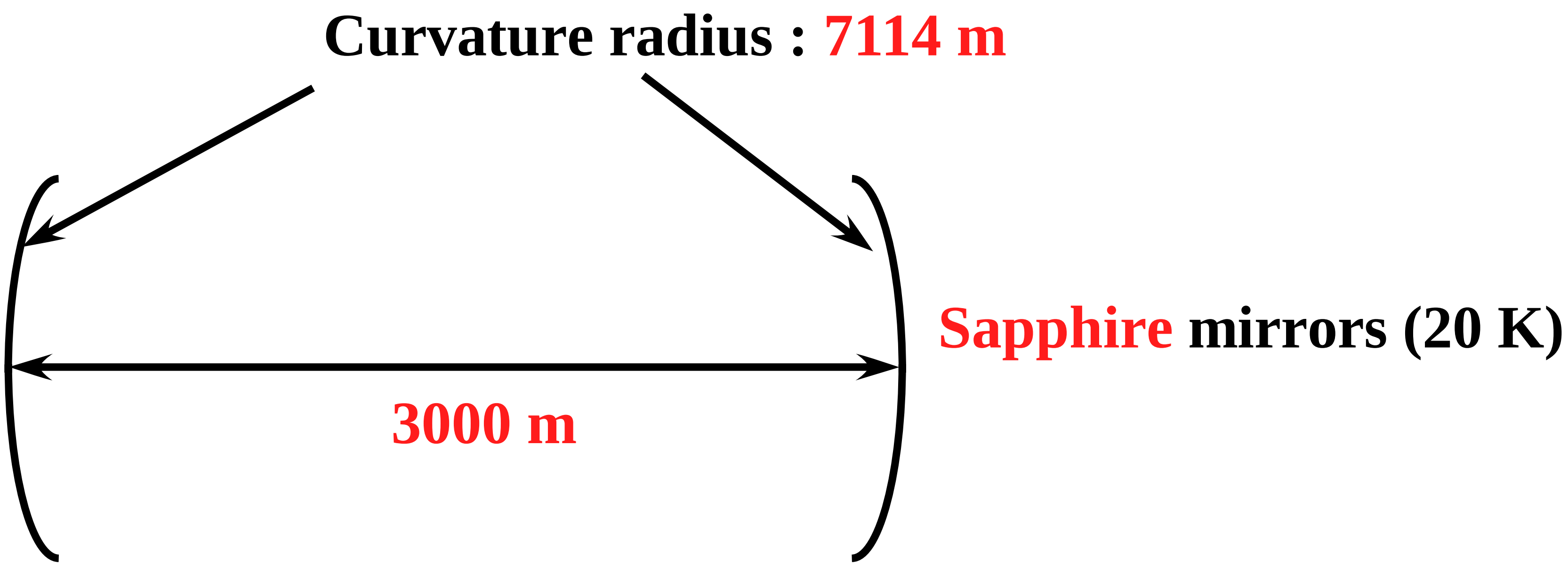
2-3. Maximum of R

Phys. Lett. A 354 (2006) 360.



3. LCGT

3-1. Specification



Curvature radius : 7114 m

Sapphire mirrors (20 K)

