

Suspension noise modeling

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Outline of the topics in the document

- **The last stage suspension of the ET interferometer;**
- **The model for the thermal noise;**
- **Parameters for the LSS in ET;**
- **Numerical evaluations and comparison with the ET goal curve.**

The mirror last stage suspension

✧ *The role of the Last Stage Suspension is to compensate the residual seismic noise and to steer the optical components maintaining the relative position of the interferometer mirrors.*

Components and roles:

- **Marionette:** Mirror control with actuators (coil-magnets, electrostatic) between the upper suspension stage and marionette;
- **Reaction Mass (RM):** Mirror steering with (coil-magnets, electrostatic) actuator between RM and mirror; Mirror protection;
- **Mirror:** monolithic silicon suspension.

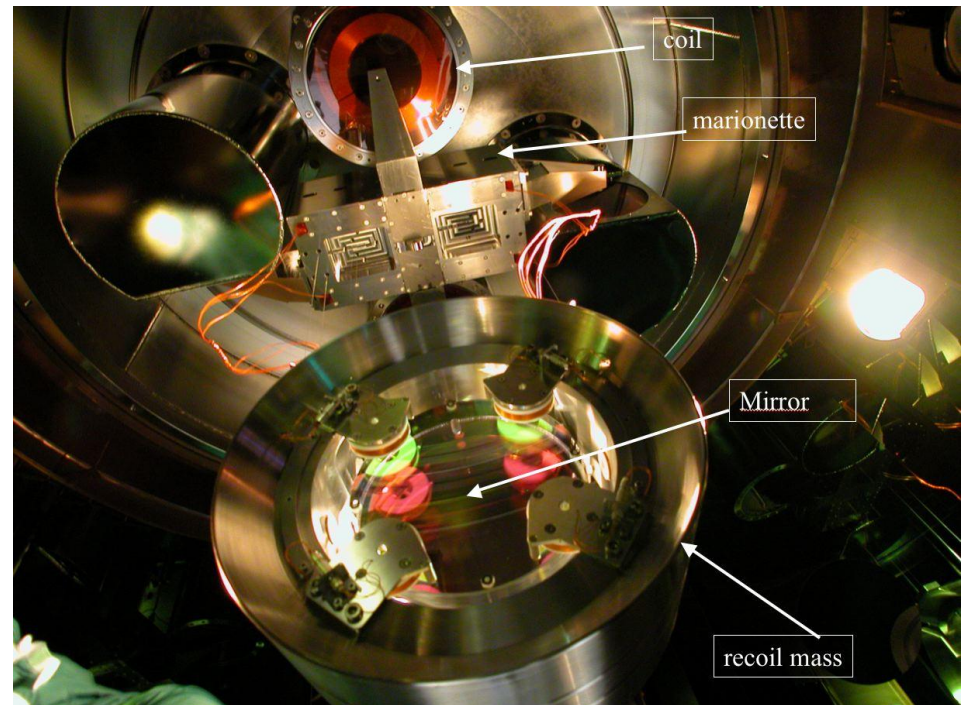
Requirements:

✧ **Materials:**

- ✧ **UHV compatible;**
- ✧ **Amagnetic;**
- ✧ **No electrostatic charges;**
- ✧ **Internal Frequencies above the antenna bandwidth;**
- ✧ **Low frequencies of the system below control bandwidth;**

✧ **Compatibility with SuperAttenuator and lower part of the tower:**

- ✧ **Weights**
- ✧ **Shape**



The ET mirror last stage suspension (Cryogenic)