3000 m

Power in cavities : 0.41 MW

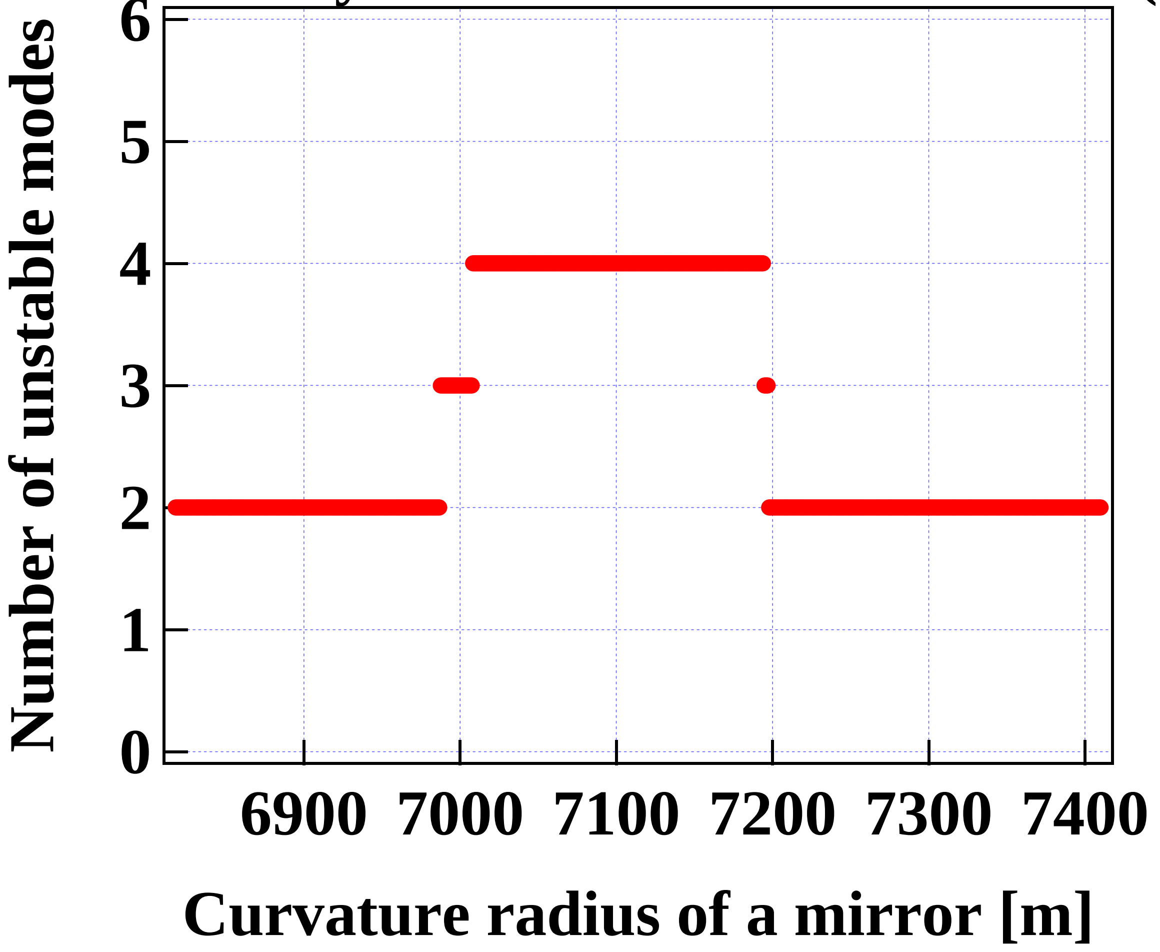
Wavelength : 1064 nm

K. Yamamoto et al., Amaldi7 proceedings
Journal of Physics : Conference Series 122 (2009) 012015

3-2. Number of unstable modes

K. Yamamoto et al., Amaldi7 proceedings

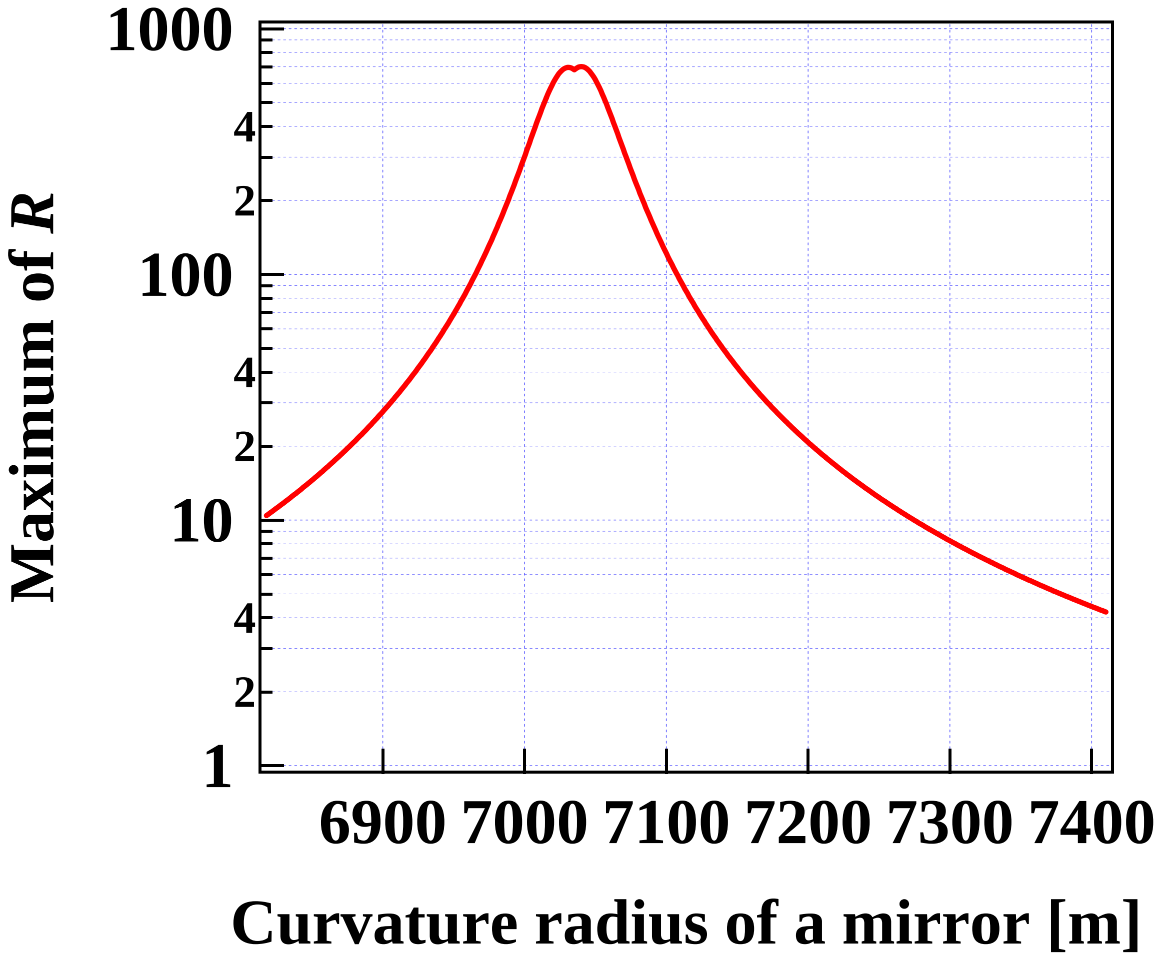
Journal of Physics : Conference Series 122 (2009) 012015



3-3. Maximum of R

K. Yamamoto et al., Amaldi7 proceedings

Journal of Physics : Conference Series 122 (2009) 012015



4. Difference between AdLIGO and LCGT

K. Yamamoto et al., Amaldi7 proceedings
Journal of Physics : Conference Series 122 (2009) 012015

4-1. Number of unstable modes

Advanced LIGO : 20 ~ 60

LCGT : 2 ~ 4

(i) Elastic mode density : ~ (Sound velocity)$^{-3}$

Advanced LIGO (Fused silica) : 6 km/s

LCGT (Sapphire) : 10 km/s

5 times smaller

(ii) Optical mode density

Advanced LIGO : 7 modes / FSR

LCGT : 3 modes / FSR

2 times smaller

Larger beam radius for thermal noise reduction (Advanced LIGO)

(iii) Summary

Product of elastic and optical mode densities : 10 times smaller

Number of unstable mode
Advanced LIGO : 20 ~ 60
LCGT : 2 ~ 4

4-2. Mirror curvature

Advanced LIGO : *R* strongly depends on mirror curvature.

LCGT : *R* weakly depends on mirror curvature.

***R* is function of optical mode frequency.**

Mirror curvature dependence of interval of transverse optical mode

Advanced LIGO : 15 Hz/m
LCGT : 0.58 Hz/m

30 times smaller

Larger beam radius for thermal noise reduction

(Advanced LIGO)

5. Einstein Telescope

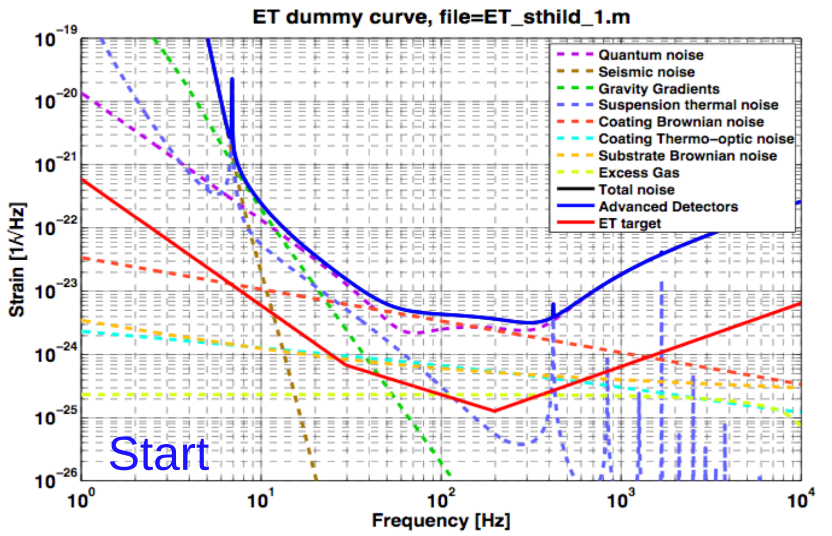
How much are parameters of Einstein Telescope ?

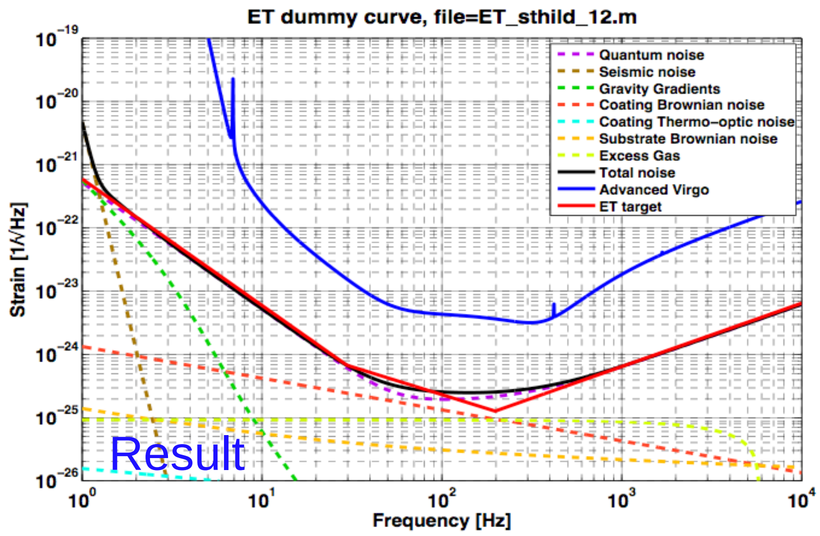
The ET sensitivity curve with ‘conventional‘ techniques

Stefan Hild and Andreas Freise

University of Birmingham

1st ET General meeting, Pisa, November 2008





	advanced detector	potential ET design
Arm length	3 km	10 km
SR-phase	detuned (0.15)	tuned (0.0)
SR transmittance	11 %	10 %
Input power (after IMC)	125 W	500 W
Arm power	0.75 MW	3 MW
Quantum noise suppression	none	10 dB
Beam radius	6 cm	12 cm
Temperature	290 K	20 K
Suspension	Superattenuator	5 stages of each 10 m length
Seismic	$1 \cdot 10^{-7}\,\mathrm{m}/f^2$ for $f > 1\,\mathrm{Hz}$ (Cascina)	$5 \cdot 10^{-9}\,\mathrm{m}/f^2$ for $f > 1\,\mathrm{Hz}$ (Kamioka)
Gravity gradient reduction	none	factor 50 required (cave shaping)
Mirror masses	42 kg	120 kg
BNS range	150 Mpc	2650 Mpc
BBH range	800 Mpc	17700 Mpc