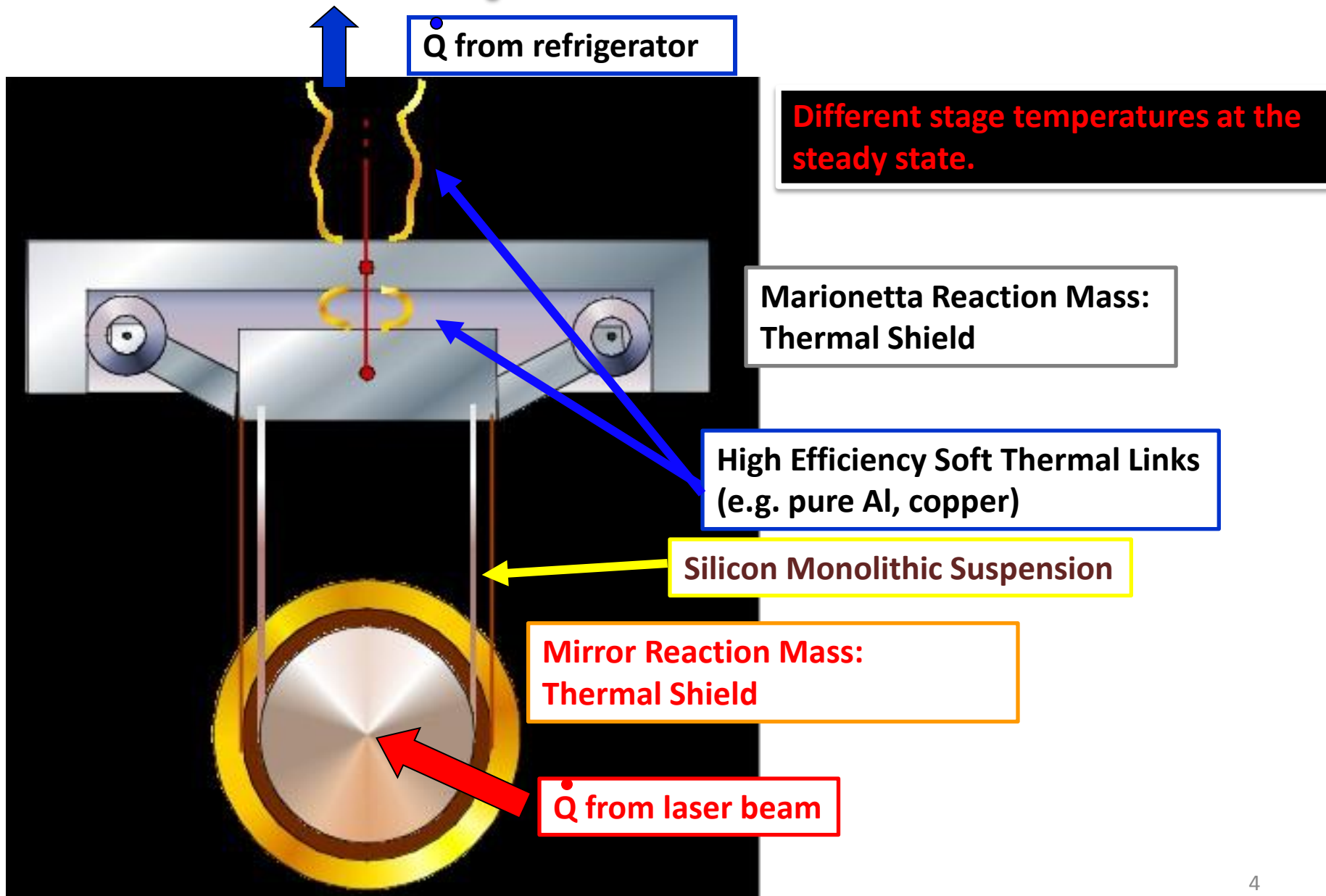
↗ Mirror and its suspension wires:

- wires and mirror materials compatible with good mechanical and thermal properties;
 - High thermal conductivities materials;
 - Low mechanical and optical losses;

a promising material both as mirror substrate and wire is silicon having

- high thermal conductivity
- very low thermal expansion (zero below 17K)

Conceptual scheme



Mechanical Issues

Big Masses:

- ☑ reduces the recoils (good for suspension thermal noise)
- ☑ increases the violin modes (good for control)
- BUT
- ✗ reduces the vertical modes (not good for control)
- ✗ look at the overall weight!!

Wires Length Increment

- ☑ reduces the pendulum frequencies (good for suspension thermal noise)
- BUT
- ✗ reduces the violin modes (not good for control)
- ✗ reduces the vertical modes (not good for control)