5-1. Upper limit of R

$$R \sim \sum \boxed{\frac{4PQ_mQ_o}{McL\omega_m^2}} \frac{\Lambda_o}{1+\Delta\omega^2/\delta_o^2}$$

Comparison with LCGT

Power (in a cavity, *P*) : 8 times larger (0.41MW ⟶ 3MW)

Cavity length (*L*) : 3 times longer (3km ⟶ 10km)

Beam radius : 4 times larger (3cm ⟶ 12cm)

Mirror mass (*M*) : 4^3 = 64 times larger
Resonant frequency of elastic modes (ω_m) : 4 times smaller

Upper limit of *R* is 0.7 times larger.

(If mirror is silicon, not sapphire, upper limit of *R* is 2 times larger)

(It is assumed that band width of cavity is same)

5-2. Number of unstable modes

(i) Elastic mode density : ~ (Mirror size/Sound velocity)^3

LCGT (Sapphire) : 10 km/s

Einstein Telescope (Silicon ?) : 6 km/s

Mirror radius : 4 times larger than that of LCGT

300 times larger (Silicon)

60 times larger (Sapphire) than that of LCGT

(ii) Optical mode density

Cavity length (L) : 3 times (3km → 10km)

Optical mode density : 3 times larger

Beam radius : $3^{1/2}$ times larger → 5 cm ! (LCGT : 3 cm)

We must make beam radius larger (12 cm) to suppress thermal noise.

Same trick as Advanced LIGO to make beam larger (Mirror curvature is a half of cavity length)

LCGT : 3 modes / FSR

Einstein Telescope : 7 modes / FSR

Optical mode density : 2 times larger

Total : Optical mode density : 6 times larger

(iii) Summary

Product of elastic and optical mode densities (number of unstable modes)

2000 times larger (Silicon)

400 times larger (Sapphire)

than that of **LCGT**.

Product of elastic and optical mode densities (number of unstable modes)

200 times larger (Silicon)

40 times larger (Sapphire)

than that of **Advanced LIGO**.

5-3. Mirror curvature

Einstein Telescope and Advanced LIGO

Mirror curvature is about a half of cavity length.

Cavity length of Einstein Telescope is about 3 times longer.

Mirror curvature dependence of interval of transverse optical mode

Einstein Telescope : 15*3 = 45 Hz/m

Advanced LIGO : 15 Hz/m

LCGT : 0.58 Hz/m

Einstein Telescope : *R* strongly depends on mirror curvature.

Larger beam radius and longer baseline for thermal noise reduction

6. Instability suppression

Investigation in LIGO (UWA)

Phys. Lett. A 355 (2006) 419.