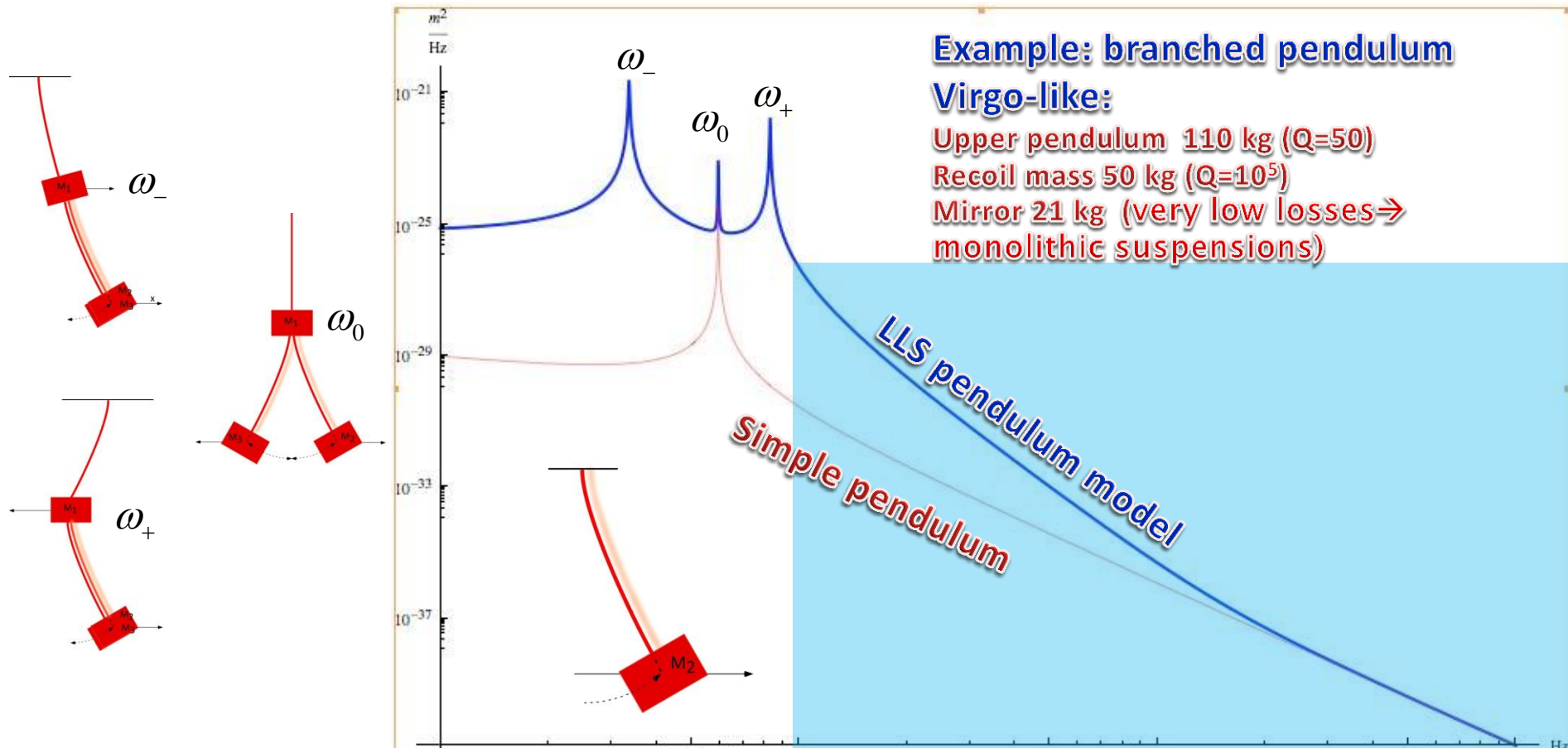
Wires Diameter Increment:

- ☑ increment of the wire sections (good for cooling)
- BUT
- ✗ reduces the violin mode frequencies (not good for control)
- ✗ reduces the dilution factor (not good for suspension thermal noise)

- The suspension thermal noise affects the sensitivity in the frequency range below 10 Hz;
- Importance of the **different temperatures of the pendulum stages** in the computation of the thermal noise
- The marionette stage thermal effect is very important
- The thermal noise of violin modes is negligible (but look at the frequencies)

THE SUSPENSION THERMAL NOISE

- In presence of low dissipative mirror suspensions, a new thermal noise estimation must be done by including the viscous and internal dissipations of the marionette and recoil mass pendulum.
- The marionette's mechanical pendulum losses give a non negligible effect via its recoil, in the off-resonance high-freq. range [*].
- For cryogenic LSS the different temperatures of the pendulum stages is important in the computation of the thermal noise [**]

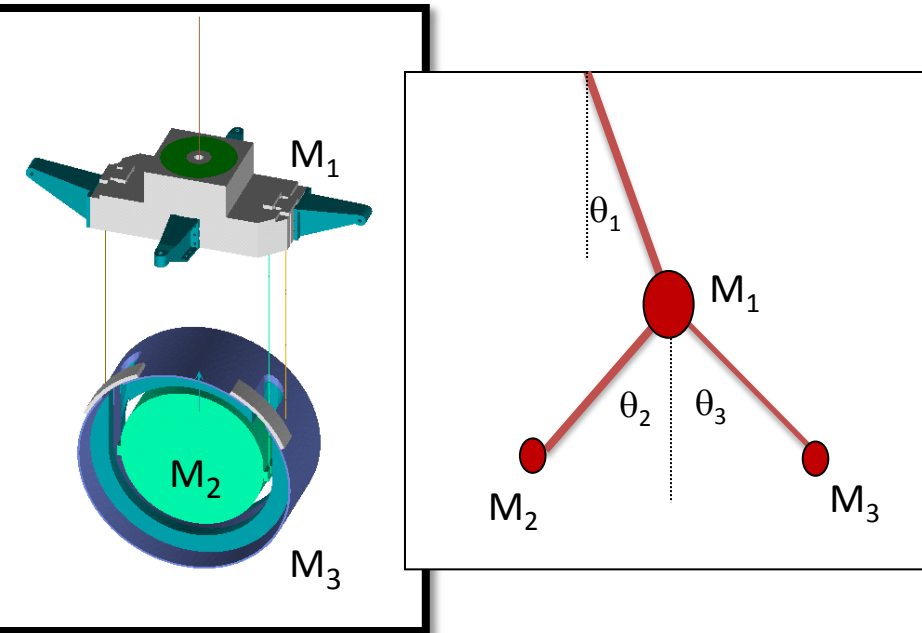


[*] VIR-015C-09, F. Piergiovanni, M. Punturo and P. Puppo, *The thermal noise of the Virgo+ and Virgo Advanced Last Stage Suspension (The PPP effect)*.

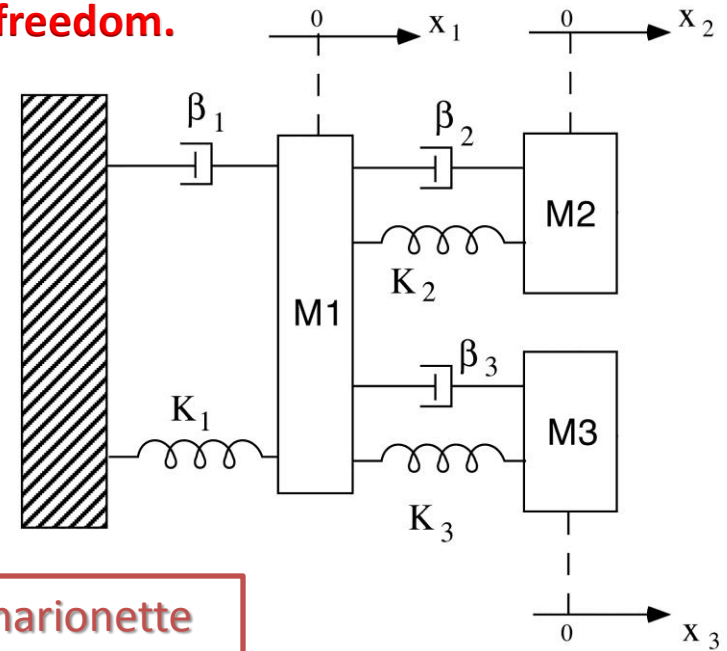
[**] P. Puppo, Amaldi 8, New York, June 2009 and MG12, Paris, July 2009, proceedings

Suspension TN Modeling

A Virgo-like last stage suspension is a cascade of three pendula. To the first pendulum (the marionette) the mirror and the recoil mass are hung as branches.



Equivalent to a branched combination of three harmonic oscillators [*]. This is true for horizontal and vertical degrees of freedom.



M_1 marionette
 M_2 mirror
 M_3 recoil mass

modal expansion model for cryo LSS [**]

[*] Bernardini A., Majorana E., Puppo P., Rapagnani P., Ricci F., Testi G. "Suspension last stages for the mirrors of the Virgo interferometric gravitational wave antenna." *Rev. Sci. Instr.* 70, no. 8 (1999): 3463.

[**] P. Puppo, Amaldi 8, New York, June 2009 and MG12, Paris, July 2009, proceedings

Parameters for the mirror

- Several different mechanisms contribute to the thermal noise of the mirror:
 - Brownian (BR)(substrate, coating) (T, ϕ)
 - Thermoelastic (TE) (substrate) (Thermal props)
 - Thermorefractive (TR) dn/dT (substrate)
 - Thermo-optic new correlated model (TR-TE) (coating)
- The coating brownian noise dominates over the other thermal sources

Issues *

Mirror Size:	Thickness 30 cm, diam: 45 cm
Beam Size:	w=9.00 cm
Substrate:	Silicon ($\phi_s = 10^{-9}$, good temp. props)
Mass:	110 kg
Coating:	Ti:Ta2O5 / SiO2
Working Temperature:	T=10K (coating losses reduced)

* see Erice's meeting talk of J. Franc and R. Nawrodt talk or ET-021-09.