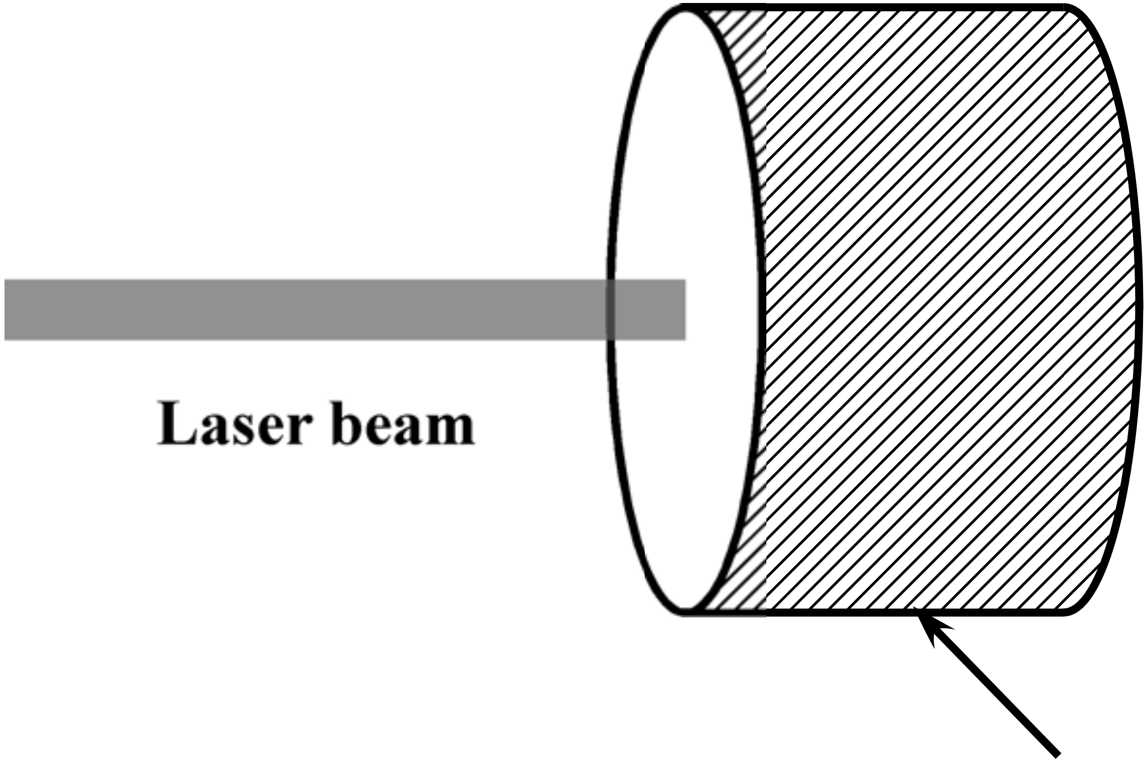
(1) Thermal tuning

(2) Q reduction

(3) Feedback control

Q reduction (elastic mode)

R **is proportional to *Q*.**



Coating (dissipation)
It reduces Q values effectively,
but does not increase thermal noise effectively.

K. Yamamoto et al., Phys. Lett. A 305 (2002) 18.
S. Gras et al., Phys. Lett. A 333 (2004) 1.
S. Gras et al., J. Phys. : Conf. Ser. 32 (2006) 251.

For LCGT

$Q = 10^8 \longrightarrow Q = 10^6$

(Almost all modes become stable)

0.25 mm thickness Ta_2O_5 coating on cylindrical surface

K. Yamamoto et al.,Phys. Rev. D 74 (2006) 022002.

Thermal noise of cylindrical surface coating

is comparable with that of reflective coating

and a few times smaller than that of LCGT goal sensitivity.

K. Yamamoto et al., Amaldi7 proceedings
Journal of Physics : Conference Series 122 (2009) 012015
K. Yamamoto et al., Phys. Lett. A 305 (2002) 18.

For Einstein Telescope

Upper limit of *R* is comparable with that of LCGT.
R is proportional to *Q*.

$$Q = 10^8 \longrightarrow Q = 10^6$$

1 mm thickness Ta_2O_5 coating on cylindrical surface

(4 times larger mirror)

Thermal noise of cylindrical surface coating

is comparable with that of ET goal sensitivity.

7. Summary

(1) Upper limit of *R* (strength of instability)

Upper limit of *R* of Einstein Telescope is almost same as that of Advanced LIGO and LCGT.

(2) Number of unstable modes and mirror curvature dependence

LCGT : Less unstable modes and weak curvature dependence

Cooled mirror for thermal noise reduction (sapphire mirror and normal beam radius)

ET : Many unstable modes and strong curvature dependence

Cooled mirror but larger beam radius and longer arm for thermal noise reduction

(3) Instability suppression

Q reduction (elastic mode) is effective, but no safety margin.

Do not be too pessimistic,

but pay attention.