Thermal Issues - (High power stored, curve B)

With liquid Helium

$$P_{HeII}(T_{mario}) = P_{abs} = 3 \text{ W} \Rightarrow T_{mario} = 2 \text{ K}$$

$$P_{abs} = 4 \frac{\sum_w}{L} K_{mean} (T_{mario} - T_{mirror})$$

$$K_{mean} = \frac{1}{\Delta T} \int_{T_{mario}}^{T_{mirror}} K_{si}(T) dT \cong 938 \frac{\text{W}}{\text{m} \cdot \text{K}}$$

$$\Rightarrow T_{mirror} = 10 \text{ K}$$

$$d_w = 8 \text{ mm}$$

With PT cooler

$$P_{cooler}(T_{mario}) = P_{abs} = 3 \text{ W} \Rightarrow T_{mario} = 5 \text{ K}$$

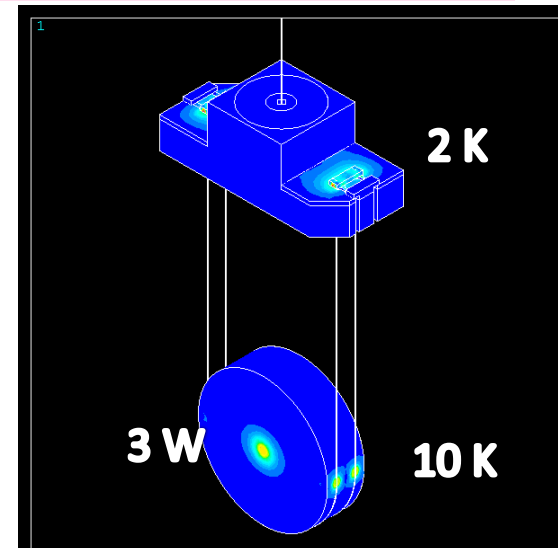
$$P_{abs} = 4 \frac{\sum_w}{L} K_{mean} (T_{mario} - T_{mirror})$$

$$K_{mean} = \frac{1}{\Delta T} \int_{T_{mario}}^{T_{mirror}} K_{si}(T) dT \cong 1383 \frac{\text{W}}{\text{m} \cdot \text{K}}$$

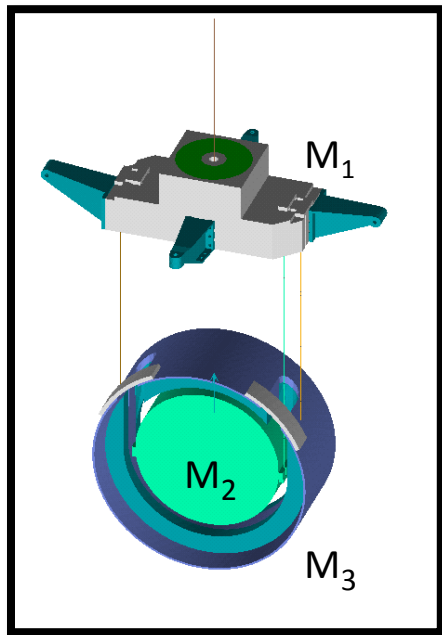
$$\Rightarrow T_{mirror} = 10 \text{ K}$$

$$d_w = 8.3 \text{ mm}$$

- To reach a temperature of 10K with a power of 3W on the mirror we need a silicon suspension wire with a diameter of 8 mm;
- The temperature of the marionette stage can be 5 K or 2K depending on which kind of refrigeration system is used



Parameters for the LSS – High Power Curve B



PAYLOAD

MARIONETTE: (Ti6Al4V WIRE)

d = 3 mm, M_1 : 400 kg,

L=2 m

T=2 K

MIRROR (SILICON WIRE)

dimensions: diam 45cm, thickness 30cm

(limit of present technology)

d = 8 mm, M_2 : 110 kg,

L=2 m

T=10 K

RECOIL MASS (SILICON WIRE)

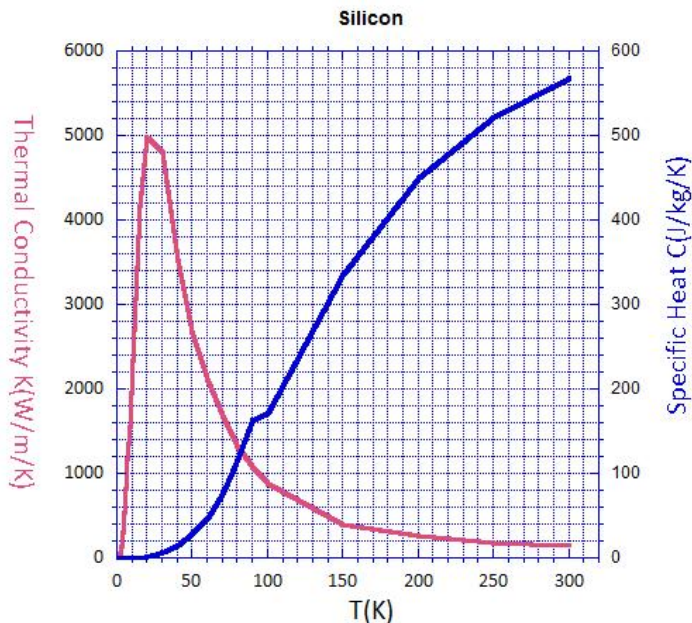
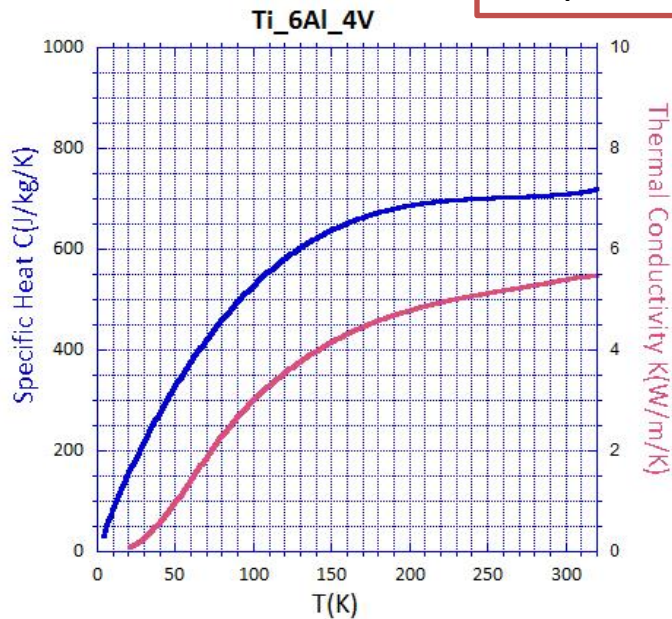
d = 5 mm, M_3 : 110 kg

L=2 m

T=10 K

Modes:	pendulum	0.28 Hz, 0.36 Hz, 0.50Hz
	vertical	0.4 Hz (blades), 23 Hz, 62 Hz
	violins	15.8 Hz, 31.6 Hz, 63.2 Hz, 126.4 Hz, ...

Properties



Wires and substrate

Mechanical @ 2 K

Density $\rho_{\text{Ti6Al4V}} = 4.4532 \cdot 10^3 \text{ kg/m}^3$
 Young Modulus $Y_{\text{Ti6Al4V}} = 127 \text{ GPa}$
 Poisson Ratio $\sigma_{\text{Ti6Al4V}} = 0.403$
 Loss Angle $\Phi_{\text{Ti6Al4V}} = 10^{-4}$
 Young Modulus gradient:
 $\beta_{\text{Ti6Al4V}} = (1/Y_{\text{Ti6Al4V}})(dY_{\text{Ti6Al4V}}/dT) = -4.6 \cdot 10^{-3} (1/K)$

Ti6Al4V

Thermal @ 2 K

Specific Heat $C_{\text{Ti6Al4V}} = 0.07 \text{ J/kg/K}$
 $(C(T) = 8.8 \cdot 10^{-4} T^3 \text{ up to } 30\text{K})$
 Heat Conductivity $K_{\text{Ti6Al4V}} = 0.14 \text{ W/m/K}$
 Thermal Expansion $\alpha_{\text{Ti6Al4V}} = -1.7 \cdot 10^{-3} \text{ m/m/K}$

Mechanical @ 10 K

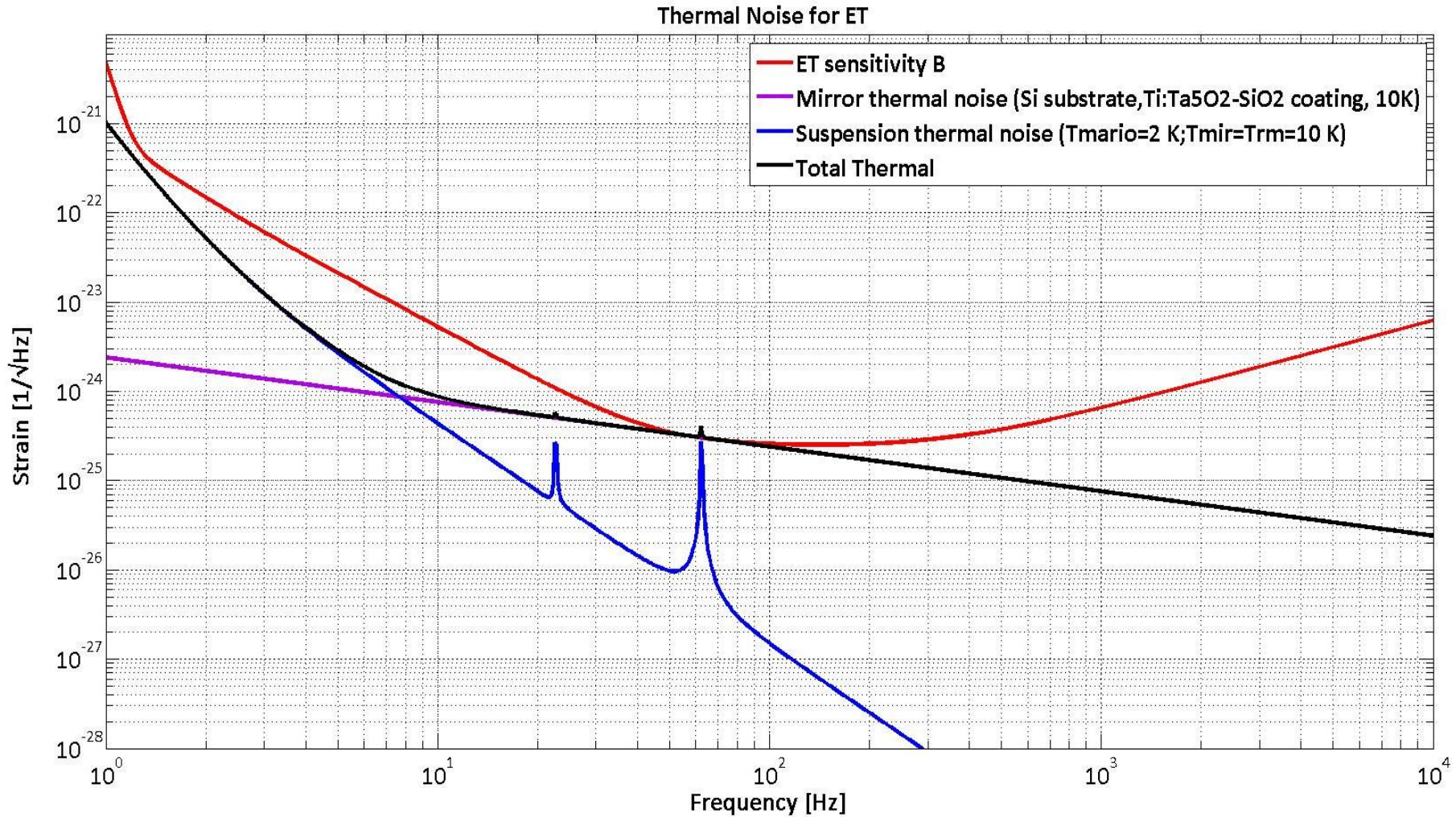
Density $\rho_{\text{si}} = 2.3315 \cdot 10^3 \text{ kg/m}^3$
 Young Modulus(100) $Y_{\text{si}} = 132 \text{ GPa (100)}$
 189 GPa (111)
 Poisson Ratio $\sigma_{\text{si}} = 0.22$
 Loss Angle $\Phi_{\text{si}} = 10^{-9}$
 Young Modulus gradient (100):
 $\beta_{\text{si}} = (1/Y_{\text{si}})(dY_{\text{si}}/dT) = -7.7 \cdot 10^{-5} (1/K)$

Si

Thermal @ 10 K

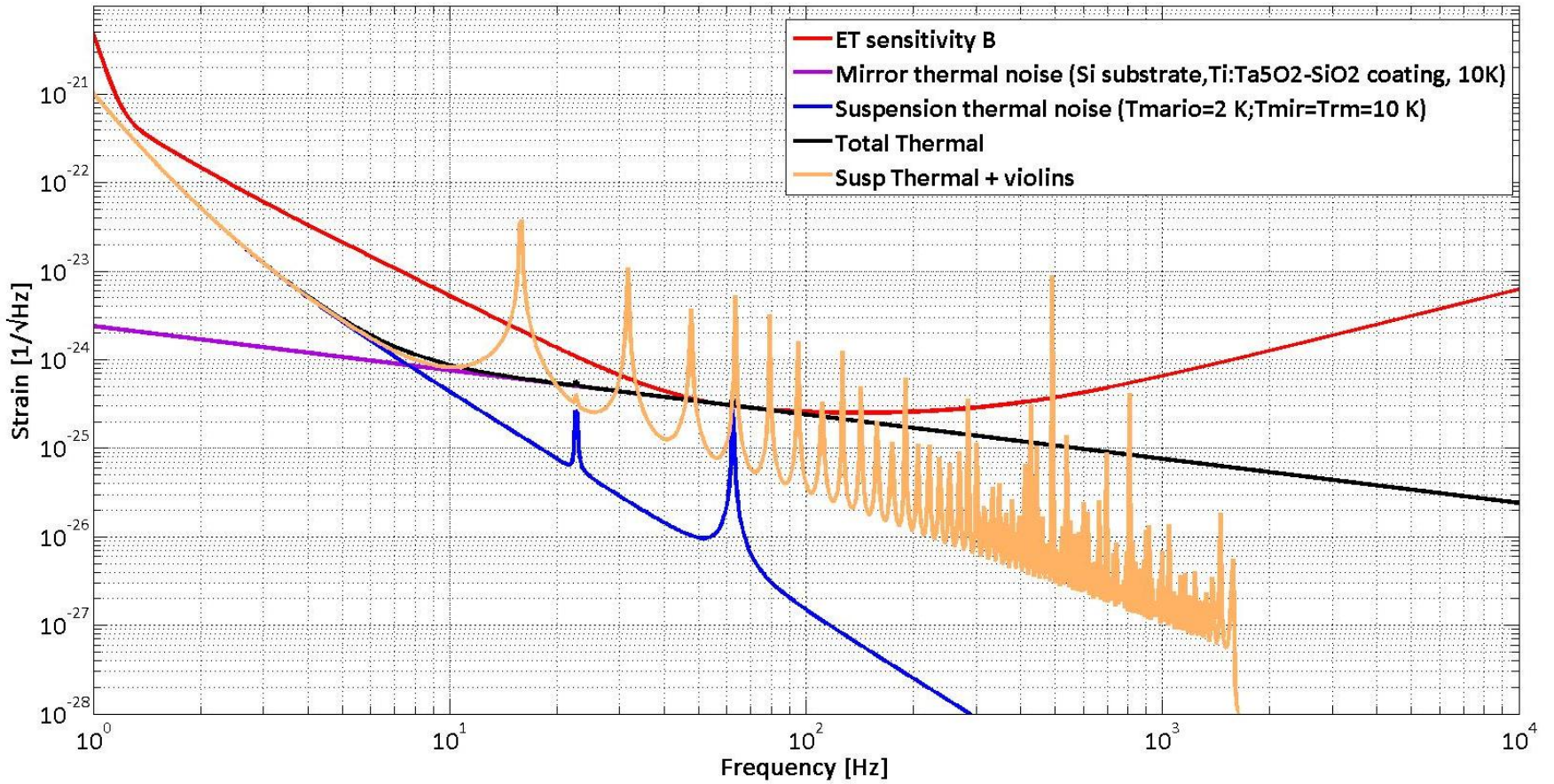
Specific Heat $C_{\text{si}} = 0.276 \text{ J/kg/K}$
 Thermo-Optic coef $dn/dT @ 30\text{K}: 5.8 \cdot 10^{-6} \text{ K}^{-1}$
 Heat Conductivity $K_{\text{si}} = 2330 \text{ W/m/K}$
 Thermal Expansion $\alpha_{\text{si}} = 4.85 \cdot 10^{-10} \text{ m/m/K}$

Thermal Noise Curve compared with the Sensitivity Curve B



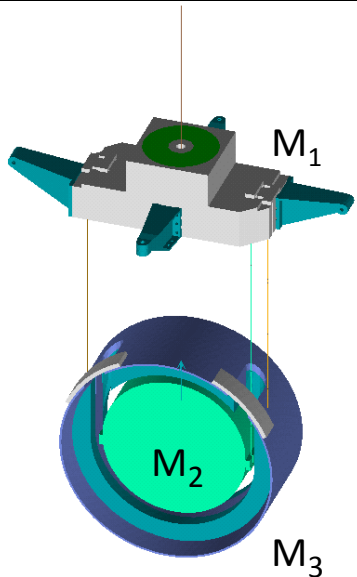
Thermal Noise Curve with violins

Thermal Noise for ET



Suspension thermal noise in the xylophone case (LF-Curve C).

d Fisica Nucleare



PAYLOAD

MARIONETTE: (Ti6Al4V WIRE)

$d = 3 \text{ mm}$, $M_1: 400 \text{ kg}$,

$L = 2 \text{ m}$ $T = 2 \text{ K}$

MIRROR (SILICON WIRE)

dimensions: diam 45cm, thickness 30cm

(limit of present technology)

$d = 3 \text{ mm}$, $M_2: 110 \text{ kg}$,

$L = 2 \text{ m}$ $T = 10 \text{ K}$ (only 18kW in cavity, 600 μm)

enough for heat extraction)

RECOIL MASS (SILICON WIRE)

$d = 3 \text{ mm}$, $M_3: 110 \text{ kg}$

$L = 2 \text{ m}$ $T = 10 \text{ K}$

Modes:	pendulum	0.28 Hz, 0.36 Hz, 0.50Hz
	vertical	0.4 Hz (blades), 20 Hz, 26 Hz
	violins	33 Hz, 67 Hz, 100 Hz, 200 Hz, ...

Thermal Noise Curve compared with the Sensitivity Curve C

Thermal Noise for ET

